Cambridge AS & A Level · Mathematics 9709

Mathematics 9709 formulas and definitions: Pure Mathematics 2

All 51 formulas, facts and definitions to learn, by syllabus topic. Formulas the exam gives you are left out.

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2 Pure Mathematics 2

2.1 Algebra

$|x| = x$ if $x \geqslant 0$, and $|x| = -x$ if $x < 0$; so $|x| \geqslant 0$ always

$|a| = |b| \iff a^2 = b^2 \iff a = b$ or $a = -b$

$|x - a| < b \iff a - b < x < a + b$

$|x - a| > b \iff x < a - b$ or $x > a + b$

$y = |ax + b|$: a V with vertex at $\left(-\frac{b}{a}, 0\right)$, y-intercept $|b|$, arm gradients $\pm a$

$p(x) = \text{divisor} \times \text{quotient} + \text{remainder}$, with the remainder of lower degree than the divisor

Remainder theorem: dividing $p(x)$ by $(x - a)$ leaves $p(a)$; by $(ax + b)$ leaves $p\left(-\frac{b}{a}\right)$

Factor theorem: $(x - a)$ is a factor $\iff p(a) = 0$; $(ax + b)$ is a factor $\iff p\left(-\frac{b}{a}\right) = 0$

$b^2 - 4ac < 0$: the quadratic has no real roots

2.2 Logarithmic and exponential functions

$\log_a b = c \iff a^c = b$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a(x^n) = n \log_a x$

$\log_a a = 1$, $\log_a 1 = 0$; in particular $\ln e = 1$, $\ln 1 = 0$

$\ln x = \log_e x$, with $e = 2.718\ldots$

$\ln(e^x) = x$ and $e^{\ln x} = x$ (for $x > 0$)

$e^x > 0$ for every x; $\ln x$ exists only for $x > 0$

$y = kx^n \Rightarrow \ln y = \ln k + n \ln x$

$y = k \times a^x \Rightarrow \ln y = \ln k + x \ln a$

Straight line: gradient $= \dfrac{Y_2 - Y_1}{X_2 - X_1}$, and $Y = mX + c$

2.3 Trigonometry

$\sec\theta = \dfrac{1}{\cos\theta}$, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\cos^2 A \equiv \frac{1}{2}(1 + \cos 2A)$ and $\sin^2 A \equiv \frac{1}{2}(1 - \cos 2A)$

$a\sin\theta \pm b\cos\theta$ or $a\cos\theta \pm b\sin\theta$ in R form: $R = \sqrt{a^2 + b^2}$, $\tan\alpha = \dfrac{b}{a}$

$R\sin(\theta \pm \alpha) + c$ and $R\cos(\theta \pm \alpha) + c$ lie between $c - R$ and $c + R$

$\sec$ and cosec: period 360° (2π), values never between -1 and 1; $\cot$: period 180° (π)

Exact values of $\sin$, $\cos$, $\tan$ at 30°, 45°, 60° (Paper 1 note)

2.4 Differentiation

Chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\dfrac{d}{dx}\, e^{f(x)} = f'(x)\, e^{f(x)}$, $\quad \dfrac{d}{dx} \ln f(x) = \dfrac{f'(x)}{f(x)}$

$\dfrac{d}{dx}\sin(ax + b) = a\cos(ax + b)$, and the same pattern for $\cos$ and $\tan$

Implicit: $\dfrac{d}{dx}\big(f(y)\big) = f'(y)\dfrac{dy}{dx}$, for example $\dfrac{d}{dx}\left(y^2\right) = 2y\dfrac{dy}{dx}$

$\dfrac{d}{dx}(xy) = y + x\dfrac{dy}{dx}$

Tangent: $y - y_1 = m(x - x_1)$; normal gradient $-\dfrac{1}{m}$

Stationary point: $\dfrac{dy}{dx} = 0$ (for a fraction, its numerator $= 0$)

Tangent parallel to the y-axis: the denominator of $\dfrac{dy}{dx}$ is 0 ($\dfrac{dx}{dt} = 0$ for parametric curves)

$\sec x = \dfrac{1}{\cos x}$

$e^x > 0$ for every x; $\ln e = 1$; $\ln 1 = 0$; $e^{\ln k} = k$

2.5 Integration

$\displaystyle\int e^{ax+b}\, dx = \frac{1}{a}e^{ax+b} + c$

$\displaystyle\int \frac{1}{ax + b}\, dx = \frac{1}{a}\ln|ax + b| + c$

$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b)+c,\ \int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b)+c,$
$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b)+c$

$\cos^2 A = \frac{1}{2}(1+\cos 2A),\ \sin^2 A = \frac{1}{2}(1-\cos 2A)$

$\tan^2 A = \sec^2 A - 1,\ \sin A\cos A = \frac{1}{2}\sin 2A$

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\left[y_0 + 2(y_1+\cdots+y_{n-1}) + y_n\right],\ h = \frac{b-a}{n}$

Curve bends upwards: over-estimate; bends downwards: under-estimate

Area $= \int_a^b (y_{\text{top}} - y_{\text{bottom}})\,dx$; volume about the x-axis $= \pi\int_a^b y^2\,dx$

$\ln a + \ln b = \ln ab,\ \ln a - \ln b = \ln \frac{a}{b},\ k\ln a = \ln a^k,\ e^{\ln k} = k$

2.6 Numerical solution of equations

If f is continuous and $f(a)$, $f(b)$ have opposite signs, then $f(x) = 0$ has a root between a and b

Iterative formula: $x_{n+1} = F(x_n)$, starting from x_1

If $x_n \to \alpha$, then $\alpha = F(\alpha)$: the limit is a root of $x = F(x)$

Checking an answer $\alpha \approx c$: a sign change of $x - F(x)$ between $c-h$ and $c+h$, where h is half a unit in the last place (for example $h = 0.005$ for 2 decimal places)

$\ln e^k = k$, $e^{\ln k} = k$, $\ln a + \ln b = \ln ab$, $\ln a - \ln b = \ln \frac{a}{b}$, $k\ln a = \ln a^k$

$\sec x = \frac{1}{\cos x}$; $|x-a| = x-a$ when $x \geqslant a$ and $a-x$ when $x < a$