Cambridge AS & A Level · Mathematics 9709

Mathematics 9709 formulas and definitions: Pure Mathematics 3

All 85 formulas, facts and definitions to learn, by syllabus topic. Formulas the exam gives you are left out.

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3 Pure Mathematics 3

3.1 Algebra

$(a+bx)^n = a^n\left(1+\frac{b}{a}x\right)^n$, valid for $|x| < \left|\frac{a}{b}\right|$

The expansion of $(1+\text{term})^n$ is valid when $|\text{term}| < 1$; for a sum or product of expansions, use the strictest condition

Partial fraction forms (the table in "Understand it"), with a constant in front when top and bottom have the same degree

$p(x) \equiv \text{divisor} \times \text{quotient} + \text{remainder}$, with the remainder of lower degree than the divisor

Remainder theorem: dividing $p(x)$ by $(ax+b)$ leaves $p\left(-\frac{b}{a}\right)$

Factor theorem: $(ax+b)$ is a factor $\iff p\left(-\frac{b}{a}\right) = 0$

$|a| = |b| \iff a^2 = b^2$; $|x-a| < b \iff a-b < x < a+b$

3.2 Logarithmic and exponential functions

$\log_a b = c \iff a^c = b$

$\log_a(pq) = \log_a p + \log_a q$

$\log_a\left(\frac{p}{q}\right) = \log_a p - \log_a q$

$\log_a(p^n) = n\log_a p$, so $\frac{1}{2}\ln p = \ln\sqrt{p}$ and $-\ln p = \ln\frac{1}{p}$

$\log_a a = 1, \log_a 1 = 0$; $\ln e = 1, \ln 1 = 0$; $n = \log_a a^n$

$\ln(e^x) = x$ and $e^{\ln x} = x$ (for $x > 0$)

$e^x > 0$ and $a^x > 0$ for every x; $\ln x$ exists only for $x > 0$

$e^{2x} = (e^x)^2$, $e^{-x} = \frac{1}{e^x}$, $a^{x+n} = a^n \times a^x$

$y = Ae^{kx}$: growth if $k > 0$, decay if $k < 0$; doubling (or halving) time $\dfrac{\ln 2}{|k|}$

$y = Ax^m \Rightarrow \ln y = \ln A + m \ln x$

$y = Ab^x \Rightarrow \ln y = \ln A + x \ln b$

Straight line: gradient $= \dfrac{Y_2 - Y_1}{X_2 - X_1}$, and $Y = mX + c$

3.3 Trigonometry

$\sec\theta = \dfrac{1}{\cos\theta}$, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\cos^2 A \equiv \frac{1}{2}(1 + \cos 2A)$ and $\sin^2 A \equiv \frac{1}{2}(1 - \cos 2A)$

$(\cos\theta \pm \sin\theta)^2 \equiv 1 \pm \sin 2\theta$

R form: $R = \sqrt{a^2 + b^2}$, $\tan\alpha = \dfrac{b}{a}$ (from matching $R\cos\alpha$ and $R\sin\alpha$)

$R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$ lie between $-R$ and R

$\sec$ and cosec: period 2π, never between -1 and 1; $\cot$: period π

Exact values of $\sin$, $\cos$, $\tan$ at $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ (30°, 45°, 60°)

3.4 Differentiation

Chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\dfrac{d}{dx}\, e^{f(x)} = f'(x)\, e^{f(x)}$, $\dfrac{d}{dx} \ln f(x) = \dfrac{f'(x)}{f(x)}$

$\dfrac{d}{dx} \tan^{-1} f(x) = \dfrac{f'(x)}{1 + \big(f(x)\big)^2}$, for example $\dfrac{d}{dx} \tan^{-1}(ax) = \dfrac{a}{1 + a^2x^2}$

Implicit: $\dfrac{d}{dx}\big(g(y)\big) = g'(y)\dfrac{dy}{dx}$, $\dfrac{d}{dx}(xy) = y + x\dfrac{dy}{dx}$

Tangent: $y - y_1 = m(x - x_1)$; normal gradient $-\dfrac{1}{m}$

$\dfrac{dy}{dx} = \dfrac{N}{D}$: tangent parallel to the x-axis where $N = 0$, parallel to the y-axis where $D = 0$

$\sec x = \dfrac{1}{\cos x}$, $\operatorname{cosec} x = \dfrac{1}{\sin x}$, $\cot x = \dfrac{\cos x}{\sin x}$

$-1 \leqslant \sin x \leqslant 1$, $-1 \leqslant \cos x \leqslant 1$, $e^x > 0$

3.5 Integration

$\displaystyle\int e^{ax+b}\, dx = \frac{1}{a}e^{ax+b}$, $\displaystyle\int \frac{1}{ax+b}\, dx = \frac{1}{a}\ln|ax+b|$

$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b),\ \int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b),$
$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b)$

$\int \frac{1}{(ax+b)^2}\,dx = -\frac{1}{a(ax+b)}$

$\int \tan x\,dx = \ln|\sec x| = -\ln|\cos x|$

$\int f'(x)\big(f(x)\big)^n\,dx = \frac{\big(f(x)\big)^{n+1}}{n+1}\ (n \neq -1)$

$\int_a^b u\frac{dv}{dx}\,dx = \Big[uv\Big]_a^b - \int_a^b v\frac{du}{dx}\,dx$

Substitution: replace dx using $\frac{du}{dx}$, replace every x, change the limits

$\cos^2 A = \frac{1}{2}(1+\cos 2A),\ \sin^2 A = \frac{1}{2}(1-\cos 2A),\ \tan^2 A = \sec^2 A - 1$

Partial fraction forms for $(ax+b)(cx+d)(ex+f)$, $(ax+b)(cx+d)^2$ and $(ax+b)(cx^2+d)$

$\ln a + \ln b = \ln ab,\ \ln a - \ln b = \ln \frac{a}{b},\ k\ln a = \ln a^k$

3.6 Numerical solution of equations

If f is continuous and $f(a)$, $f(b)$ have opposite signs, then $f(x) = 0$ has a root between a and b

Iterative formula: $x_{n+1} = F(x_n)$, starting from x_1

If $x_n \to \alpha$, then $\alpha = F(\alpha)$: the limit is a root of $x = F(x)$

Checking an answer c: a sign change of $x - F(x)$ between $c-h$ and $c+h$, where h is half a unit in the last place ($h = 0.005$ for 2 decimal places, 0.0005 for 3)

$\sec x = \frac{1}{\cos x}$, $\operatorname{cosec} x = \frac{1}{\sin x}$, $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

$\ln e^k = k$, $e^{\ln k} = k$, $\ln a + \ln b = \ln ab$, $k\ln a = \ln a^k$

Triangle area $\frac{1}{2}ab\sin C$, so segment area $= \frac{1}{2}r^2(\theta - \sin\theta)$

3.7 Vectors

$|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2+y^2+z^2}$

Unit vector in the direction of $\mathbf{v}$: $\frac{\mathbf{v}}{|\mathbf{v}|}$

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ ("end minus start")

Midpoint of AB: $\frac{1}{2}(\mathbf{a}+\mathbf{b})$

Line: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$; through A and B: $\mathbf{r} = \mathbf{a} + t(\mathbf{b}-\mathbf{a})$

Parallel: one direction vector is a multiple of the other

Perpendicular: $\mathbf{a} \cdot \mathbf{b} = 0$

Acute angle between lines: $\cos\theta = \dfrac{|\mathbf{b}_1 \cdot \mathbf{b}_2|}{|\mathbf{b}_1|\,|\mathbf{b}_2|}$

Foot of perpendicular from A: $\overrightarrow{AP} \cdot \mathbf{b} = 0$ for the general point P

Reflection of A in a line with foot F: $\overrightarrow{OA'} = 2\overrightarrow{OF} - \overrightarrow{OA}$

Area of triangle ABC: $\frac{1}{2}\,AB \times AC \times \sin A$; parallelogram: twice that

$\sin\theta = \sqrt{1 - \cos^2\theta}$ for $0^\circ < \theta < 180^\circ$

3.8 Differential equations

Separating: $\dfrac{dy}{dx} = f(x)g(y) \Rightarrow \displaystyle\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$

One constant of integration, found from the initial condition

$\ln|y| = f(x) + c \Rightarrow y = Ae^{f(x)}$

"Rate of change of x" $= \dfrac{dx}{dt}$; "proportional to" $= k\times$; "decreasing" means a negative rate

Connected rates: $\dfrac{dV}{dt} = \dfrac{dV}{dh} \times \dfrac{dh}{dt}$

$\displaystyle\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|$, $\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b}$

$\int \tan x\,dx = -\ln|\cos x| = \ln|\sec x|$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ (rearranged from the given $\cos 2A$)

Vertical-sided tank with base area A: $V = Ah$; cylinder $V = \pi r^2 h$

3.9 Complex numbers

$\mathrm{i}^2 = -1$; $z = x + \mathrm{i}y$, $\operatorname{Re} z = x$, $\operatorname{Im} z = y$

$a + \mathrm{i}b = c + \mathrm{i}d \iff a = c$ and $b = d$

$z^* = x - \mathrm{i}y$; $zz^* = x^2 + y^2 = |z|^2$

Divide: multiply top and bottom by the conjugate of the bottom

$(z+w)^* = z^* + w^*$, $(zw)^* = z^*w^*$

$|z| = \sqrt{x^2+y^2}$; $\tan(\arg z) = \dfrac{y}{x}$, quadrant from a sketch

$z = r(\cos\theta + \mathrm{i}\sin\theta) = r\mathrm{e}^{\mathrm{i}\theta}$, $x = r\cos\theta$, $y = r\sin\theta$

$|z_1 z_2| = |z_1||z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2\ (\pm 2\pi)$

$\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$, $\arg\dfrac{z_1}{z_2} = \arg z_1 - \arg z_2\ (\pm 2\pi)$

Real coefficients: roots $a \pm b\mathrm{i}$ in pairs, factor $z^2 - 2az + a^2 + b^2$

$\lvert z_1 - z_2 \rvert$ = distance between the points z_1 and z_2
$\lvert z - a \rvert = k$: circle; $\lvert z - a \rvert = \lvert z - b \rvert$: perpendicular bisector; $\arg(z - a) = \alpha$: half-line from a
Circle centre c, radius r, $d = \lvert c \rvert$: $\lvert z \rvert$ from $d - r$ to $d + r$; extreme $\arg z = \arg c \pm \sin^{-1} \frac{r}{d}$