Cambridge AS & A Level · Mathematics (9709) · Notes

Kinematics of motion in a straight line

AS

Read online at pastopic.com/cambridge/mathematics-9709/notes/kinematics-of-motion-in-a-straight-line

Last checked 28 September 2026

Read first: Differentiation, Integration

What you need to know

By the end of this topic you should be able to:

1. **Tell scalars from vectors.** Distance and speed are scalars: they have a size only. Displacement, velocity and acceleration are vectors: they have a size and a direction. Everything happens along one straight line, so a direction is just a sign, $+$ or $-$. "Deceleration" means the speed is going down.
2. **Sketch and read displacement–time and velocity–time graphs**, knowing that
   - the gradient of a displacement–time graph is the velocity,
   - the gradient of a velocity–time graph is the acceleration,
   - the area under a velocity–time graph is the displacement.
3. **Use calculus with respect to time.** Differentiate displacement to get velocity and velocity to get acceleration; integrate to go back. Only the Paper 1 techniques are needed: powers of t (including fractional and negative powers) and brackets such as $(2t+1)^{3/2}$.
4. **Use the constant acceleration formulae** ("suvat") for motion in a straight line, including questions that need more than one equation, for one particle in several stages or for two particles at once.

Understand it

Distance, displacement, speed and velocity

Pick a fixed point O on the line and a positive direction. The **displacement** s is where the object is, measured from O: $s=-5$ m means 5 m from O on the negative side. The **distance** travelled is how much ground it has covered, whatever the direction, so it is never negative and it never goes down.

A ball that rolls 6 m forwards and then 2 m back has a displacement of $+4$ m but has travelled a distance of 8 m.

Velocity is the rate of change of displacement, with a sign that shows the direction of motion. **Speed** is the size of the velocity, with no sign: a velocity of -3 m s^{-1} is a speed of 3 m s^{-1}, moving in the negative direction. **Acceleration** is the rate of change of velocity. When acceleration and velocity have opposite signs the object is slowing down, which is called **deceleration**: "decelerates at 2 m s^{-2}" means $a = -2$ when the motion is in the positive direction.

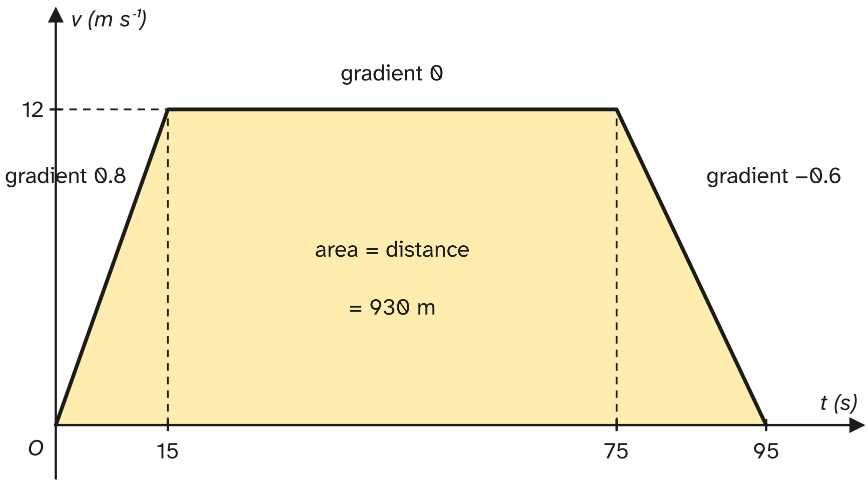
Instantaneous rest means $v = 0$ for an instant. This is where an object can turn round and change direction.

Velocity–time graphs

On a velocity–time graph:

- the **gradient** is the acceleration (change in velocity ÷ time taken);
- the **area** between the graph and the t-axis is the displacement. Area above the axis is motion in the positive direction; area below the axis is motion in the negative direction.

A tram starts from rest and speeds up steadily to 12 m s^{-1} in 15 s. It keeps this speed for 60 s, then slows steadily to rest in 20 s.



The acceleration in the first stage is $\frac{12}{15} = 0.8$ m s^{-2}, and in the last stage $\frac{0-12}{20} = -0.6$ m s^{-2}, a deceleration of 0.6 m s^{-2}. The shape is a trapezium, so the distance is

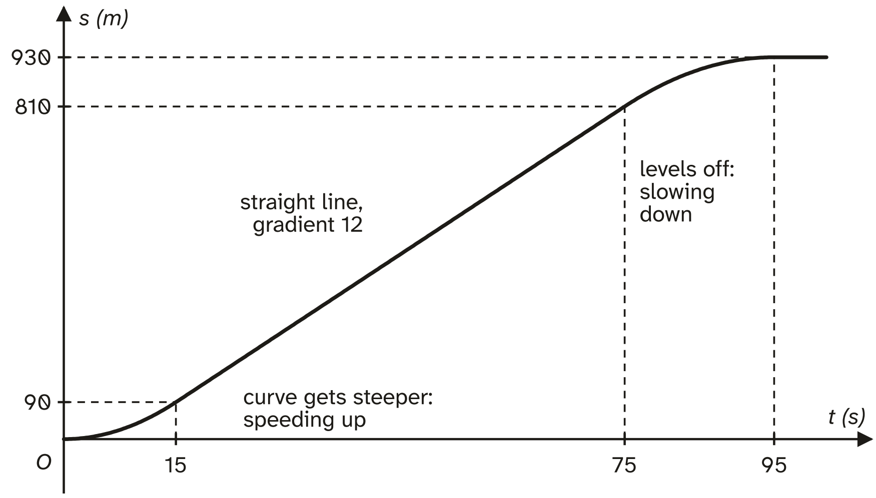
$$\tfrac{1}{2}(95 + 60) \times 12 = 930 \text{ m},$$

the same as the two triangles and the rectangle added up: $90 + 720 + 120 = 930$ m.

A graph made of straight lines means the acceleration is constant in each stage. If the velocity is a curve (a quadratic in t, say) the acceleration keeps changing, and you must not treat that part as a straight line.

Displacement–time graphs

On a displacement–time graph the **gradient** is the velocity. A straight line means constant velocity; a horizontal line means the object is at rest; a line sloping down means it is moving back towards O (negative velocity). A curve means the velocity is changing: getting steeper means speeding up.



When the tram accelerates, its displacement–time graph curves upwards; at constant speed it is straight; while it slows down it bends over until it is flat, because at rest the gradient is 0. There are no sudden corners where the velocity changes smoothly.

Constant acceleration: the suvat formulae

When the acceleration is **constant**, five quantities describe one stage of the motion: s (displacement), u (velocity at the start), v (velocity at the end), a (acceleration) and t (time). Each formula leaves out one of them:

Formula	Leaves out
$v = u + at$	s
$s = \frac{1}{2}(u + v)t$	a
$s = ut + \frac{1}{2}at^2$	v
$v^2 = u^2 + 2as$	t
$s = vt - \frac{1}{2}at^2$	u

Write down the three you know and the one you want, and choose the formula without the fifth. They all come from the velocity–time graph: $v = u + at$ says the gradient is a, and $s = \frac{1}{2}(u + v)t$ is the area of the trapezium under the line.

These formulae only work when a is constant. If a, v or s is given as a function of t such as $a = 2t - 8$, use calculus instead.

Motion under gravity

An object moving freely up or down (no air resistance) has a constant acceleration of g downwards. Cambridge uses $g = 10$ m s^{-2}. Take **upwards as positive**, so $a = -10$ throughout, going up and coming down.

- At the **greatest height**, $v = 0$.

- s is the displacement from the point of projection, so it is **negative** below that point.
- The motion is symmetric: the time up to the top equals the time back down to the same level, and the speed when it passes a level is the same going up and coming down.

For a ball thrown up at 20 m s-1: $0 = 20 - 10t$ gives $t = 2$ s at the top, $0 = 400 - 20s$ gives a greatest height of 20 m, and it is back at the start after 4 s.

Variable acceleration: calculus

Velocity is the rate of change of displacement, and acceleration the rate of change of velocity:

$$v = \frac{ds}{dt}, \qquad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

So **differentiate** to go from s to v to a, and **integrate** to go back:

$$v = \int a\,dt, \qquad s = \int v\,dt.$$

Each integration gives a constant, found from a starting condition ("starts from rest" means $v = 0$ when $t = 0$; "leaves O" means $s = 0$ when $t = 0$). Don't assume the constant is 0.

Differentiating and integrating use the same rules as in Paper 1, with t in place of x: $\frac{d}{dt}(t^n) = nt^{n-1}$ and $\int t^n\,dt = \frac{t^{n+1}}{n+1} + c$. For example $v = 6t - t^{3/2}$ gives $a = 6 - \frac{3}{2}t^{1/2}$ and $s = 3t^2 - \frac{2}{5}t^{5/2}$ (if $s = 0$ at $t = 0$).

A bracket raised to a power, $(at+b)^n$, works like t^n with one extra step for the a inside the bracket:

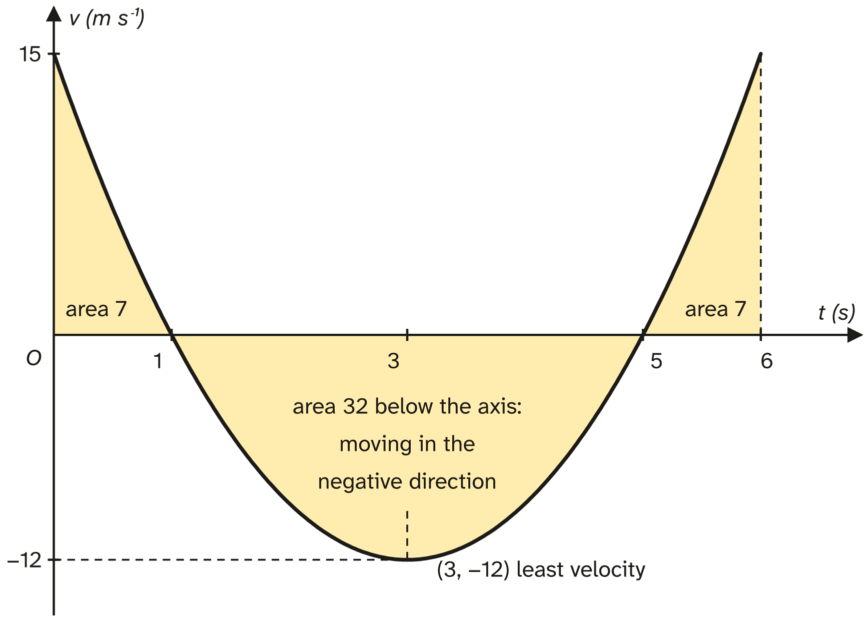
- **differentiate:** bring the power down, lower it by 1, then **multiply** by a: $\dfrac{d}{dt}(4t+1)^{3/2} = \frac{3}{2}(4t+1)^{1/2} \times 4 = 6(4t+1)^{1/2}$;
- **integrate:** raise the power by 1, divide by the new power, then **divide** by a: $\displaystyle\int (4t+1)^{1/2}\,dt = \frac{(4t+1)^{3/2}}{\frac{3}{2}\times 4} + c = \frac{1}{6}(4t+1)^{3/2} + c.$

Differentiating the answer gets you back to $(4t+1)^{1/2}$, which is a quick check. So a particle with $v = (4t+1)^{1/2}$ that leaves O at $t = 0$ has $s = \frac{1}{6}(4t+1)^{3/2} - \frac{1}{6}$ (the constant makes $s = 0$ at $t = 0$), and at $t = 2$ it is $\frac{1}{6}(27-1) = \frac{13}{3}$ m from O.

The **greatest or least velocity** is where the velocity is stationary, that is where $a = 0$, just like a stationary point on a curve.

Distance is not always displacement

Take a particle leaving O with $v = 3t^2 - 18t + 15 = 3(t-1)(t-5)$. Integrating, $s = t^3 - 9t^2 + 15t$.



The particle moves in the positive direction until $t = 1$, in the negative direction from $t = 1$ to $t = 5$ (passing through O on the way and ending 25 m on the negative side), then in the positive direction again. From s: $s(1) = 7$, $s(5) = -25$ and $s(6) = -18$. So

- the **displacement** at $t = 6$ is -18 m (it ends 18 m on the negative side of O);
- the **distance** travelled in the first 6 s is $7 + 32 + 7 = 46$ m, since it goes 7 m in the positive direction, 32 m in the negative direction and 7 m in the positive direction again.

To find a distance, **split the time at every instant where** $v = 0$, find the displacement for each piece and add their sizes. Integrating straight from 0 to 6 gives only the displacement, -18.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it.

Formula	Status
$v = u + at$	Given
$s = \frac{1}{2}(u + v)t$	Given
$s = ut + \frac{1}{2}at^2$	Given
$v^2 = u^2 + 2as$	Given
$s = vt - \frac{1}{2}at^2$	Learn it
The suvat formulae need **constant** acceleration	Learn it
$v = \dfrac{ds}{dt}, \quad a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$	Learn it
$v = \displaystyle\int a\,dt, \quad s = \int v\,dt$ (find the constant each time)	Learn it

Formula	Status
$\frac{d}{dt}(t^n) = nt^{n-1}$; $\int t^n \, dt = \frac{t^{n+1}}{n+1} + c \; (n \neq -1)$	Given
Gradient of a displacement–time graph = velocity	Learn it
Gradient of a velocity–time graph = acceleration; area under it = displacement	Learn it
Greatest or least velocity: $a = 0$; instantaneous rest: $v = 0$	Learn it
Average speed $= \frac{\text{total distance}}{\text{total time}}$	Learn it
Free motion under gravity: $a = -g = -10$ m s^{-2} (upwards positive)	Learn it

How to do it

In the Paper 4 questions we have tagged for this topic (78 questions, 2021–2025), the types came up this often. 12 of the 78 questions mix two types (for example a calculus question that asks for a sketch of the velocity–time graph), so the counts add up to more than 78.

Question type	Questions
Variable acceleration: calculus with s, v or a given in terms of t	37
Journeys in stages and velocity–time or displacement–time graphs	27
Vertical motion under gravity	15
Constant acceleration formulae in a straight line	11

1. Variable acceleration: calculus (37 questions)

1. **Spot it.** a, v or s is a formula in t (not a constant), so use calculus, never suvat.
2. **Move between s, v and a.** Differentiate to go down ($s \to v \to a$) and integrate to go up ($a \to v \to s$). Rewrite roots as powers first: $\sqrt{t} = t^{1/2}$, $\frac{4}{t^2} = 4t^{-2}$.
3. **Find each constant** from a condition on that quantity: "starts from rest" gives $v = 0$ at $t = 0$; "passes O with velocity 5 m s^{-1}" gives $v = 5$ and $s = 0$ at $t = 0$.
4. **Answer the question asked** using the table:

The question says	Do this
"at instantaneous rest", "changes direction"	Solve $v = 0$
"greatest / least / maximum / minimum velocity"	Solve $a = 0$, then find v there
"returns to O", "passes O again"	Solve $s = 0$ (with s measured from O)
"displacement from O at time T"	Find $s(T)$, or $\int_0^T v \, dt$
"total distance travelled"	Split at each $v = 0$ in the interval, add the sizes

For example, with $v = 3t^2 - 18t + 15$ from "Understand it": it is at rest when $3(t-1)(t-5) = 0$, at $t = 1$ and $t = 5$; its least velocity is where $a = 6t - 18 = 0$, at $t = 3$, giving $v = -12$ m s^{-1}, a speed of 12 m s^{-1}.

Variations you will meet:

- **Brackets such as $(5t+4)^{-1/2}$.** Differentiate by multiplying by the 5 inside; integrate by dividing by it. For $v = 2(5t+4)^{-1/2}$: $a = 2 \times (-\frac{1}{2})(5t+4)^{-3/2} \times 5 = -5(5t+4)^{-3/2}$, and $s = \frac{2(5t+4)^{1/2}}{\frac{1}{2} \times 5} + c = \frac{4}{5}(5t+4)^{1/2} + c$. If $s = 0$ at $t = 0$, then $c = -\frac{4}{5} \times 2 = -\frac{8}{5}$. Don't expand the bracket or treat it as t^n.
- **Fractional powers in the equation.** Treat $\sqrt{t}$ as the unknown. $4\sqrt{t} - t = 0$ gives $\sqrt{t}\,(4 - \sqrt{t}) = 0$, so $t = 0$ or $t = 16$. $2t - 7\sqrt{t} + 3 = 0$ is a quadratic in $\sqrt{t}$: $(2\sqrt{t} - 1)(\sqrt{t} - 3) = 0$, so $\sqrt{t} = \frac{1}{2}$ or 3, giving $t = \frac{1}{4}$ or $t = 9$.
- **Unknown constants** such as k in $v = kt^2 + 2t$. Each piece of information ("the acceleration at $t = 2$ is 7 ", "it travels 12 m in the first 3 s", "its greatest velocity is ...") gives an equation. Differentiate or integrate first, then substitute. Two unknowns need two equations, solved together.
- **Motion in stages with different formulae**, such as $v = 2t^2$ for $0 \leqslant t \leqslant 3$ and $v = 30 - 4t$ for $t \geqslant 3$. "No instantaneous change in velocity" means both formulae give the same v at the switch time: use it to find a constant. Work out the distance for each stage separately with its own formula, then add. If the formulae give different accelerations at the switch, the acceleration changes suddenly there.
- **Show there is no return to O / no instant of rest.** Show the equation $s = 0$ (or $v = 0$) has no roots in the interval, for example a quadratic with a negative discriminant, and say what that means.
- **"Show that the particle never comes to rest"** with an unknown: the quadratic for v must have a negative discriminant.
- **Two particles** each with their own formula: they meet when their displacements from the same point are equal (or add up to the gap between their starting points if they move towards each other).
- **Sketch the velocity–time graph** from the formula: use its shape (a quadratic is a U or an upside-down U), where it crosses the t-axis ($v = 0$), its value at $t = 0$ and its turning point ($a = 0$). Label these values.
- **Speed and distance numerically equal**: set the expression for v equal to the expression for s and solve.

2. Journeys in stages and graphs (27 questions)

1. **Sketch the velocity–time graph** even if the question doesn't ask for one. Mark each time and speed on it, using letters for unknowns (V, T).
2. **Use the gradients** for accelerations: stage time $= \frac{\text{change in speed}}{\text{acceleration}}$, so a car that decelerates at 0.5 m s^{-2} from V m s^{-1} to rest takes $\frac{V}{0.5} = 2V$ seconds.
3. **Use the areas** for distances: triangles, rectangles and trapezia. Write "total area $=$ total distance" as an equation.
4. **Solve** for the unknown and check it makes sense (times positive, speeds within reason, one stage not longer than the whole journey).

For example, a runner speeds up from rest to 6 m s^{-1} in 4 s, runs at 6 m s^{-1} for T s and slows to rest in 3 s, covering 180 m. Area: $\frac{1}{2}(4)(6) + 6T + \frac{1}{2}(3)(6) = 180$, so $21 + 6T = 180$ and $T = 26.5$.

Variations you will meet:

- **Unknown speed and time together.** Write the times of the speeding-up and slowing-down stages in terms of V, then the constant-speed time as "total time minus the others". Substituting into the area equation gives a quadratic in V. Reject the root that gives a negative time.
- **Motion back towards the start.** Parts of the graph below the t-axis are motion in the negative direction. Returning to the start means area above $=$ area below; total distance means area above $+$ area below.
- **Displacement–time graphs.** Straight parts: velocity $=$ gradient $= \frac{\text{change in } s}{\text{change in } t}$ (negative when s is falling). Curved parts starting from rest with constant acceleration: use $s = \frac{1}{2}at^2$. You may be asked to turn one kind of graph into the other: straight lines on an s–t graph become horizontal lines on a v–t graph.
- **Sketching.** Straight-line stages give straight lines; constant speed is horizontal; a stage where a depends on t is a curve. Start and finish in the right places (from rest means from $(0, 0)$) and label the key times and speeds. For a displacement–time sketch, accelerating stages curve upwards, constant speed is straight and the curve flattens as the object comes to rest, with no corners between stages.
- **Average speed** $= \frac{\text{total distance}}{\text{total time}}$, including any time at rest.
- **Two people or vehicles** compared: find each distance (or write it in terms of the unknown) and set up the equation the question describes.
- **Elevators** moving up and down: upward and downward stages are areas above and below the axis. "Starts and finishes on the ground floor" means the two areas are equal.

3. Vertical motion under gravity (15 questions)

1. **Choose upwards as positive**, so $a = -10$. Measure s from the point of projection.
2. **List** s, u, v, a, t for the part of the motion you want and pick the suvat formula without the unknown you don't need.
3. **Use the key facts:** $v = 0$ at the greatest height; $s = 0$ when it is back at the start; $s = -h$ when it lands h m below the start.
4. **Quadratics in** t give two times, for example going up past a height and coming down past it. Keep both if the question needs them.

For example, a stone thrown up at 20 m s^{-1} is 15 m above its starting point when $15 = 20t - 5t^2$, that is $t^2 - 4t + 3 = 0$, so $t = 1$ (on the way up) and $t = 3$ (on the way down).

Variations you will meet:

- **Time spent above a height** h**:** find the two times it is at height h and subtract, or use symmetry: the time from height h up to the top equals the time back down, so double one of them.
- **Time for which the speed is at least some value:** by symmetry, find the time it takes to slow from the launch speed to that speed and use the same time on the way down.
- **Two particles in the same vertical line** released at the same instant: write each one's height above the ground in terms of t and set them equal. For one projected up at 18 m s^{-1} from the ground and one dropped from 27 m: $18t - 5t^2 = 27 - 5t^2$, so $t = 1.5$ and they meet $27 - 5(1.5)^2 = 15.75$ m above the ground. If one starts later, its time is $t - 1$ (or whatever the delay is).
- **Given a speed at a time** "and it is moving downwards": that velocity is negative.

- **After a collision or a bounce** (other topics give the new speed), start a new stage with the new u and the same $a = -10$.

4. Constant acceleration formulae in a straight line (11 questions)

1. **Draw a line** with the start, the end and the positive direction marked, and list s, u, v, a, t for each stage.
2. **The end of one stage is the start of the next**: its final velocity is the next stage's u.
3. **Choose the formula** that leaves out the quantity you don't know and don't need. With two unknowns, write two equations and solve them together.

For example, $u = 4$ m s^{-1}, $a = 1.5$ m s^{-2}, $t = 6$ s: $v = 4 + 1.5(6) = 13$ m s^{-1} and $s = 4(6) + \frac{1}{2}(1.5)(36) = 51$ m.

Variations you will meet:

- **Distance in the nth second:** displacement at time n minus displacement at time $n - 1$. With $u = 3$ and $a = 2$, the 5th second covers $(15 + 25) - (12 + 16) = 12$ m.
- **Answers in terms of a letter:** if a is unknown, carry it through, for example $s = 8u + 32a$ for $t = 8$, and use a second piece of information to link u and a.
- **Two particles on the same line:** write each displacement from one fixed point in terms of the same time t (if one starts 4 s later, its time is $t - 4$). They are side by side when the displacements are equal.
- **Distance or time to stop:** put $v = 0$ with a negative a.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A delivery van passes a point P with speed 5 m s^{-1} and accelerates uniformly for 10 s until its speed is 13 m s^{-1}. It then travels at 13 m s^{-1} for T s, before decelerating uniformly at 0.65 m s^{-2} and coming to rest at Q. The distance PQ is 1000 m.

(a) Sketch a velocity–time graph for the van's motion from P to Q. **[2]**

(b) Find the acceleration in the first 10 s and the time for which the van is decelerating. **[2]**

(c) Find T. **[3]**

(d) Find the van's average speed for the journey from P to Q. **[2]**

(a) Three straight lines: from $(0, 5)$ up to $(10, 13)$, horizontal at 13 until $t = 10 + T$, then down to the t-axis. **B1** for the shape starting at 5 on the v-axis, **B1** for 5, 13 and 10 labelled.

(b) Acceleration $= \dfrac{13 - 5}{10} = 0.8$ m s^{-2}. **B1**

Deceleration time $= \dfrac{13}{0.65} = 20$ s. **B1**

(c) Area under the graph $=$ distance:

$$\tfrac{1}{2}(5 + 13)(10) + 13T + \tfrac{1}{2}(20)(13) = 1000.$$

M1 for adding the three areas and setting them equal to 1000, **A1** for the correct equation.

$90 + 13T + 130 = 1000$, so $13T = 780$ and $T = 60$. **A1**

(d) Total time $= 10 + 60 + 20 = 90$ s. **M1**

Average speed $= \dfrac{1000}{90} = 11.1$ m s^{-1} (3 s.f.). **A1**

Example 2

A stone is thrown vertically upwards with speed 12 m s^{-1} from the top of a cliff 32 m above the sea. It then falls past the cliff top into the sea.

(a) Find the greatest height of the stone above the sea. **[2]**

(b) Find the length of time for which the stone is at least 35 m above the sea. **[3]**

(c) Find the time the stone takes to reach the sea and its speed as it reaches the sea. **[3]**

Take upwards as positive, $a = -10$, and measure s from the cliff top.

(a) At the top, $v = 0$: $0 = 12^2 - 2(10)s$, so $s = 7.2$ m. **M1** for $v^2 = u^2 + 2as$ with $v = 0$.

Greatest height $= 32 + 7.2 = 39.2$ m. **A1**

(b) 35 m above the sea is $s = 3$:

$$3 = 12t - 5t^2 \;\Rightarrow\; 5t^2 - 12t + 3 = 0.$$

M1 for $s = ut + \frac{1}{2}at^2$ with $s = 3$.

$t = \dfrac{12 \pm \sqrt{84}}{10}$, so $t = 0.283$ or $t = 2.117$. **A1**

Time above 35 m $= \dfrac{\sqrt{84}}{5} = 1.83$ s (3 s.f.). **A1**

(Or by symmetry: the stone is above 35 m while within $39.2 - 35 = 4.2$ m of the top. Falling 4.2 m from rest takes t with $4.2 = 5t^2$, $t = 0.9165$ s, and twice this is 1.83 s.)

(c) The sea is $s = -32$:

$$-32 = 12t - 5t^2 \;\Rightarrow\; 5t^2 - 12t - 32 = 0 \;\Rightarrow\; (5t+8)(t-4) = 0.$$

M1 for using $s = -32$ with the correct signs. $t = 4$ s (reject the negative root). **A1**

$v = 12 - 10(4) = -28$, so the speed is 28 m s^{-1}. **A1** (Check: $v^2 = 144 + 2(-10)(-32) = 784$.)

Example 3

A particle P moves in a straight line. It passes through a point O with velocity 12 m s^{-1}, and t s later its acceleration is a m s^{-2}, where $a = 2t - 8$.

(a) Find an expression for the velocity of P in terms of t, and the values of t at which P is instantaneously at rest. **[3]**

(b) Find the least velocity of P. **[2]**

(c) Show that P is back at O when $t = 6$. **[3]**

(d) Find the total distance travelled by P in the first 7 seconds. **[3]**

(a) $v = \displaystyle\int (2t-8)\,dt = t^2 - 8t + c$. **M1** for integrating.

$v = 12$ when $t = 0$, so $c = 12$ and $v = t^2 - 8t + 12$. **A1**

$v = 0$: $(t-2)(t-6) = 0$, so $t = 2$ and $t = 6$. **A1**

(b) Least velocity where $a = 0$: $2t - 8 = 0$, $t = 4$. **M1**

$v = 16 - 32 + 12 = -4$ m s^{-1}. **A1**

(c) $s = \displaystyle\int (t^2 - 8t + 12)\,dt = \tfrac{1}{3}t^3 - 4t^2 + 12t + k$, and $s = 0$ at $t = 0$ gives $k = 0$. **M1 A1**

$s(6) = 72 - 144 + 72 = 0$, so P is at O. **A1**

(d) P changes direction at $t = 2$ and $t = 6$, so split there. **M1**

$s(2) = \tfrac{8}{3} - 16 + 24 = \tfrac{32}{3},\;\; s(6) = 0,\;\; s(7) = \tfrac{343}{3} - 196 + 84 = \tfrac{7}{3}.$

Distance $= \tfrac{32}{3} + \left|0 - \tfrac{32}{3}\right| + \left(\tfrac{7}{3} - 0\right) = \tfrac{71}{3}$. **M1** for adding the sizes of the three pieces.

Total distance $= 23.7$ m (3 s.f.). **A1**

Integrating straight from 0 to 7 gives $\tfrac{7}{3}$, the displacement, not the distance.

Example 4

A particle starts from rest at a point O and moves in a straight line. At time t s after leaving O:

- for $0 \leqslant t \leqslant 4$, its acceleration is $(6 - 1.5t)$ m s^{-2};
- for $t \geqslant 4$, its velocity is $(36 - k\sqrt{t})$ m s^{-1}, where k is a constant.

(a) Find the velocity of the particle when $t = 4$. **[3]**

(b) Given that there is no instantaneous change in velocity at $t = 4$, find k and the time at which the particle comes to rest. **[3]**

(c) Show that the acceleration changes suddenly at $t = 4$. **[2]**

(d) Find the distance travelled from O until the particle comes to rest. **[4]**

(a) $v = \displaystyle\int (6 - 1.5t)\,dt = 6t - 0.75t^2 + c$, and $v = 0$ at $t = 0$ gives $c = 0$. **M1 A1**

At $t = 4$: $v = 24 - 12 = 12$ m s^{-1}. **A1**

(b) $36 - k\sqrt{4} = 12$, so $2k = 24$ and $k = 12$. **B1**

At rest: $36 - 12\sqrt{t} = 0$, so $\sqrt{t} = 3$ and $t = 9$. **M1 A1**

(c) Just before $t = 4$: $a = 6 - 1.5(4) = 0$. **B1**

For $t \geqslant 4$: $v = 36 - 12t^{1/2}$, so $a = -6t^{-1/2}$, which is -3 at $t = 4$. The acceleration jumps from 0 to -3 m s^{-2}. **B1**

(d) The velocity is positive for $0 < t < 9$, so distance $=$ displacement for each stage.

First stage: $\displaystyle\int_0^4 (6t - 0.75t^2)\,dt = \left[3t^2 - 0.25t^3\right]_0^4 = 48 - 16 = 32$. **M1 A1**

Second stage: $\displaystyle\int_4^9 (36 - 12t^{1/2})\,dt = \left[36t - 8t^{3/2}\right]_4^9 = (324 - 216) - (144 - 64) = 28$. **M1** for integrating and using limits 4 and 9.

Total distance $= 32 + 28 = 60$ m. **A1**

Common mistakes

- **Using suvat when the acceleration is not constant.** If a or v is a formula in t, the constant acceleration formulae give wrong answers and earn nothing. Examiners stress this every year: integrate or differentiate instead.
- **Multiplying instead of integrating.** "Distance $=$ velocity $\times$ time" and "velocity $=$ acceleration $\times$ time" only work for constant values. With v in terms of t, $s = vt$ scores no marks.
- **Forgetting or guessing the constant of integration.** Setting every constant to 0 is a common error, especially in the second or third stage of a journey. Find each one from a condition, such as the velocity being the same at the switch time.

- **Giving the displacement when the distance is asked for.** If the particle turns round in the interval, integrating across the whole interval lets the backwards part cancel the forwards part. Find where $v = 0$ first and split there. A negative "distance" is always wrong.
- **Using the wrong limits**, for example starting from $t = 0$ when the question asks for the distance from when the acceleration is zero, or from the switch time $t = 4$ when the stage of interest starts later.
- **Differentiating when you should integrate** (or the other way round). Going from a to v or from v to s is integration.
- **Forgetting the number inside a bracket.** $\int (2t+3)^4\,dt = \frac{(2t+3)^5}{5\times 2} = \frac{1}{10}(2t+3)^5 + c$, not $\frac{1}{5}(2t+3)^5$; and $\frac{d}{dt}(2t+3)^5 = 10(2t+3)^4$, not $5(2t+3)^4$. Divide by the 2 when integrating and multiply by it when differentiating. Check by differentiating your integral.
- **Mistakes with fractional powers.** $\int t^{1/2}\,dt = \frac{2}{3}t^{3/2}$, not $\frac{3}{2}t^{3/2}$; and when solving, square correctly: $3\sqrt{t} = 2t$ gives $9t = 4t^2$.
- **Drawing straight lines for curved stages.** When a changes with t, the velocity–time graph is a curve; finding its area with triangles is wrong.
- **Giving a time or a speed when a velocity is asked for.** "Find the minimum velocity" wants the value of v (which may be negative), not the time, and not the speed.
- **Assuming the object is at rest at a given height.** In vertical motion, $v = 0$ only at the greatest height. Being told a height and a time does not mean the object is at its top then; use $s = ut + \frac{1}{2}at^2$ instead.
- **Mixing up displacement from the start with height above the ground**, for example forgetting to add the height of a cliff or a platform.
- **Rounding too early or to 2 significant figures.** Keep full accuracy until the end and give answers to 3 s.f.

How to get the marks

- **Show the calculus.** Write the derivative or integral before substituting. The method mark is for raising (or lowering) the power by 1 and changing the coefficient; a correct integral with the constant found earns the accuracy mark.
- **Show the constant.** Write "$v = 0$ when $t = 0$, so $c = \ldots$", even when c turns out to be 0.
- **Show the limits.** Write $\left[\ \right]_1^3$ or $F(3) - F(1)$ with the numbers in; for a total distance, show each piece and that you are adding sizes.
- **"Show that" and "verify"** have given answers: every step must be visible, and "verify" means substitute and show the result holds, then say so.
- **Sketches** earn marks for the right shape (straight or curved, right way up) and for key values labelled on the axes: speeds, times, where $v = 0$, turning points. Scale doesn't matter.
- **Rejecting a root** needs a reason: "this would make the constant-speed time negative" or "it gives a time longer than the whole journey".
- **Use** $g = 10$ unless told otherwise, and state your positive direction if it isn't obvious.
- **Accuracy:** give non-exact answers to 3 significant figures, and angles to 1 decimal place. Exact fractions such as $\frac{71}{3}$ are also fine.

- **"Hence"** means use the result you just found, for example the time from part (a) as a limit in part (b).
- **Speed is never negative.** If asked for a speed, give the size of the velocity; if asked for a velocity, keep its sign.

Quick check

1. A particle moves in a straight line with constant acceleration. It passes point A with velocity 5 m s^{-1} and 6 s later passes point B with velocity 17 m s^{-1}. Find its acceleration and the distance AB.
2. A ball is projected vertically upwards with speed 35 m s^{-1}. Find the times at which it is 60 m above its point of projection.
3. A particle moves in a straight line so that t s after leaving O its displacement from O is $s = t^3 - 12t$ m. Find its velocity and acceleration when $t = 3$, and the total distance it travels in the first 3 s.
4. A particle leaves O with velocity $v = 6\sqrt{t} - t$ m s^{-1}, t s after leaving O. Find its greatest velocity and its displacement from O at that time.
5. A particle leaves O from rest. Its velocity increases uniformly to 6 m s^{-1} in 4 s, then decreases uniformly to -3 m s^{-1} at $t = 10$. Find the distance travelled and the displacement from O at $t = 10$.

Answers

1. $a = \dfrac{17 - 5}{6} = 2$ m s-2. $AB = \frac{1}{2}(5 + 17)(6) = 66$ m.
2. $60 = 35t - 5t^2$, so $t^2 - 7t + 12 = 0$ and $(t - 3)(t - 4) = 0$: $t = 3$ s (going up) and $t = 4$ s (coming down).
3. $v = 3t^2 - 12$, $a = 6t$. At $t = 3$: $v = 15$ m s-1, $a = 18$ m s-2. $v = 0$ at $t = 2$, where $s = 8 - 24 = -16$; at $t = 3$, $s = 27 - 36 = -9$. Distance $= 16 + 7 = 23$ m.
4. $a = 3t^{-1/2} - 1 = 0$ gives $\sqrt{t} = 3$, $t = 9$, and $v = 18 - 9 = 9$ m s-1. $s = 4t^{3/2} - \frac{1}{2}t^2$ (with $s = 0$ at $t = 0$), so $s(9) = 108 - 40.5 = 67.5$ m.
5. From $t = 4$ to 10 the velocity falls by 9 in 6 s, 1.5 m s-1 each second, so it is 0 at $t = 8$. Area above the axis $= \frac{1}{2}(8)(6) = 24$; area below $= \frac{1}{2}(2)(3) = 3$. Distance $= 24 + 3 = 27$ m; displacement $= 24 - 3 = 21$ m.

Past questions to try

- **9709/43/M/J/25 Q3**: a velocity–time graph where the particle travels back to its start.
- **9709/45/M/J/25 Q6**: calculus with velocity and acceleration both zero, then displacement by integration.
- **9709/42/F/M/24 Q2**: vertical motion, finding the speed of projection and a distance between two speeds.
- **9709/43/O/N/22 Q4**: constant acceleration, using distances covered in single seconds.