Cambridge AS & A Level · Mathematics 9709 · Notes

Linear combinations of random variables

A2

Read online at **pastopic.com/cambridge/mathematics-9709/notes/linear-combinations-of-random-variables**

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Read first: **Discrete random variables**, **The normal distribution**, **The Poisson distribution**

What you need to know

By the end of this topic you should be able to use these results to solve problems (you don't need to prove any of them):

1. **Scaling and shifting one variable:** $E(aX+b)=aE(X)+b$ and $\operatorname{Var}(aX+b)=a^2\operatorname{Var}(X)$.
2. **The mean of a combination of two variables:** $E(aX+bY)=aE(X)+bE(Y)$.
3. **The variance of a combination of two independent variables:** $\operatorname{Var}(aX+bY)=a^2\operatorname{Var}(X)+b^2\operatorname{Var}(Y)$, when X and Y are independent.
4. **Normal stays normal:** if X is normally distributed, so is $aX+b$.
5. **Independent normals combine to a normal:** if X and Y are independent and normally distributed, then $aX+bY$ is normally distributed.
6. **Independent Poissons add to a Poisson:** if X and Y are independent Poisson variables, then $X+Y$ is a Poisson variable.

In practice this means: find the mean and variance of a sum, difference, total or multiple, then (for normal variables) use the normal table to find a probability.

Understand it

Scaling and shifting one variable

Suppose X has mean 10 and variance 4 (standard deviation 2), and $W=3X+5$.

- **Multiplying by** 3 multiplies every value by 3, so the mean is multiplied by 3, and so is the spread: the standard deviation becomes $3\times 2=6$. Variance is the standard deviation squared, so it is multiplied by $3^2=9$.
- **Adding** 5 moves every value up by 5. The mean goes up by 5, but the values are no more spread out than before, so the variance doesn't change.

$$E(3X+5) = 3 \times 10 + 5 = 35, \qquad \mathrm{Var}(3X+5) = 3^2 \times 4 = 36.$$

This is exactly what happens when a mass in kilograms is turned into a price: at 1.20 dollars per kg plus a fixed charge of 5 dollars, the price is $1.2X + 5$.

Adding and subtracting two variables

For any two random variables, **means simply combine**: $E(aX + bY) = aE(X) + bE(Y)$. This is true even if X and Y are not independent.

For **independent** variables, **variances add, after squaring the coefficients**:

$$\mathrm{Var}(aX + bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y).$$

The most important case is a difference, $a = 1$ and $b = -1$:

$$\mathrm{Var}(X - Y) = \mathrm{Var}(X) + (-1)^2\,\mathrm{Var}(Y) = \mathrm{Var}(X) + \mathrm{Var}(Y).$$

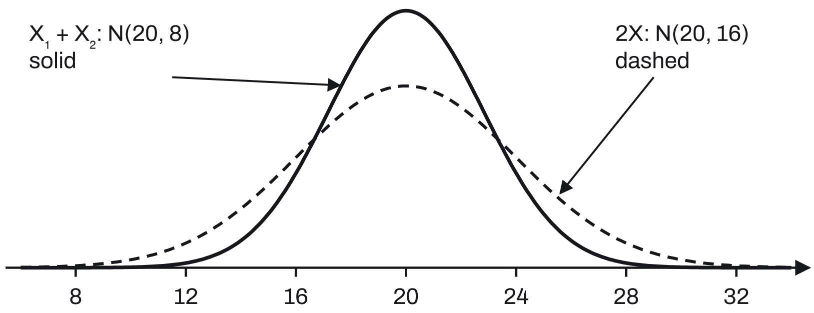
Variances always add, even when you subtract the variables. That makes sense: each variable brings its own uncertainty, and taking one away from the other doesn't cancel the uncertainty, it combines it. If X has mean 50 and variance 9, and Y has mean 30 and variance 16, then $X - Y$ has mean 20 and variance $9 + 16 = 25$.

The rules extend to any number of independent variables: $E(X_1 + X_2 + \cdots) = E(X_1) + E(X_2) + \cdots$ and $\mathrm{Var}(X_1 + X_2 + \cdots) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \cdots$.

Several values versus one value multiplied

This is the idea that causes the most lost marks in this topic. Let $X \sim N(10, 4)$.

- $X_1 + X_2$ means **two separate, independent values** of X added, like the total mass of two bags picked at random. Mean $10 + 10 = 20$, variance $4 + 4 = 8$.
- $2X$ means **one value doubled**, like the price of one bag at 2 dollars per kg. Mean $2 \times 10 = 20$, variance $2^2 \times 4 = 16$.



Both have mean 20, but $2X$ is more spread out. With two separate bags, a heavy one and a light one often partly balance each other; with one bag doubled, any error is doubled too. So:

In words	Write it as	Variance
total of n separate items	$X_1 + X_2 + \cdots + X_n$	$n\,\mathrm{Var}(X)$
one item multiplied by n (a price, "three times the mass of one")	nX	$n^2\,\mathrm{Var}(X)$

Where the distributions come from

Knowing the mean and variance is only half the job; to find a probability you also need the **shape**.

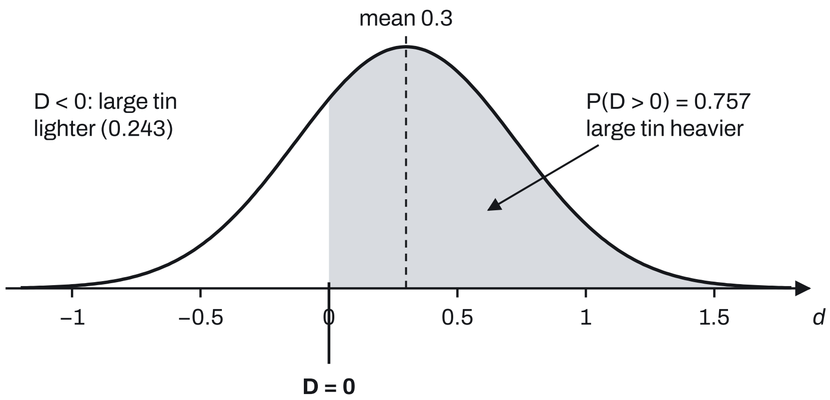
- If X is normal, $aX + b$ is normal. If X and Y are independent and normal, $aX + bY$ is normal (and so is any combination of several of them).
- If $X \sim Po(\lambda)$ and $Y \sim Po(\mu)$ are independent, $X + Y \sim Po(\lambda + \mu)$. Cars at 2 per minute plus trucks at 0.5 per minute give vehicles at 2.5 per minute.
- **Only a sum of Poissons is Poisson.** $2X$ is not: if $X \sim Po(3)$, $2X$ has mean 6 but variance $2^2 \times 3 = 12$, and a Poisson variable has equal mean and variance (also, $2X$ can only be even). $X - Y$ is not Poisson either; it can be negative.
- For any other variables (a binomial, or one given only by its mean and variance) you can still find the mean and variance of a combination with the rules above, but not its distribution.

Comparing two quantities: one new variable

"Is a large tin heavier than three small tins?" compares two random quantities. The trick is to make **one new variable** and ask whether it is positive. Large tins have $L \sim N(4.2,\ 0.05)$ and small tins $S \sim N(1.3,\ 0.015)$ (masses in kg). "Three times the mass of a small tin" is $3S$, so let $D = L - 3S$:

$$E(D) = 4.2 - 3 \times 1.3 = 0.3, \qquad \mathrm{Var}(D) = 0.05 + 3^2 \times 0.015 = 0.185.$$

D is normal, so $P(L > 3S) = P(D > 0) = P\left(Z > \frac{0 - 0.3}{\sqrt{0.185}}\right) = \Phi(0.697) = 0.757$.



A quick sketch like this tells you whether the answer is more or less than 0.5: here 0 is to the left of the mean, so the shaded area is more than half.

Key facts and formulas

"Given" means it is in the MF19 list of formulae and tables in the exam. "Learn it" means you must remember it.

Formula	Status
$E(aX+b)=aE(X)+b$	Learn it
$\mathrm{Var}(aX+b)=a^2\,\mathrm{Var}(X)$	Learn it
$E(aX+bY)=aE(X)+bE(Y)$	Learn it
$\mathrm{Var}(aX+bY)=a^2\,\mathrm{Var}(X)+b^2\,\mathrm{Var}(Y)$, for independent X and Y	Learn it
$\mathrm{Var}(X-Y)=\mathrm{Var}(X)+\mathrm{Var}(Y)$ for independent X and Y	Learn it
Total of n independent values: mean $n\mu$, variance $n\sigma^2$; one value times n: mean $n\mu$, variance $n^2\sigma^2$	Learn it
X normal $\Rightarrow aX+b$ normal; X, Y independent normal $\Rightarrow aX+bY$ normal	Learn it
$X\sim Po(\lambda), Y\sim Po(\mu)$ independent $\Rightarrow X+Y\sim Po(\lambda+\mu)$	Learn it
$Po(\lambda)$: mean λ, variance λ; $B(n,p)$: mean np, variance $np(1-p)$	Given
$\Phi(z)=P(Z\leqslant z)$ in the table, $\Phi(-z)=1-\Phi(z)$, and the critical values (2.326 for 0.99)	Given
Standardising: $z=\dfrac{x-\mu}{\sigma}$	Learn it

$N(\mu,\sigma^2)$ gives the **variance** second. If a question gives a standard deviation, square it before combining.

How to do it

In the Paper 6 questions we have tagged for this topic (36 questions, 2021–2025; 31 different ones, as 5 are shared by two papers), the types came up this often. 22 of the 31 questions mix types, so the counts add up to more than 31.

Question type	Questions
Comparing two quantities ($X>kY$, one item against several)	18
Totals of several items	11
Money and scaled quantities ($aX+b$, prices, profit)	10
Mean, variance or standard deviation only	8
A difference in either direction	3
Sample means and finding n	3
Combinations with Poisson or binomial variables	2

1. Comparing two quantities (18 questions)

1. **Translate the words into an inequality.** "A large sack is more than twice the mass of a small sack" is $L > 2S$. "Three small bags weigh more than one large bag" is $S_1 + S_2 + S_3 > L$ (three **separate** bags, not $3S$).
2. **Move everything to one side** and name it: $D = L - 2S$, and you want $P(D > 0)$.
3. **Mean**: combine the means with the coefficients. **Variance**: add the variances, each times its coefficient **squared**; separate items each add their own variance.
4. **Standardise**: $z = \frac{0 - E(D)}{\sqrt{\operatorname{Var}(D)}}$, then find the area on the correct side. Sketch the curve with 0 and the mean on it.

For example, with $X \sim N(50, 9)$ and $Y \sim N(30, 16)$ independent: $P(X > Y + 15) = P(X - Y > 15)$. $X - Y \sim N(20, 25)$, so $P = P\left(Z > \frac{15-20}{5}\right) = P(Z > -1) = \Phi(1) = 0.841$.

Variations you will meet:

- **A constant in the comparison**: "Ali's time is at least 0.3 s more than Ben's" is $P(A - B \geqslant 0.3)$. Keep the constant out of the variance; it only moves the value you standardise (or the mean, if you use $A - B - 0.3$).
- **Many items on each side**: "5 large bags weigh more than 10 small bags": $D = (L_1 + \cdots + L_5) - (S_1 + \cdots + S_{10})$, with $E(D) = 5\mu_L - 10\mu_S$ and $\operatorname{Var}(D) = 5\sigma_L^2 + 10\sigma_S^2$.
- **Two values of the same variable**: $X_1 - 3X_2$ has variance $\operatorname{Var}(X) + 9\operatorname{Var}(X) = 10\operatorname{Var}(X)$. And $P(X_1 > X_2) = 0.5$ straight away, by symmetry.
- **One quantity is a total itself**, such as a full carton = empty carton $+$ 20 packets. Find the carton's mean and variance first (the 20 packets add $20\sigma^2$), then compare.
- **Estimates first**: sometimes μ and σ^2 come from unbiased estimates of a sample (topic 6.4), then you use them as in step 3.

2. Totals of several items (11 questions)

1. **List what is added**, for example 3 bags of $N(25, 0.4)$ and one crate of $N(4, 0.1)$.
2. **Mean** $= 3 \times 25 + 4 = 79$. **Variance** $= 3 \times 0.4 + 0.1 = 1.3$ (each bag adds its own 0.4).
3. **Standardise and find the area**: for example "the total is more than 80 kg" is $1 - \Phi\left(\frac{80-79}{\sqrt{1.3}}\right)$.

Variations you will meet:

- **A safe load**: "a trolley can safely carry 8 large and 20 small crates" means $P(\text{total} \leqslant 600)$.
- **Journey in parts**: add the means and variances of the parts; don't divide by the number of parts.
- **Several weeks or months**: the total over 3 months is $X_1 + X_2 + X_3$, variance $3\sigma^2$; the standard deviation is the square root of that.
- **A mixture of boxes**: if 70% of crates hold 12 large bottles and 30% hold 24 small ones, find each total's distribution, the probability for each kind of box, then combine: $0.7 \times P_1 + 0.3 \times P_2$.
- **No continuity correction**: these are continuous variables.

3. Money and scaled quantities (10 questions)

1. **Write the amount as a combination.** At 1.20 dollars per kg and a fee of 5 dollars, the price of a mass $X \sim N(40, 9)$ is $1.2X + 5$.
2. **Mean**: $1.2 \times 40 + 5 = 53$. **Variance**: $1.2^2 \times 9 = 12.96$ (the 5 doesn't change the spread), so the standard deviation is 3.6.
3. If there are two products, use $aX + bY$: price per kg times each mass, and square each price in the variance. A **loss** on one product means a minus sign in the mean; its variance still adds.
4. **Profit** $=$ income $-$ costs. Fixed amounts (a rent of 40 dollars a day, a one-off fee of 150 dollars) change only the mean.

Variations you will meet:

- **Two ways to the same answer**: "the income in a month is less than $9\,000$ dollars" at 30 dollars per tonne can be done in tonnes ($9\,000 \div 30 = 300$ tonnes) or in dollars. Pick one and don't mix them.
- **Weighted scores**: an overall score $T + 2.5P$ has variance $\mathrm{Var}(T) + 2.5^2\,\mathrm{Var}(P)$.
- **"Comment on the assumption that the marks are independent"**: say whether it is realistic in context, for example "unlikely: a student who does well in theory probably does well in the practical".

4. Mean, variance or standard deviation only (8 questions)

1. Write the combination, for example $2W + P$ with W (mean 5, variance 2) and $P \sim Po(4)$ independent.
2. **Mean**: $2 \times 5 + 4 = 14$. **Variance**: $2^2 \times 2 + 4 = 12$ (a Poisson variance equals its mean).
3. **Read the question**: if it asks for the **standard deviation**, take the square root: $\sqrt{12} = 3.46$.

Variations you will meet:

- **A total of n people each paying aX**: mean $n \times a\mu$ and variance $n \times a^2\sigma^2$; for example, at 0.25 dollars a minute for times with mean 20 and variance 16, one person pays mean 5 and variance 1, and 24 people give mean 120 and variance 24.
- **"Find the mean and variance of the income"** usually leads into a probability in the next part: keep full accuracy.

5. A difference in either direction (3 questions)

"The two values differ by more than 6" means $X - Y > 6$ **or** $Y - X > 6$, that is $|X - Y| > 6$.

1. Let $D = X - Y$; find its mean and variance. For two values of the same variable, $E(D) = 0$.
2. Find **both** tails, $P(D > 6)$ and $P(D < -6)$, and **add** them.

For two independent values of $N(40, 18)$: $D \sim N(0, 36)$, so $P(D > 6) = 1 - \Phi(1) = 0.1587$, and by symmetry the answer is $2 \times 0.1587 = 0.317$. When $E(D) \neq 0$ the two tails are different: work out each one.

"Differ by less than 5" is the middle: $P(-5 < D < 5)$ (see Example 3).

6. Sample means and finding n (3 questions)

- **Finding the largest** n: the total of n independent items is $N(n\mu,\ n\sigma^2)$, so its standard deviation is $\sigma\sqrt{n}$ (not σn). Turn the probability condition into $\frac{\text{limit}-n\mu}{\sigma\sqrt{n}} = z$, with z from the critical values table, and solve it as a quadratic in $\sqrt{n}$. Round **down** and check the next whole number fails. State the assumption: the items are a random sample, independent of each other.
- **The mean of a combination over a sample**: if $F = 2A + B$ with $A \sim N(30, 4)$ and $B \sim N(50, 9)$, then $F \sim N(110, 25)$, and the mean of 20 independent values of F is $N\left(110, \frac{25}{20}\right) = N(110, 1.25)$ (topic 6.4).
- **The mean of** n **journeys**: the mean of 15 journeys has the same mean as one, and variance $\frac{\sigma^2}{15}$.

7. Combinations with Poisson or binomial variables (2 questions)

- **Means and variances**: use λ for both the mean and the variance of $Po(\lambda)$, and $np, np(1-p)$ for $B(n,p)$. For example, $\mathrm{Var}(X - 3Y) = \mathrm{Var}(X) + 9\,\mathrm{Var}(Y)$.
- **Sums of Poissons**: $X + Y \sim Po(\lambda + \mu)$, then Poisson probabilities as in the Poisson distribution topic.
- **A probability linking two discrete variables**, such as $P(Y = 15X)$ with X Poisson and $Y \sim B(20, p)$: list the pairs that are possible ($X = 0, Y = 0$ and $X = 1, Y = 15$), multiply the probabilities in each pair (they are independent), and add. Don't use a normal distribution.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

The random variable X has mean 8 and standard deviation 1.5. The independent random variable Y has the distribution $Po(2.5)$.

(a) Find $E(4X - 3Y + 2)$ and $\mathrm{Var}(4X - 3Y + 2)$. **[4]**

(b) T is the sum of 4 independent values of X. Find the standard deviation of T, and compare it with the standard deviation of $4X$. **[3]**

(a) $E(4X - 3Y + 2) = 4 \times 8 - 3 \times 2.5 + 2 = 26.5$. **B1**

$\mathrm{Var}(X) = 1.5^2 = 2.25$ and $\mathrm{Var}(Y) = 2.5$. **B1** for both variances.

$$\text{Var}(4X - 3Y + 2) = 4^2 \times 2.25 + 3^2 \times 2.5 = 36 + 22.5 = 58.5.$$

M1 for squared coefficients and adding, **A1**.

(b) $\text{Var}(T) = 4 \times 2.25 = 9$, so the standard deviation of T is 3. **M1 A1**

The standard deviation of $4X$ is $4 \times 1.5 = 6$, twice as big: one value multiplied by 4 is more spread out than 4 separate values added. **B1**

Example 2

A gift hamper is an empty basket packed with 5 jars of jam. The mass of an empty basket, in kilograms, has the distribution $N(1.6,\ 0.03)$, and the mass of a jar of jam has the distribution $N(0.52,\ 0.004)$. All masses are independent.

(a) Find the probability that the total mass of a randomly chosen full hamper is more than 4.5 kg. **[4]**

(b) The postage for a hamper costs 2.50 dollars plus 3 dollars per kilogram. A customer also posts a parcel whose postage, in dollars, has the independent distribution $N(14,\ 1.2)$. Find the probability that the postage for a randomly chosen hamper is more than the postage for the parcel. **[5]**

(a) Hamper mass $H = B + J_1 + J_2 + \cdots + J_5$.

$E(H) = 1.6 + 5 \times 0.52 = 4.2$. **B1**

$\text{Var}(H) = 0.03 + 5 \times 0.004 = 0.05$. **B1** (not 5^2: these are separate jars.)

$$P(H > 4.5) = 1 - \Phi\left(\frac{4.5 - 4.2}{\sqrt{0.05}}\right) = 1 - \Phi(1.342)$$

M1 for standardising with the combined values, $= 1 - 0.9101 = 0.0899$ **A1**.

(b) The hamper's postage is $C = 3H + 2.5$: one mass multiplied by 3, so its variance uses 3^2.

$E(C) = 3 \times 4.2 + 2.5 = 15.1$ and $\text{Var}(C) = 3^2 \times 0.05 = 0.45$. **B1 B1**

With the parcel's postage Q, let $D = C - Q$: $E(D) = 15.1 - 14 = 1.1$ and $\text{Var}(D) = 0.45 + 1.2 = 1.65$. **B1**

$$P(D > 0) = P\left(Z > \frac{0 - 1.1}{\sqrt{1.65}}\right) = P(Z > -0.856) = \Phi(0.856)$$

M1 for standardising and the correct area (more than 0.5, as 0 is below the mean), $= 0.804$ **A1**.

Example 3

A café sells X litres of coffee and Y litres of tea each day, where X and Y have the independent distributions $N(40, 25)$ and $N(30, 16)$. The café makes 3 dollars per litre of coffee and 2 dollars per litre of tea, and its fixed costs are 150 dollars a day.

(a) Find the probability that the café's profit on a randomly chosen day is more than 50 dollars. **[5]**

(b) Find the probability that, on a randomly chosen day, the amounts of coffee and tea sold differ by less than 5 litres. **[5]**

(a) Profit $P = 3X + 2Y - 150$.

$E(P) = 3 \times 40 + 2 \times 30 - 150 = 30$. **B1**

$\mathrm{Var}(P) = 3^2 \times 25 + 2^2 \times 16 = 225 + 64 = 289$, so the standard deviation is 17. **B1** (the 150 doesn't affect the variance.)

$$P(P > 50) = 1 - \Phi\left(\frac{50 - 30}{17}\right) = 1 - \Phi(1.176)$$

M1 M1, $= 0.120$ **A1**.

(b) $D = X - Y \sim N(40 - 30,\ 25 + 16) = N(10, 41)$. **B1 B1**

"Differ by less than 5" is $-5 < D < 5$:

$$P(-5 < D < 5) = \Phi\left(\frac{5 - 10}{\sqrt{41}}\right) - \Phi\left(\frac{-5 - 10}{\sqrt{41}}\right) = \Phi(-0.781) - \Phi(-2.343)$$

M1 for standardising both ends,

$= 0.2174 - 0.0096$ **M1**, $= 0.208$ **A1**.

Example 4

A van delivers 30 bags of cement and 12 crates of tiles. The masses, in kilograms, of the bags and the crates are normally distributed with means 25 and 18 and standard deviations 0.8 and 1.5 respectively.

(a) Stating a necessary assumption, find the probability that the total mass of the load is more than 980 kg. **[4]**

(b) The van's load limit is m kg. Find the smallest whole number m for which the probability that the load is more than m kg is less than 0.01. **[4]**

(a) Assume the masses of all the bags and crates are independent of each other (a random sample). **B1**

Total $T = B_1 + \cdots + B_{30} + C_1 + \cdots + C_{12}$.

$E(T) = 30 \times 25 + 12 \times 18 = 750 + 216 = 966$.

$\mathrm{Var}(T) = 30 \times 0.8^2 + 12 \times 1.5^2 = 19.2 + 27 = 46.2$. **B1** for both (square each standard deviation first).

$$P(T > 980) = 1 - \Phi\left(\frac{980 - 966}{\sqrt{46.2}}\right) = 1 - \Phi(2.060)$$

M1, $= 0.0197$ **A1**.

(b) $P(T > m) < 0.01$ needs $\dfrac{m - 966}{\sqrt{46.2}} > 2.326$, the critical value for 0.99. **B1** for 2.326, **M1** for an equation or inequality with the combined mean and variance.

$$m > 966 + 2.326 \times 6.797 = 981.81$$

M1 for solving. Check: $m = 981$ gives $1 - \Phi(2.207) = 0.0137$, too big; $m = 982$ gives $1 - \Phi(2.354) = 0.0093$. The smallest limit is $m = 982$ kg. **A1**

Common mistakes

- **Subtracting variances.** $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$. The reports mention this repeatedly; a variance can never be made smaller by combining with something uncertain.
- **Mixing up $X_1 + X_2 + X_3$ and $3X$.** Three separate bags: variance $3\sigma^2$. One value tripled ("three times the mass of a small sack"): variance $9\sigma^2$. Getting this the wrong way round is the most common error in the reports.
- **Forgetting to square the coefficient**, for example $\text{Var}(2.5P) = 2.5 \times 40$ instead of $2.5^2 \times 40$, or using 0.30 instead of 0.30^2 for a price per kilogram.
- **Mixing standard deviation and variance.** $N(12, 1.3^2)$ has variance 1.69. Square any standard deviation before combining; take the square root of the combined variance before standardising.
- **Putting a constant into the variance.** Fixed costs, the 0.3 s in "at least 0.3 s more", and the $+3$ in $X_1 - 2X_2 + 3$ change only the mean.
- **The wrong area.** After standardising, many answers come out as 0.83 instead of 0.17. Sketch the curve with the mean and the value you standardised, then decide.
- **Only one tail for "differ by".** "Differ by more than 2" needs $P(X - Y > 2) + P(Y - X > 2)$. Add the two, don't multiply.
- **Giving the variance when the standard deviation was asked for.** Read the last line of the question again.
- **Dividing when you should add**: the total of a journey in three parts is the sum of the means and the sum of the variances, not their average.
- **Mixing methods**: working part in kilograms and part in dollars in the same standardisation.
- **Rounding too early.** Keep values such as $\sqrt{46.2} = 6.797\ldots$ in full until the end, not 6.8.

How to get the marks

- **State the distribution you are using**, with its mean and variance, before you standardise: "$D = L - 3S \sim N(0.3,\ 0.185)$". The mean and the variance are often a mark each, and they can be given at an early stage.
- **Standardise with your combined values.** The method mark needs the mean and variance of the combination, not of one of the original variables.
- **Show the area**: write $1 - \Phi(1.342)$ or $\Phi(0.856)$, then the value. A final answer is given to 3 significant figures.
- **"Find the probability that the difference is..."**: define the new variable in words or symbols ($D = X - Y$) so the examiner can follow.

- **"Stating a necessary assumption"**: one sentence in context: "the masses of the bags and crates in the van are independent of each other".
- **"Comment on the assumption"**: give a view (realistic or not) and a reason from the context.
- **"Write down"** means no working is needed (for example $P(X_1 > X_2) = 0.5$).
- **Don't use a continuity correction** for normal variables; it can cost the accuracy mark.
- **Method marks follow through.** A wrong mean or variance used correctly still earns the standardising and area marks.

Quick check

1. $X \sim N(20, 9)$. Find the mean and variance of (a) $3X - 4$ and (b) $X_1 + X_2 + X_3$, the sum of three independent values of X.
2. $X \sim N(50, 16)$ and $Y \sim N(45, 9)$ are independent. Find $P(X > Y)$.
3. W has mean 6 and variance 2; V has mean 4 and variance 3; they are independent. Find $E(2W - 3V)$ and $\mathrm{Var}(2W - 3V)$.
4. $X \sim Po(1.5)$ and $Y \sim Po(2.5)$ are independent. Find $P(X + Y = 3)$, and explain why $2X$ does not have a Poisson distribution.
5. Two independent values X_1 and X_2 are taken from $N(10, 4)$. Find the probability that they differ by more than 3.

Answers

1. (a) Mean $3 \times 20 - 4 = 56$, variance $3^2 \times 9 = 81$. (b) Mean $3 \times 20 = 60$, variance $3 \times 9 = 27$.
2. $D = X - Y \sim N(5,\ 16 + 9) = N(5, 25)$. $P(D > 0) = P\left(Z > \frac{0-5}{5}\right) = P(Z > -1) = \Phi(1) = 0.841$.
3. $E(2W - 3V) = 12 - 12 = 0$. $\mathrm{Var}(2W - 3V) = 2^2 \times 2 + 3^2 \times 3 = 8 + 27 = 35$.
4. $X + Y \sim Po(4)$, so $P(X + Y = 3) = e^{-4}\dfrac{4^3}{3!} = 0.01832 \times \dfrac{64}{6} = 0.195$. $2X$ has mean 3 but variance $2^2 \times 1.5 = 6$; a Poisson variable's mean and variance are equal (and $2X$ can take only even values).
5. $D = X_1 - X_2 \sim N(0,\ 4 + 4) = N(0, 8)$. $P(D > 3) = 1 - \Phi\left(\frac{3}{\sqrt{8}}\right) = 1 - \Phi(1.061) = 1 - 0.8556$. Both tails: $2 \times 0.1444 = 0.289$.

Past questions to try

- **9709/62/O/N/22 Q6**: a total of six bags, then four small bags against two large ones.
- **9709/62/M/J/23 Q5**: values that differ by more than 2, then a weighted score and independence.
- **9709/62/F/M/23 Q5**: one packet against double another, then a mixture of boxes.
- **9709/61/M/J/22 Q4**: the mean and standard deviation of $X - 3Y$ with Poisson and binomial variables.