Cambridge AS & A Level · Mathematics (9709) · Notes

Logarithmic and exponential functions

AS

Read online at pastopic.com/cambridge/mathematics-9709/notes/logarithmic-and-exponential-functions-p2

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Read first: Functions, Coordinate geometry, Algebra

What you need to know

By the end of this topic you should be able to:

1. **Link logarithms and indices, and use the laws of logarithms.** Know that a logarithm is a power: $\log_a b = c$ means exactly the same as $a^c = b$. Use the three laws (adding logs, subtracting logs, and bringing a power down in front) to simplify and to solve equations. Changing from one base to another is **not** required.
2. **Know the functions e^x and $\ln x$.** Know what the number e is, that $\ln x$ is the logarithm to base e, and that e^x and $\ln x$ are inverse functions, so their graphs are reflections of each other in the line $y = x$. Know the shape of $y = e^{kx}$ both when k is positive and when k is negative.
3. **Solve equations and inequalities where the unknown is in a power**, such as $4^x = 11$ or $0.9^x < 0.25$, by taking logarithms of both sides.
4. **Turn a relationship into a straight line.** Take logarithms of a relationship like $y = kx^n$ or $y = k \times a^x$ to get the equation of a straight line (in $\ln x$ and $\ln y$, or in x and $\ln y$), then find the unknown constants from the line's gradient and intercept.

Understand it

A logarithm is a power

$2^5 = 32$ says "2 to the power 5 is 32". The same fact written the other way round is $\log_2 32 = 5$: "the power you raise 2 to, to get 32, is 5". So

$$\log_a b = c \iff a^c = b.$$

The small number a is the **base**. For example $\log_{10} 1000 = 3$ because $10^3 = 1000$, and $\log_3 \frac{1}{9} = -2$ because $3^{-2} = \frac{1}{9}$. Two values follow straight from the definition: $\log_a a = 1$ (because $a^1 = a$) and $\log_a 1 = 0$ (because $a^0 = 1$). You can only take the logarithm of a **positive** number: no power of a positive base gives 0 or a negative number.

The laws of logarithms

Powers of the same base multiply by adding the powers ($a^m \times a^n = a^{m+n}$). Logs are powers, so the rules for indices become rules for logs:

$$\log_a(xy) = \log_a x + \log_a y, \qquad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y, \qquad \log_a(x^n) = n\log_a x.$$

For example $\log_2 8 + \log_2 4 = 3 + 2 = 5$, and $\log_2(8 \times 4) = \log_2 32 = 5$ too. The third law is the one that solves equations: it brings a power down to the front, where you can reach it.

There is **no** law for $\log(x+y)$ or $\log(x-y)$, and $\frac{\log x}{\log y}$ is not $\log x - \log y$.

The number e and the natural logarithm

Every graph $y = a^x$ (with $a > 1$) passes through $(0, 1)$ and gets steeper as x increases. There is exactly one base for which the gradient at every point equals the height of the graph there. That base is $e = 2.71828\ldots$, and $y = e^x$ is called **the exponential function**. At $(0, 1)$ its gradient is 1.

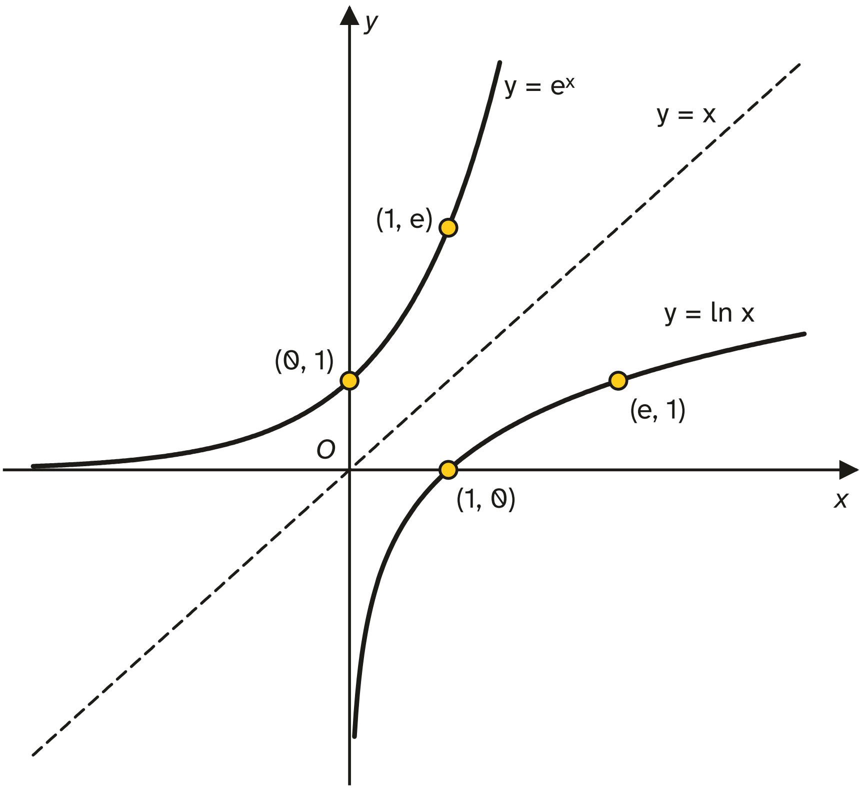
The logarithm to base e is the **natural logarithm**, written $\ln x$ (so $\ln x = \log_e x$). Everything above holds for it: $\ln e = 1$, $\ln 1 = 0$, and the three laws. Your calculator has e^x and $\ln$ keys.

e to the x and ln x undo each other

e^x and $\ln x$ are **inverse functions**: one undoes the other.

$$\ln(e^x) = x \text{ for every } x, \qquad e^{\ln x} = x \text{ for } x > 0.$$

For example $\ln(e^3) = 3$ and $e^{\ln 7} = 7$. As with any function and its inverse, the two graphs are reflections of each other in the line $y = x$.



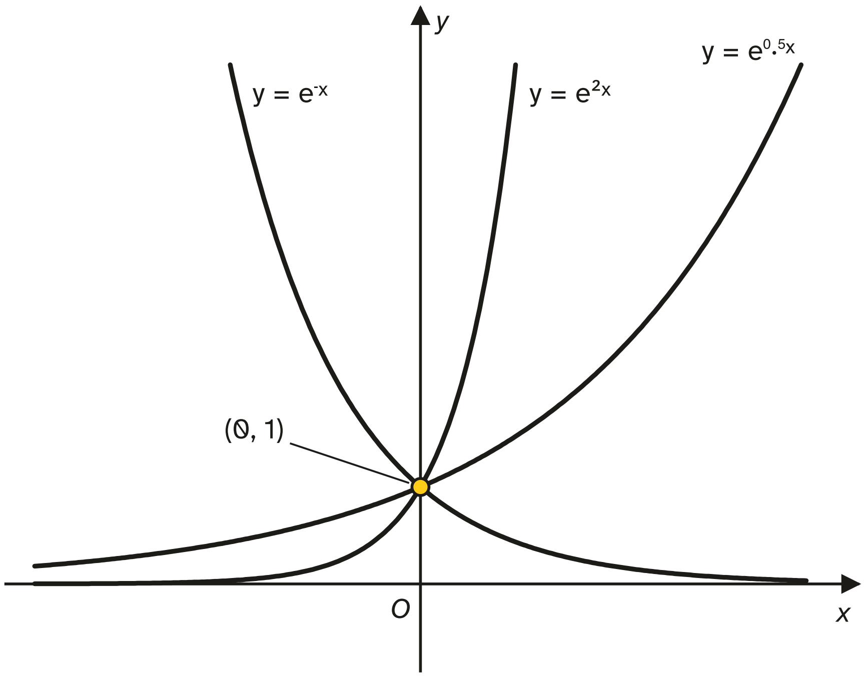
What to remember about the two graphs:

- $y = e^x$: domain all real x; always positive (range $y > 0$); crosses the y-axis at $(0, 1)$; the x-axis is an **asymptote** on the left; always increasing.
- $y = \ln x$: domain $x > 0$ only; range all real numbers; crosses the x-axis at $(1, 0)$; the y-axis is an asymptote; always increasing, but slowly. $\ln x$ is **negative** for $0 < x < 1$.

Graphs of e to the kx

In $y = e^{kx}$ the constant k controls how fast the graph grows. Every one of these graphs passes through $(0, 1)$ (because $e^0 = 1$) and stays above the x-axis.

- $k > 0$: the graph rises from left to right, **exponential growth**. The bigger k is, the steeper it rises.
- $k < 0$: the graph falls from left to right, **exponential decay**, towards the x-axis. $y = e^{-x}$ is the reflection of $y = e^x$ in the y-axis.



Solving when the unknown is a power

To get x out of a power, take logs of both sides and use the power law. For $3^x = 20$:

$$\ln(3^x) = \ln 20 \;\Rightarrow\; x\ln 3 = \ln 20 \;\Rightarrow\; x = \frac{\ln 20}{\ln 3} = 2.73 \text{ (3 s.f.)}.$$

When the base is e, taking $\ln$ undoes it straight away: $e^{2x} = 5$ gives $2x = \ln 5$, so $x = \frac{1}{2}\ln 5 = 0.805$.

The reverse also works. To remove a logarithm, raise e to both sides: $\ln(x-1) = 3$ gives $x - 1 = e^3$, so $x = e^3 + 1$.

Inequalities work the same way, with one trap. $\ln$ is an increasing function, so taking $\ln$ of both sides keeps the inequality sign. But when you then **divide by a negative number**, the sign turns round, and $\ln$ of any number between 0 and 1 is negative. For $0.6^x < 0.2$:

$$x\ln 0.6 < \ln 0.2, \quad \text{and } \ln 0.6 < 0, \text{ so } x > \frac{\ln 0.2}{\ln 0.6} = 3.15 \text{ (3 s.f.)}.$$

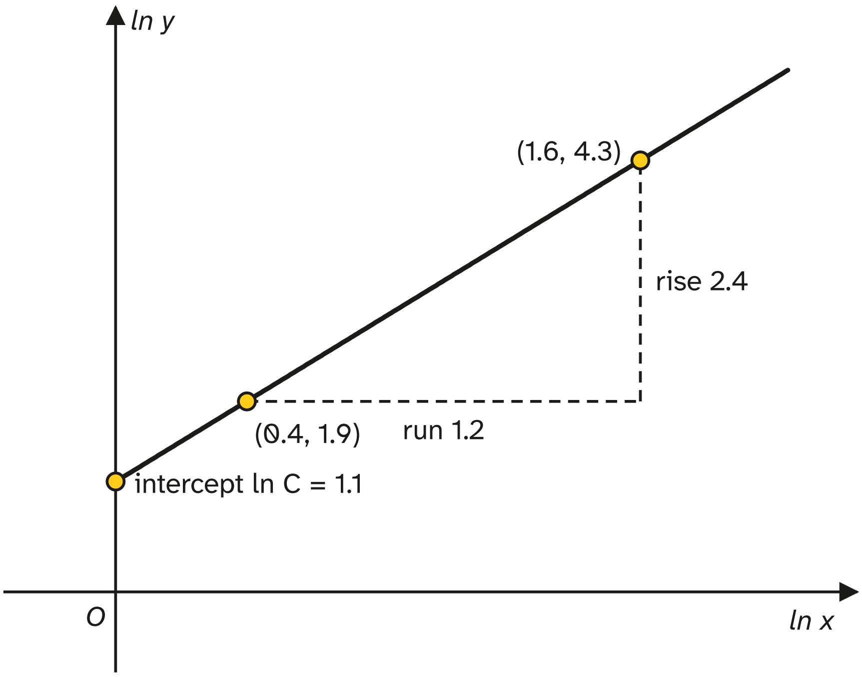
That makes sense: 0.6^x gets smaller as x grows, so it is below 0.2 for all large enough x.

Turning a curve into a straight line

In experiments you often suspect a rule such as $y = kx^n$ or $y = k \times a^x$ but don't know the constants. Taking logs turns each rule into the equation of a straight line, $Y = mX + c$:

$$y = kx^n \;\Rightarrow\; \ln y = \ln k + n\ln x \qquad (Y = \ln y,\ X = \ln x,\ \text{gradient } n,\ \text{intercept } \ln k)$$
$$y = k \times a^x \;\Rightarrow\; \ln y = \ln k + x\ln a \qquad (Y = \ln y,\ X = x,\ \text{gradient } \ln a,\ \text{intercept } \ln k)$$

So plot the right things and you get a straight line. Its **gradient** and its **intercept** on the vertical axis (where the horizontal variable is 0) give the constants. For $y = Ae^{bx}$ the same idea gives $\ln y = \ln A + bx$: gradient b, intercept $\ln A$.



The points on such a graph are already logarithms: the point $(0.4, 1.9)$ on a graph of $\ln y$ against $\ln x$ means $\ln x = 0.4$ and $\ln y = 1.9$. Use the numbers as they are; never take their logs again.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. Nothing in this topic is in MF19, so you must learn all of it.

Formula	Status
$\log_a b = c \iff a^c = b$	Learn it
$\log_a(xy) = \log_a x + \log_a y$	Learn it
$\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$	Learn it
$\log_a(x^n) = n\log_a x$	Learn it
$\log_a a = 1$, $\log_a 1 = 0$; in particular $\ln e = 1$, $\ln 1 = 0$	Learn it
$\ln x = \log_e x$, with $e = 2.718\ldots$	Learn it
$\ln(e^x) = x$ and $e^{\ln x} = x$ (for $x > 0$)	Learn it
$e^x > 0$ for every x; $\ln x$ exists only for $x > 0$	Learn it
$y = kx^n \Rightarrow \ln y = \ln k + n\ln x$	Learn it

Formula	Status
$y = k \times a^x \Rightarrow \ln y = \ln k + x \ln a$	Learn it
Straight line: gradient $= \dfrac{Y_2 - Y_1}{X_2 - X_1}$, and $Y = mX + c$	Learn it

Useful consequences: $2\ln x = \ln x^2$, $-\ln x = \ln \frac{1}{x}$, $\frac{1}{2}\ln x = \ln \sqrt{x}$, and $\ln(ke^{bx}) = \ln k + bx$.

How to do it

In the Paper 2 questions we have tagged for this topic (39 questions, 2021–2025; many appear in two or three versions of the same exam, so there are 28 different ones), the types came up this often. Each question has one main type here, so the counts add up to 39. Many of them start with a modulus or polynomial part from the Algebra topic.

Question type	Questions
Equations in e^x or a^x that become quadratics or polynomials	10
Exponential equations and inequalities	9
Straight-line graphs from logarithms	9
Using a modulus answer with a power or a logarithm	6
Logarithmic equations	5

1. Equations in e^x or a^x that become quadratics or polynomials (10 questions)

1. **Spot the hidden letter.** $e^{2x} = (e^x)^2$, $e^{4x} = (e^{2x})^2$ and $9^x = (3^x)^2$. Write $u = e^x$ (or whatever power repeats), so the equation becomes a quadratic or a polynomial in u.
2. **Solve for u:** factorise, use the formula, or use the factors you found in an earlier part.
3. **Reject impossible values.** e^x and a^x are always positive, so $u = -4$ or $u = 0$ gives no solution. Say so.
4. **Take logs** of each positive value to find x.

For example, $e^{2x} - 5e^x + 6 = 0$ becomes $u^2 - 5u + 6 = 0$, so $u = 2$ or $u = 3$, giving $x = \ln 2$ or $x = \ln 3$.

Variations you will meet:

- **"Hence solve $p(e^{2y}) = 0$"** after factorising a polynomial $p(x)$. Every root of $p(x) = 0$ is a possible value of e^{2y}. Keep the positive ones only, then $2y = \ln(\text{root})$.
- **The equation comes as a product**, such as $e^x(e^x - 3) = 10$. Expand first to get $u^2 - 3u - 10 = 0$.
- **$e^y + e^{-y} = c$.** Multiply every term by e^y: $e^{2y} + 1 = ce^y$, a quadratic in e^y. Both roots may be positive, giving two answers.
- **A negative power, such as e^{-3y}**, is still just a letter u. If $u = 5$, then $-3y = \ln 5$ and $y = -\frac{1}{3}\ln 5$.
- **"Show that there is only one real root."** Show that the other factor gives no positive value: a quadratic factor with a negative discriminant has no real roots at all.

- **"Exact"** means leave the answer as a logarithm, simplified: $\frac{1}{3}\ln\frac{1}{2}$ is better written $-\frac{1}{3}\ln 2$.

2. Exponential equations and inequalities (9 questions)

1. **Take $\ln$ of both sides**, keeping each side in brackets: $\ln\left(5^{x+1}\right) = \ln\left(2^{3x}\right)$.
2. **Use the laws.** The power law brings each index down: $(x+1)\ln 5 = 3x\ln 2$. A product such as $4e^{2x}$ splits first: $\ln 4 + \ln e^{2x} = \ln 4 + 2x$.
3. **Collect the x terms** on one side, factorise out x and divide.
4. **Give the accuracy asked for** (usually 3 or 4 significant figures). Keep the logs in your calculator until the last step.

Variations you will meet:

- **An inequality** such as $3^{2x-5} < 400$: the same steps, but watch the sign when you divide. $\ln$ of a base above 1 is positive (sign stays); $\ln$ of a base between 0 and 1 is negative (sign turns round).
- **"Show that the equation can be written as $y = kx$"**, for example from $a^{my} = b^{nx}$: take logs to get $my\ln a = nx\ln b$, so $y = \frac{n\ln b}{m\ln a}x$, and work out k.
- **After a modulus or other part**, list the integers that satisfy both inequalities: find each range, then pick the whole numbers in both.

3. Straight-line graphs from logarithms (9 questions)

1. **Take logs of the given relationship** and write it in the form $Y = mX + c$, where Y and X are what the graph's axes show. Keep brackets: $y = 3^{x-2}$ gives $\ln y = (x-2)\ln 3$, not $x - 2\ln 3$.
2. **Match up**: the gradient m equals some expression in the constants (such as n, $\ln a$ or $\frac{1}{\ln a}$), and so does the intercept c.
3. **Find the gradient** from the two points given: $\frac{Y_2 - Y_1}{X_2 - X_1}$.
4. **Find the intercept** by putting one point into $Y = mX + c$.
5. **Solve for the constants**, using e to undo any $\ln$: $\ln k = 1.2$ gives $k = e^{1.2}$. Give the accuracy asked for.

Instead of steps 3 and 4 you can put both points into the log equation and solve the two equations simultaneously.

Variations you will meet:

- **Different axes.** $\ln y$ against x (for $y = ka^x$ or $y = Ae^{bx}$); $\ln y$ against $\ln x$ (for $y = kx^n$); y against $\ln x$ (for $b^y = cx^2$, which gives $y\ln b = \ln c + 2\ln x$, so gradient $\frac{2}{\ln b}$); even y against x (for $b^{4y} = e^{x+c}$, which gives $4y\ln b = x + c$, so gradient $\frac{1}{4\ln b}$). Always rearrange so the vertical variable is on its own.
- **Constants in unusual places**, such as $y = Ce^{(C-2d)x}$: the gradient is $C - 2d$ and the intercept is $\ln C$. Find C from the intercept first, then d.
- **"Show that the gradient is ..."** (1 or 2 marks): write the log equation, say which term is the gradient, and finish with the value given.
- **Only the gradient or one point given**: you get one equation per fact, so there is usually only one unknown.

- **"a is an integer"**: give a whole number, even if your working gives 14.97.
- **Using the line afterwards**, for example "find x when $y = 36$": put $\ln 36$ into the line's equation.

4. Using a modulus answer with a power or a logarithm (6 questions)

These questions solve a modulus equation or inequality first (see the Algebra note), then say "hence" and replace x with a power or a log.

1. **Match the two problems.** In "hence solve $|2 \times 5^y - 1| = 5^y + 4$", the x of the first part has become 5^y.
2. **Use the answer from the first part**: set 5^y equal to each value of x you found (or put it into the range for an inequality).
3. **Throw out impossible values**: 5^y can never be negative or zero.
4. **Take logs** to find y, to the accuracy asked.

Variations you will meet:

- **"Find the largest (or smallest) integer N"**, for example with $e^{0.2N}$ in place of x: turn the range for x into one for $e^{0.2N}$, take $\ln$ (the sign stays, since $\ln$ is increasing), then choose the whole number. $N < 11.5$ gives $N = 11$; $N > 16.2$ gives $N = 17$.
- **$\ln N$ in place of x**: $\ln N > 2.8$ means $N > e^{2.8} = 16.4$.

5. Logarithmic equations (5 questions)

1. **Collect the logs into one log on each side** using the laws: $\ln A - \ln B = \ln \frac{A}{B}$, $\ln A + \ln B = \ln AB$, and $2 \ln A = \ln A^2$ first.
2. **Remove the logs.** $\ln P = \ln Q$ gives $P = Q$. $\ln P = 4$ (a plain number) gives $P = e^4$. Write $2 \ln 3$ as $\ln 9$ so both sides are single logs.
3. **Solve** the equation that is left (often linear or quadratic).
4. **Check** each answer makes every bracket inside a log positive, and reject any that doesn't.

For example, $\ln(2x + 1) - \ln(x - 3) = 2$: $\frac{2x+1}{x-3} = e^2$, so $2x + 1 = e^2x - 3e^2$, giving $x = \frac{3e^2+1}{e^2-2}$.

Variations you will meet:

- **"Give your answer in an exact form"**: keep e and $\ln$ in the answer, as in the example.
- **"Show that $x = \ldots$"**, where the rearranged equation is then solved by iteration (see the Numerical solution of equations note).
- **"Hence"** with x replaced by something else, such as $\cot y$: set $\cot y$ equal to your earlier answer instead of starting again.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

Solve the equation $\ln(x+2)+\ln(x-3)=\ln 3+3\ln 2$. **[4]**

Use the laws on both sides: $\ln\big((x+2)(x-3)\big)=\ln 3+\ln 8=\ln 24$. **M1** for using a law of logarithms correctly.

$$(x+2)(x-3)=24.$$

A1 for a correct equation with no logarithms.

$x^2-x-6=24 \Rightarrow x^2-x-30=0 \Rightarrow (x-6)(x+5)=0$, so $x=6$ or $x=-5$. **M1** for solving the quadratic.

$x=-5$ would need $\ln(-3)$, which does not exist, so $x=6$ only. **A1**

Example 2

(a) Use logarithms to solve the equation $5^{2x-1}=3^{x+2}$, giving your answer correct to 3 significant figures. **[4]**

(b) Find the smallest integer n for which $6\times 0.75^n<0.2$. **[3]**

(a) Take $\ln$ of both sides and use the power law:

$$(2x-1)\ln 5=(x+2)\ln 3.$$

M1 for taking logs and bringing the powers down, **A1** for this equation.

$2x\ln 5-\ln 5=x\ln 3+2\ln 3 \Rightarrow x(2\ln 5-\ln 3)=2\ln 3+\ln 5$. **M1** for collecting the x terms and solving.

$$x=\frac{2\ln 3+\ln 5}{2\ln 5-\ln 3}=1.80\ (3\text{ s.f.}).$$

A1

(b) $0.75^n<\frac{0.2}{6}=\frac{1}{30}$, so $n\ln 0.75<\ln\frac{1}{30}$. **M1** for taking logs correctly.

$\ln 0.75$ is negative, so dividing by it turns the sign round:

$$n > \frac{\ln \frac{1}{30}}{\ln 0.75} = 11.8\ldots$$

M1 for dividing and reversing the inequality. The smallest integer is $n = 12$. **A1**

Example 3

The variables x and y satisfy the equation $y = Cx^m$, where C and m are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points $(0.4, 1.9)$ and $(1.6, 4.3)$. Find the value of m, and the value of C correct to 3 significant figures. **[5]**

Take $\ln$ of both sides: $\ln y = \ln C + m \ln x$. **B1**

This is a straight line in $\ln x$ and $\ln y$ with gradient m. **M1** for equating m to the gradient:

$$m = \frac{4.3 - 1.9}{1.6 - 0.4} = \frac{2.4}{1.2} = 2.$$

A1

Put $(0.4, 1.9)$ into the line: $1.9 = \ln C + 2 \times 0.4$, so $\ln C = 1.1$. **M1** for substituting a point to find $\ln C$.

$C = e^{1.1} = 3.00$ (3 s.f.). **A1**

The third diagram in "Understand it" shows this line. Note that $(0.4, 1.9)$ already means $\ln x = 0.4$ and $\ln y = 1.9$; you do not take $\ln 0.4$.

Example 4

The polynomial $p(x)$ is defined by $p(x) = 2x^3 + ax^2 - 11x + 6$, where a is a constant. It is given that $(x - 3)$ is a factor of $p(x)$.

(a) Find the value of a. **[2]**

(b) Factorise $p(x)$ completely. **[3]**

(c) Hence find the exact values of y that satisfy the equation $p(e^{2y}) = 0$. **[3]**

(a) $p(3) = 54 + 9a - 33 + 6 = 0$, so $9a = -27$ and $a = -3$. **M1 A1**

(b) Dividing $2x^3 - 3x^2 - 11x + 6$ by $(x - 3)$ gives the quotient $2x^2 + 3x - 2$. **M1 A1**

$$p(x) = (x - 3)(2x - 1)(x + 2).$$

A1

(c) Put $x = e^{2y}$: $e^{2y} = 3$, $e^{2y} = \frac{1}{2}$ or $e^{2y} = -2$.

$e^{2y} = -2$ is impossible, because e^{2y} is always positive.

$e^{2y} = 3 \Rightarrow 2y = \ln 3 \Rightarrow y = \frac{1}{2} \ln 3$. **M1** for taking $\ln$ of a positive value correctly.

$e^{2y} = \frac{1}{2} \Rightarrow 2y = \ln \frac{1}{2} = -\ln 2 \Rightarrow y = -\frac{1}{2} \ln 2.$

A1 A1, the second one only if no answer comes from $e^{2y} = -2$.

Common mistakes

- **Splitting a log of a sum.** $\ln(x+2)$ is **not** $\ln x + \ln 2$, and $\ln(2+e^x)$ is not $\ln 2 + x$. The laws only work for products, quotients and powers. Examiners report this every year, and it usually ends the question.
- **Turning a difference of logs into a fraction of logs.** $\ln(x+2) - \ln x = \ln \frac{x+2}{x}$, not $\frac{\ln(x+2)}{\ln x}$.
- **Dropping brackets when taking logs.** $\ln\left(5^{3x-b}\right) = (3x-b)\ln 5$. Writing $3x - b\ln 5$ gives the wrong gradient and intercept, and costs the marks that follow.
- **Mishandling a product with e.** $\ln(9e^{-3x}) = \ln 9 - 3x$. Taking logs of $9e^{-3x}$ as a single lump, or writing $\ln 9 \times (-3x)$, is a common slip.
- **Taking logs of coordinates that are already logs.** On a graph of $\ln y$ against $\ln x$, the point $(0.62, 3.05)$ means $\ln x = 0.62$ and $\ln y = 3.05$. Don't write $\ln 0.62$ or $\ln 3.05$.
- **Getting the gradient wrong in terms of the constants.** Write the log equation in full first, then match it to $Y = mX + c$. If the gradient is $C - 2d$ or $\frac{2}{\ln b}$, set that whole expression equal to the number.
- **Taking logs of each term of a hidden quadratic.** $\ln(e^{2x}) - \ln(5e^x) = \ln(-6)$ is wrong: $\ln$ does not work term by term across a $+$ or $-$. Substitute $u = e^x$ and solve the quadratic first.
- **Keeping impossible roots.** $e^{2y} = -2$ or $3^x = 0$ has no solution. Answers from them lose the final mark.
- **Forgetting to turn the inequality round** when dividing by the log of a number less than 1, or by any negative number.
- **Giving decimals when the question says exact**, or 3 decimal places when it asks for 3 significant figures.

How to get the marks

- **Show the log step.** The first method mark is for taking logs of both sides and using the power law correctly at least once, with x no longer in a power. Write that line out, for example $(2x-1)\ln 5 = (x+2)\ln 3$.
- **Then show the linear equation being solved.** A dependent method mark needs to see you collect the x terms; an answer straight from a calculator's equation solver can lose it.
- **Any base is fine.** You can take $\log_{10}$ instead of $\ln$ and get the same answer, but $\ln$ is simplest when e appears.
- **Straight-line questions:** state the log equation (it earns a mark by itself), say what the gradient equals, then use a point. Keep 3 or 4 significant figures in working and round only at the end.
- **"Show that"** means the answer is given. Every step must be visible, for example state the log equation **and** say which expression is the gradient before writing the given value.
- **"Hence"** means use your previous answer. Replacing x by e^{2y} in your factors earns the marks; starting again from scratch usually doesn't.

- **"Exact"** means $\ln$ and e stay in the answer, simplified: $\frac{1}{2}\ln 3$, $\frac{3e^2+1}{e^2-2}$. Otherwise give answers to 3 significant figures unless the question says otherwise (some ask for 4).
- **Say which answers you reject** and why ("$e^{2y} > 0$"), and give **only** the valid ones; extra wrong answers lose the final mark.
- **Integer answers** ("a is an integer", "the largest integer N") must be a single whole number.

Quick check

1. Write $3\ln 2 + \frac{1}{2}\ln 81 - \ln 6$ as a single logarithm, as simply as possible.
2. Solve the equation $3^{2x} - 4(3^x) - 21 = 0$, giving your answer correct to 3 significant figures.
3. Solve the equation $\ln(4x-3) = 1 + \ln x$, giving your answer in an exact form.
4. The variables x and y satisfy $y = Pe^{-qx}$, where P and q are constants. The graph of $\ln y$ against x is a straight line through $(0, 1.5)$ and $(4, 0.3)$. Find q, and find P correct to 3 significant figures.
5. Find the set of values of x for which $5e^{-2x} > 1.5$, giving the boundary correct to 3 significant figures.

Answers

1. $3\ln 2 = \ln 8$ and $\frac{1}{2}\ln 81 = \ln 9$, so the expression is $\ln\dfrac{8\times 9}{6} = \ln 12$.
2. Let $u = 3^x$: $u^2 - 4u - 21 = 0$, so $(u-7)(u+3) = 0$. $3^x = -3$ is impossible, so $3^x = 7$ and $x = \dfrac{\ln 7}{\ln 3} = 1.77$ (3 s.f.).
3. $\ln(4x-3) - \ln x = 1$, so $\dfrac{4x-3}{x} = e$. Then $4x - 3 = ex$, so $x = \dfrac{3}{4-e}$. (This is about 2.34, which makes $4x-3$ and x positive, so it is valid.)
4. $\ln y = \ln P - qx$. Gradient $= \dfrac{0.3-1.5}{4-0} = -0.3$, so $q = 0.3$. The intercept is $\ln P = 1.5$, so $P = e^{1.5} = 4.48$ (3 s.f.).
5. $e^{-2x} > 0.3$, so $-2x > \ln 0.3$. Dividing by -2 turns the sign round: $x < -\frac{1}{2}\ln 0.3 = 0.602$ (3 s.f.).

Past questions to try

- **9709/21/M/J/23 Q1**: an exponential equation with different bases, to 3 significant figures.
- **9709/22/F/M/22 Q3**: a straight line of $\ln y$ against x with a known gradient, then finding x for a given y.
- **9709/22/O/N/22 Q4**: factorising a polynomial, then solving $p(e^{4y}) = 0$ with logarithms.
- **9709/22/F/M/23 Q4**: a modulus inequality, then the smallest integer N with $\ln N$ in place of x.