Cambridge AS & A Level · Mathematics (9709) · Notes

Momentum AS

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Read first: Kinematics of motion in a straight line, Forces and equilibrium

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What you need to know

By the end of this topic you should be able to:

1. **Use the definition of momentum**, momentum $=$ mass $\times$ velocity, and understand that it is a **vector**: it has a direction as well as a size. Everything happens along one straight line, so the direction is just a sign, $+$ or $-$.
2. **Use conservation of momentum** for the **direct impact** of two bodies: two particles moving along the same straight line collide, and the total momentum just before the collision equals the total momentum just after. This includes collisions in which the two bodies **coalesce** (stick together and move on as one).

Impulse and the coefficient of restitution are **not** in the syllabus. Many questions also ask about the kinetic energy lost in a collision, or put the collision in the middle of a longer motion problem, so this note also shows how the kinetic energy formula and the constant acceleration formulae fit with momentum.

Understand it

What momentum is

The **momentum** of a particle of mass m kg moving with velocity v m s^{-1} is

$$\text{momentum} = mv.$$

Its unit is the newton second (N s), which is the same as kg m s^{-1}. A heavy object moving slowly can have as much momentum as a light one moving fast: 2 kg at 3 m s^{-1} and 0.5 kg at 12 m s^{-1} both have momentum 6 N s.

Momentum is a **vector**. Choose a positive direction along the line (to the right, say). A particle moving the other way has a **negative** velocity, so it has **negative** momentum. A 0.4 kg particle moving to the left at 5 m s^{-1} has velocity -5 and momentum $0.4 \times (-5) = -2$ N s. Speed is the size of the velocity, so it is never negative; velocity carries the sign.

Why momentum is conserved

When two particles collide, each pushes on the other. By Newton's third law the two pushes are equal in size and opposite in direction, and they last for exactly the same (very short) time. So whatever momentum one particle gains, the other loses. The **total** momentum of the two particles does not change:

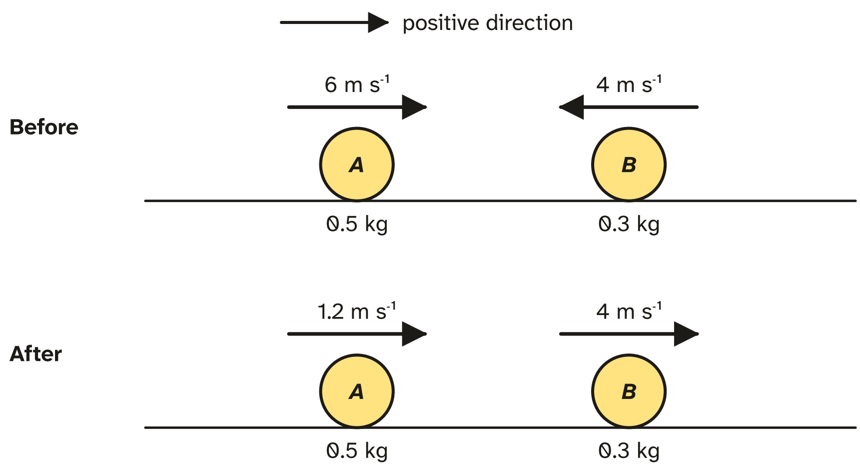
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2,$$

where u_1, u_2 are the velocities just before the impact and v_1, v_2 are the velocities just after. This is the **principle of conservation of momentum**.

It needs no outside force to act along the line during the collision. On a smooth horizontal plane that is exactly true. When the collision happens in the air, or on a slope or a rough floor, the impact is so quick that gravity and friction have no time to change the momentum during it, so you still use conservation of momentum **at the instant of the collision**. Gravity and friction matter before and after, which is where the constant acceleration formulae come in.

Drawing the collision

Always draw two pictures, "before" and "after", with each particle's mass and an arrow for its velocity, and mark your positive direction.



In the diagram, B moves left, so its velocity before is -4. Total momentum before $= 0.5 \times 6 + 0.3 \times (-4) = 3 - 1.2 = 1.8$ N s. After the collision, $0.5 \times 1.2 + 0.3 \times 4 = 0.6 + 1.2 = 1.8$ N s: the same. B's speed is 4 m s^{-1} both times, but its direction has reversed, so its momentum has changed from -1.2 to $+1.2$ N s. (This is Example 1 below.)

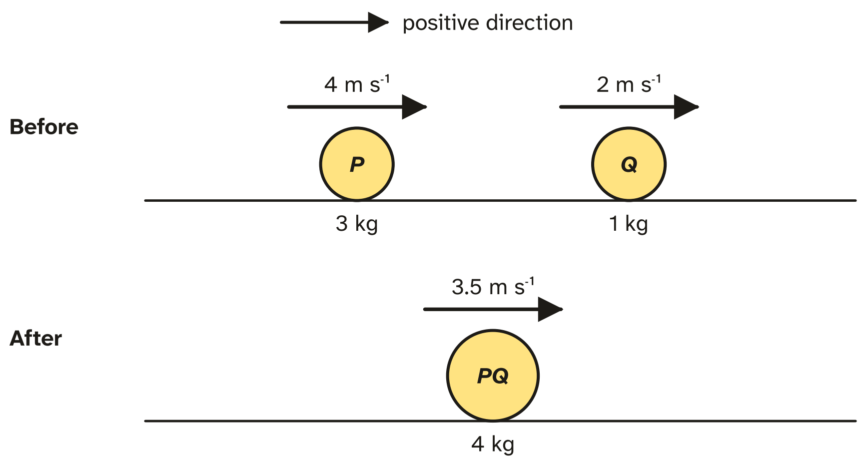
Some words tell you a velocity straight away:

- **"at rest"** or **"stationary"**: velocity 0, so that particle's momentum term is 0;
- **"is brought to rest"** in the collision: velocity 0 afterwards;
- **"rebounds"** or **"its direction is reversed"**: the sign of its velocity changes;
- **"continues in the same direction"**: the sign stays the same.

Coalescing particles

When particles **coalesce**, they move off together as one particle whose mass is the sum of the two masses. There is only one velocity after the impact:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v.$$



In the diagram P catches up with Q: $3 \times 4 + 1 \times 2 = 4v$, so $14 = 4v$ and $v = 3.5$ m s^{-1}.

Kinetic energy in a collision

Momentum is conserved in every collision, but **kinetic energy usually is not**. The kinetic energy (KE) of a particle is

$$\mathrm{KE} = \tfrac{1}{2}mv^2.$$

It is measured in joules (J) and it is a scalar: it has no direction, and it is never negative (squaring removes the sign of v). In a collision some kinetic energy turns into sound, heat and changes of shape, so the total KE afterwards is usually **less** than before. It can stay the same, but it can **never increase**.

For the diagram above: KE before $= \tfrac{1}{2}(3)(4^2) + \tfrac{1}{2}(1)(2^2) = 24 + 2 = 26$ J, KE after $= \tfrac{1}{2}(4)(3.5^2) = 24.5$ J, so 1.5 J of kinetic energy is lost.

This means you find velocities with **momentum**, never by assuming that kinetic energy is conserved. The KE is only used when the question talks about it.

If you know a particle's momentum p and its kinetic energy E, you can find its speed and mass: $E = \tfrac{1}{2}mu^2$ and $p = mu$, so $\dfrac{E}{p} = \dfrac{u}{2}$, which gives $u = \dfrac{2E}{p}$ and then $m = \dfrac{p}{u}$. For P in the diagram before the collision, $p = 12$ and $E = 24$ give $u = 4$ and $m = 3$, as they should.

Is a result possible?

After a collision the particles cannot pass through each other. So if A was behind B (on the left, with right positive), then afterwards A's velocity must be **less than or equal to** B's velocity: $v_A \leqslant v_B$. If A ended up faster to the right than B, A would have had to go through B. Together with "KE cannot increase", this lets you rule out impossible answers.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it. MF19 gives no momentum formulas at all.

Formula	Status
Momentum $= mv$, in N s (or kg m s^{-1}); a vector, so its sign shows its direction	Learn it
Conservation of momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$	Learn it
Coalescing: $m_1u_1 + m_2u_2 = (m_1 + m_2)v$	Learn it
Kinetic energy $= \frac{1}{2}mv^2$, in J	Learn it
KE lost in a collision $=$ total KE before $-$ total KE after	Learn it
From momentum p and kinetic energy E: $u = \dfrac{2E}{p}$, $m = \dfrac{p}{u}$	Learn it
$v = u + at,\ \ s = \frac{1}{2}(u+v)t,\ \ s = ut + \frac{1}{2}at^2,\ \ v^2 = u^2 + 2as$	Given
Weight $= mg$, with $g = 10$ m s^{-2}	Learn it
Newton's second law: resultant force $= ma$	Learn it
Friction when sliding: $F = \mu R$ (on a horizontal floor $R = mg$, so the deceleration is μg)	Learn it
Sliding freely on a smooth slope at angle α: acceleration $g\sin\alpha$ down the slope	Learn it
Rough slope: $R = mg\cos\alpha$, $F = \mu R$; acceleration $g\sin\alpha - \mu g\cos\alpha$ sliding down, deceleration $g\sin\alpha + \mu g\cos\alpha$ sliding up	Learn it
Total energy lost over a motion $=$ (PE $+$ KE at start) $-$ (PE $+$ KE at end) $=$ work against friction $+$ KE lost at impacts	Learn it
Potential energy gained when rising h: mgh; falling h from rest on a smooth track gives $v^2 = 2gh$	Learn it
Constant speed (no acceleration): distance $=$ speed $\times$ time	Learn it

How to do it

In the Paper 4 questions we have tagged for this topic (36 questions, 2021–2025), the types came up this often. 14 of the 36 questions mix two types, so the counts add up to 50.

Question type	Questions
A collision inside a longer motion problem	15
Kinetic energy lost in a collision	12
Several collisions, and whether or when particles meet again	9
One collision: find a speed, velocity, mass or constant	8
Two possible directions after a collision	6

Every type starts the same way, so learn this routine first:

1. **Draw** "before" and "after" diagrams with masses and velocity arrows. Choose the positive direction (usually the way the first particle is moving).
2. **Write the momentum equation**: total momentum before $=$ total momentum after, with a minus sign for every velocity pointing the negative way. Use **masses** only, never weights.
3. **Solve**, then read the sign of your answer: a positive velocity moves the positive way, a negative one moves the other way.
4. **Answer the question asked**: a speed is positive; a velocity needs its direction.

1. A collision inside a longer motion problem (15 questions)

The collision sits between two pieces of ordinary motion, so work in three stages.

1. **Before the collision:** find each particle's velocity **at the instant of the collision**, not its starting velocity. Use the constant acceleration formulae, Newton's second law, or energy.
2. **The collision:** conservation of momentum with those velocities.
3. **After the collision:** the new velocity (and, if they coalesced, the new combined mass) is the starting point for the next piece of motion.

Variations you will meet:

- **Two particles moving vertically.** Both have acceleration g downwards whatever their masses. Take upwards as positive and write each particle's height with $s = ut + \frac{1}{2}at^2$, using $a = -10$. They collide when their heights are equal; the $-5t^2$ terms cancel, so the equation is linear in t. Then find each velocity with $v = u + at$ at that time.

  For example, A (0.3 kg) is projected upwards from the ground at 20 m s^{-1}, and at the same instant B (0.2 kg) is projected downwards at 5 m s^{-1} from a point 25 m directly above. Heights: $20t - 5t^2 = 25 - 5t - 5t^2$, so $25t = 25$ and $t = 1$. They meet at height $20 - 5 = 15$ m. Velocities: A has $20 - 10 = 10$ (up), B has $-5-10=-15$ (down). If they coalesce, $0.3\times 10+0.2\times(-15)=0.5v$ gives $v=0$: the combined particle is momentarily at rest 15 m up, then falls, reaching the ground after $15=5t^2$, that is $t=\sqrt{3}=1.73$ s.

- **Projected at different times.** If B starts 1 second after A, B has been moving for $(t-1)$ seconds at time t. Use t in A's equations and $t-1$ in B's.
- **Speed at the ground afterwards.** Use $v^2=u^2+2as$ from the collision point to the ground, with the velocity just after the collision as u. With up positive, $a=-10$ and $s=-h$ (the ground is h below the collision point), so $v^2=u^2+20h$, whether the particle starts off moving up or down.
- **A smooth slope** at angle α: a particle sliding freely has acceleration $g\sin\alpha$ down the slope, whatever its mass. Take one direction along the slope as positive and keep to it for every velocity and acceleration.
- **A rough horizontal floor** with coefficient of friction μ: friction $F=\mu mg$ opposes the motion, so a sliding particle decelerates at μg (the mass cancels). Use $v^2=u^2+2as$ for the speed just before the collision and the distance to stop afterwards.
- **A rough slope** at angle α with coefficient of friction μ: the normal reaction is $R=mg\cos\alpha$ and friction is $F=\mu R$, acting against the motion. Sliding **down**, the resultant force down the slope is $mg\sin\alpha-F$; sliding **up**, the resultant force down the slope is $mg\sin\alpha+F$ (so the particle slows down). Divide by m: the accelerations are $g\sin\alpha-\mu g\cos\alpha$ and $g\sin\alpha+\mu g\cos\alpha$, whatever the mass, so a coalesced particle with the same μ has the same acceleration as before. Working backwards, a speed after a known distance gives a from $v^2=u^2+2as$, and then $\mu=\dfrac{g\sin\alpha-a}{g\cos\alpha}$.

  For example, with $\sin\alpha=0.6$, $\cos\alpha=0.8$ and $\mu=0.25$: sliding down, $a=10(0.6)-0.25(10)(0.8)=6-2=4$ m s^{-2}; sliding up, the deceleration is $6+2=8$ m s^{-2}. A 0.5 kg particle released from rest slides 2 m down this slope: $v^2=2(4)(2)=16$, so $v=4$ m s^{-1}. It then hits a 1.5 kg particle that was held at rest and is released at the instant of impact, and they coalesce: $0.5\times 4=2v$, so $v=1$ m s^{-1}.
- **"Total loss of energy" over a whole motion** means (PE $+$ KE at the start) $-$ (PE $+$ KE at the end). It counts the potential energy lost going downhill and the work done against friction, not just the KE lost at the impact. In the example above, from release until just after the collision: PE lost $=0.5\times 10\times(2\times 0.6)=6$ J and the KE at the end is $\frac{1}{2}(2)(1^2)=1$ J, so the total loss is $6-1=5$ J. As a check, the work against friction is $F\times d=(0.25\times 0.5\times 10\times 0.8)\times 2=2$ J, and the KE lost at the impact is $4-1=3$ J: $2+3=5$ J.
- **A falling object drives something into the ground** (a hammer on a nail, a weight on a stake). The object and the thing it hits move on together, so they coalesce: for example, an 80 kg block hitting a 20 kg stake at 6 m s^{-1} gives $80\times 6=100v$, so $v=4.8$ m s^{-1}. Then use Newton's second law on the **combined** mass, with **its weight** and the resistance: taking down as positive, $100\times 10-5000=100a$ gives $a=-40$, and $0=4.8^2+2(-40)s$ gives $s=0.288$ m. (A work–energy equation, $\frac{1}{2}(100)(4.8^2)+100\times 10\times s=5000s$, gives the same answer.) Working backwards, a given distance gives the deceleration and then the resistance.

- **A smooth curved track** before the collision: a particle released from rest that drops a height h reaches the bottom with $\frac{1}{2}mv^2 = mgh$, so $v^2 = 2gh$.
- **Meeting again later**: write each particle's displacement from the collision point in terms of the time since the collision, and set them equal (or make the gap between them zero).
- **Bouncing off the ground or a barrier**: the question tells you how the speed changes ("half the speed it had", "reduced by 90%"); the direction reverses. There is no momentum equation for hitting a fixed wall or the ground.

2. Kinetic energy lost in a collision (12 questions)

1. Find every unknown velocity first, using momentum.
2. **Total KE before** $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$, and **total KE after** in the same way (one term if they coalesced, with the combined mass).
3. **Loss** $=$ **before** $-$ **after.** Add both particles' energies before subtracting: the loss belongs to the whole system, not to one particle.

A negative velocity squared is positive: $(-6)^2 = 36$.

Variations you will meet:

- **The loss is given and there is an unknown.** Write the KE before and after in terms of the unknown (after using momentum to link the unknowns), then set before $-$ after equal to the given loss. A fraction or percentage lost means KE after $=$ (the fraction kept) $\times$ KE before; for 40% lost, KE after $= 0.6\times$ KE before. This often gives a **quadratic**. Reject a root that makes a particle move the wrong way, pass through the other particle, or have a negative mass, and say why (Example 2).
- **"Find the greater (or largest possible) loss"**: when the collision has two possible outcomes (type 5), find the loss for each and state which is bigger.
- **Two collisions**: the total loss is the KE at the very start minus the KE at the very end. If you are given the total loss, that gives an equation in the unknown.
- **"Show that the loss is ..."**: find both totals and subtract, with every value shown.

3. Several collisions, and whether or when particles meet again (9 questions)

1. **One momentum equation per collision**, with only the two particles in that collision. A particle that was not involved (for example the first one, now moving on its own) does not appear.
2. Use the velocity from the earlier collision as the "before" velocity in the next one.
3. Between collisions on a smooth horizontal plane there is **no acceleration**, so each particle moves at **constant speed**: distance $=$ speed $\times$ time. Don't use the constant acceleration formulae with a made-up acceleration.

Timing and distances:

- Time for a particle to cover a gap: gap $\div$ speed.

- Two particles moving the same way, one catching the other: time $=$ gap $\div$ (difference of speeds).
- Two particles moving towards each other: time $=$ gap $\div$ (sum of speeds).
- Work out **where each particle is** at the instant of each collision (measured from a fixed point), so you know the gap for the next stage. Example 4 does this.

Conditions:

- **"Its direction is reversed"** or **"it rebounds"**: if a particle was moving the negative way, its velocity afterwards is positive, so it is > 0; this gives an inequality. For example, A (2 kg, 3 m s^{-1}) hits B (m kg, at rest), and B moves off at 1.5 m s^{-1}. Then $6 = 2v_A + 1.5m$, so $v_A = \frac{6-1.5m}{2}$. A rebounds only if $v_A < 0$, which needs $6 - 1.5m < 0$, that is $m > 4$.
- **"There are no further collisions"**: after the last collision, the particle behind must not be moving faster in the same direction as the one in front (or they must be moving apart). If B bounces off a wall and comes back towards A at speed w_B, while A is moving away from the wall at speed w_A, there is no further collision when $w_B \leqslant w_A$. With unknowns, this gives an inequality; solve it and combine it with any other condition (such as the one above). If the question says an unknown is an integer, pick the integer values inside the range.
- **A wall or barrier**: the particle's direction reverses and its speed is multiplied by the fraction given ("a quarter of its speed" means $\times\frac{1}{4}$; "reduced by 75%" also means $\times\frac{1}{4}$).
- **Answers in terms of a letter** ("show that the speed of E is . . ."): carry the letter through each momentum equation exactly as if it were a number.

4. One collision: find a speed, velocity, mass or constant (8 questions)

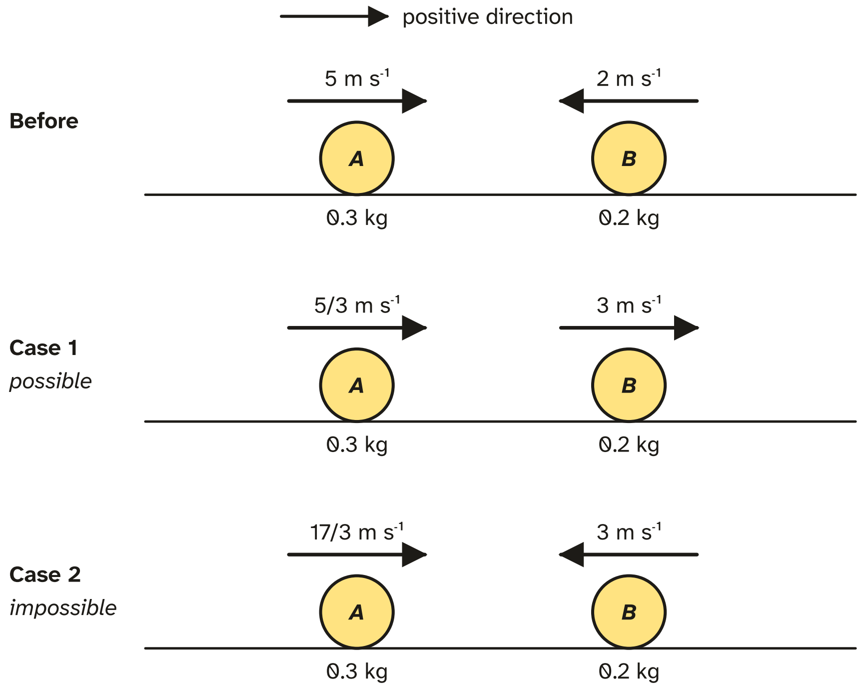
This is the basic routine on its own: draw, write the momentum equation, solve, give the answer with its direction.

Variations you will meet:

- **An unknown mass**, m or km: it appears in the momentum equation in the same way as a number; solve for it. If every mass is a multiple of m, divide through by m.
- **"Both are brought to rest"**: the total momentum after is 0, so the momentum before was 0 too.
- **Given momentum and kinetic energy**: use $u = \frac{2E}{p}$, then $m = \frac{p}{u}$.
- **"Both move in the same direction, and the difference in their speeds is d"**: the particle in front must be the faster one, so its velocity is the other's $+d$. For example, A (4 kg, 6 m s^{-1}) hits B (2 kg, at rest), and afterwards the difference in their speeds is 1.5 m s^{-1}: $24 = 4v_A + 2(v_A + 1.5)$, so $6v_A = 21$, $v_A = 3.5$ and $v_B = 5$ m s^{-1}.
- **Answers in terms of u**: if the speeds are given as multiples of u, the answer is a multiple of u.

5. Two possible directions after a collision (6 questions)

When the question gives a particle's **speed** after the collision but not its direction, there are two cases: it keeps its direction, or it is reversed. Write a momentum equation for each case, with $+$ and then $-$ for that velocity.



In the diagram, momentum before $= 0.3 \times 5 + 0.2 \times (-2) = 1.1$ N s, and B has speed 3 afterwards.

- B reversed ($+3$): $1.1 = 0.3v_A + 0.6$, so $v_A = \frac{5}{3}$ m s^{-1}. B is in front and faster, so this is fine.
- B not reversed (-3): $1.1 = 0.3v_A - 0.6$, so $v_A = \frac{17}{3}$ m s^{-1}. Now A (behind) moves right and B moves left: they would still be approaching each other, so A would have to pass through B. The KE after would also be 5.72 J, more than the 4.15 J before. **Impossible.**

Most exam questions of this type have **both** cases possible, and "find the two possible speeds" wants both answers, as speeds (positive). Only reject a case if it breaks one of the rules in "Is a result possible?", and say which.

Variations you will meet:

- **"The speeds of P and Q are equal after the collision"**: either they move together with the same velocity v, or they move apart with velocities $-v$ and v. Two equations, two values.
- **"The speed of Q is three times the speed of P"**: write P's velocity as v (or $-v$) and Q's as $3v$. For example, A (1 kg, 4 m s^{-1}) hits B (2 kg, at rest) and afterwards B's speed is three times A's. Same direction: $4 = v + 6v$, so A's speed is $\frac{4}{7} = 0.571$ m s^{-1}. A rebounds: $4 = -v + 6v$, so A's speed is 0.8 m s^{-1}.
- **The unknown is a starting speed**: with an after-speed whose direction is unknown, you get two values of the starting speed. Keep only the positive ones if it is a speed.
- **Then the KE**: find the loss in each case (type 2), or the one the question asks for.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **DM1** method mark that needs the M mark before it.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Take $g = 10$ m s^{-2} throughout.

Example 1

Particles A and B, of masses 0.5 kg and 0.3 kg, move towards each other along a straight line on a smooth horizontal plane, A with speed 6 m s^{-1} and B with speed 4 m s^{-1}. After they collide, A moves in the same direction as before with speed 1.2 m s^{-1}.

(a) Find the speed and direction of B after the collision. **[3]**

(b) Find the kinetic energy lost in the collision. **[3]**

(a) Take A's direction as positive, so B's velocity before is -4. Let B's velocity after be v (first diagram in "Understand it").

$$0.5 \times 6 + 0.3 \times (-4) = 0.5 \times 1.2 + 0.3v$$

M1 for a momentum equation with four terms and the masses (not weights), **A1** for the correct signs.

$1.8 = 0.6 + 0.3v$, so $v = 4$. B moves at 4 m s^{-1} **in A's original direction** (it has rebounded). **A1**

(b) KE before $= \frac{1}{2}(0.5)(6^2) + \frac{1}{2}(0.3)(4^2) = 9 + 2.4 = 11.4$ J. **B1**

KE after $= \frac{1}{2}(0.5)(1.2^2) + \frac{1}{2}(0.3)(4^2) = 0.36 + 2.4 = 2.76$ J. **M1** for before $-$ after with both particles in each total.

Loss $= 11.4 - 2.76 = 8.64$ J. **A1**

Example 2

A particle A of mass 2 kg moves with speed 6 m s^{-1} on a smooth horizontal plane towards a particle B of mass 1 kg, which is at rest. After the collision both particles move in the direction A was originally moving, and one quarter of the total kinetic energy is lost.

Find the speed of each particle after the collision. **[7]**

Take A's original direction as positive, and let the velocities after be v_A and v_B.

Momentum: $2 \times 6 = 2v_A + v_B$, so $v_B = 12 - 2v_A$. **M1 A1**

KE before $= \frac{1}{2}(2)(6^2) = 36$ J, so KE after $= \frac{3}{4} \times 36 = 27$ J. **B1**

$$\tfrac{1}{2}(2)v_A^2 + \tfrac{1}{2}(1)v_B^2 = 27 \;\Rightarrow\; v_A^2 + \tfrac{1}{2}(12 - 2v_A)^2 = 27.$$

M1 for the KE equation with v_B replaced.

$v_A^2 + 72 - 24v_A + 2v_A^2 = 27$, so $3v_A^2 - 24v_A + 45 = 0$, that is $v_A^2 - 8v_A + 15 = 0$ and $(v_A - 3)(v_A - 5) = 0$. **DM1** for reaching a three-term quadratic and solving it.

$v_A = 3$ gives $v_B = 6$; $v_A = 5$ gives $v_B = 2$. In the second case A, which is behind B, would be moving faster than B in the same direction, so A would have passed through B: reject it. **A1**

A moves at 3 m s^{-1} and B at 6 m s^{-1}. **A1** (Check: $\frac{1}{2}(2)(9) + \frac{1}{2}(1)(36) = 9 + 18 = 27$.)

Example 3

Particles A and B, of masses 2 kg and 3 kg, lie on a rough horizontal floor, 7.8 m apart. The coefficient of friction between each particle and the floor is 0.25. B is at rest and A is projected directly towards B with speed 8 m s^{-1}.

(a) Show that the speed of A just before it hits B is 5 m s^{-1}. **[3]**

After the collision A moves in the same direction with speed 1 m s^{-1}.

(b) Find the speed of B just after the collision. **[2]**

(c) Find the distance between A and B when both have come to rest. **[4]**

(a) Friction on A $= \mu R = 0.25 \times 2 \times 10 = 5$ N, opposing the motion. **B1**

Newton's second law: $-5 = 2a$, so $a = -2.5$ m s^{-2}. Then $v^2 = 8^2 + 2(-2.5)(7.8) = 64 - 39 = 25$. **M1**

$v = 5$ m s^{-1}. **A1**, the given answer with every step shown.

(b) Use A's speed **at the collision**, not its starting speed: $2 \times 5 = 2 \times 1 + 3v_B$. **M1**

$v_B = \frac{8}{3} = 2.67$ m s^{-1}. **A1**

(c) Each particle decelerates at $\mu g = 2.5$ m s^{-2} (friction on B is $0.25 \times 30 = 7.5$ N, and $7.5 \div 3 = 2.5$).

A: $0 = 1^2 - 2(2.5)s_A$, so $s_A = 0.2$ m. **M1**

B: $0 = \left(\frac{8}{3}\right)^2 - 2(2.5)s_B$, so $s_B = \frac{64}{45} = 1.42$ m. **M1**

B stays ahead of A the whole time (it starts faster and both slow down at the same rate), so they do not collide again. Both start from the collision point, so the distance between them is $\frac{64}{45} - \frac{9}{45} = \frac{55}{45} = \frac{11}{9}$. **M1**

Distance $= 1.22$ m (3 s.f.). **A1**

Example 4

Three particles A, B and C, of masses 0.5 kg, 0.2 kg and 0.3 kg, lie in a straight line in that order on a smooth horizontal plane. B and C are at rest, 1.5 m apart. A is projected towards B with speed 3 m s^{-1}. After A collides with B, A continues in the same direction with speed 1 m s^{-1}.

(a) Find the speed of B after this collision. **[2]**

B then collides with C, and B is brought to rest.

(b) Find the speed of C after this collision. **[2]**

(c) Find the time from the first collision until A reaches B again. **[3]**

(d) When A reaches B they coalesce. Show that the combined particle never catches C. **[2]**

(a) $0.5 \times 3 = 0.5 \times 1 + 0.2v_B$, so $1.5 = 0.5 + 0.2v_B$. **M1**

$v_B = 5$ m s^{-1}. **A1**

(b) Only B and C are in this collision: $0.2 \times 5 = 0.2 \times 0 + 0.3v_C$. **M1**

$v_C = \frac{10}{3} = 3.33$ m s^{-1}. **A1**

(c) Measure positions from the point of the first collision. B covers the 1.5 m to C in $1.5 \div 5 = 0.3$ s and then stops there. In those 0.3 s A moves $1 \times 0.3 = 0.3$ m, so A is not there yet. **B1**

The gap is now $1.5 - 0.3 = 1.2$ m and B is at rest, so A needs $1.2 \div 1 = 1.2$ s more. **M1**

Time $= 0.3 + 1.2 = 1.5$ s. **A1** (A simply travels the whole 1.5 m at 1 m s^{-1}.)

(d) $0.5 \times 1 + 0.2 \times 0 = 0.7w$, so $w = \frac{5}{7} = 0.714$ m s^{-1}. **M1**

C moves in the same direction at 3.33 m s^{-1}, which is faster than 0.714 m s^{-1}, and there is no friction, so the gap only grows: the combined particle never catches C. **A1**

Common mistakes

- **Getting the sign wrong when the particles move towards each other.** The examiner reports say this every year. If A moves right at 5 and B moves left at 3, the momentum before is $m_A(5) + m_B(-3)$, not $m_A(5) + m_B(3)$. Draw the diagram and mark the positive direction first.
- **Missing a rebound.** "A rebounds with speed 1" means A's velocity is -1. Writing $+1$ gives a wrong answer that can look reasonable.
- **Giving a velocity when a speed is asked for.** A speed is never negative. If your velocity comes out as -2.25, the speed is 2.25 m s^{-1}, and you should say which way it moves if the question wants a velocity.
- **Using kinetic energy instead of momentum** to find a velocity after a collision. Kinetic energy is usually lost, so "KE before $=$ KE after" is wrong.

- **Multiplying by g.** Momentum is mass $\times$ velocity. Writing mg instead of m in the momentum equation loses the accuracy marks.
- **Using the starting velocity instead of the velocity at the collision.** If a particle has been slowed by friction, or has been moving up or down under gravity, find its velocity at the instant of impact first.
- **Assuming the speed does not change at a collision.** A particle that hits another one comes out with a different velocity: always use momentum at a collision.
- **Finding the KE loss of one particle only**, or subtracting one particle's KE from the other's. The loss is (total before) $-$ (total after), for the whole system. Watch $(-6)^2$: it is $+36$.
- **Using the constant acceleration formulae when nothing is accelerating.** On a smooth horizontal plane each particle moves at constant speed between collisions: distance $=$ speed $\times$ time.
- **Getting the gap wrong** when working out when two particles meet. Find where each particle is at the moment the second one starts moving, then work with the gap between them.
- **Putting a third particle into a momentum equation.** Each collision involves two particles only.
- **Assuming particles coalesce** when the question does not say so (or forgetting the combined mass when they do).
- **Forgetting the weight or using the wrong mass** when a falling object drives something into the ground: use the combined mass and include its weight in Newton's second law.
- **Misreading "the speed of Q is twice the speed of P".** Q's speed is $2v$ when P's is v, and you still need to think about both directions.
- **Rounding too early.** Keep exact values (fractions, surds) until the end, and give the final answer to 3 significant figures.

How to get the marks

- **The momentum equation earns the method mark on sight**, even with a sign slip, as long as it has the right number of terms and each mass is paired with its own velocity. So write the whole equation before tidying it; don't jump to a rearranged fraction.
- **State your positive direction** (a diagram does this), and show directions in the answer: "4 m s^{-1} in the direction A was moving".
- **Speed or velocity?** A speed must be positive. A velocity needs a sign or a direction.
- **"Show that"** means the answer is given: show every step, including the momentum equation with the numbers in, and reach the printed value exactly.
- **"Hence"** or **"use the result from (a)"**: use that value in the next part.
- **Two possible answers**: when the question says "two possible", give both, each clearly labelled. If you reject a value (a negative mass, a particle passing through another, $m = 0$ when the question says $m \neq 0$), say why.
- **Inequalities**: a condition such as "the direction is reversed" or "no further collisions" needs an inequality, not an equation. Solve it and state the range (and the integer, if asked).
- **Kinetic energy**: write KE before and KE after in full so each can earn its mark, then subtract. A loss can be given as a positive number of joules.

- **Accuracy**: $g = 10$. Give answers exactly or to 3 significant figures, unless the question asks for something else (such as "3 decimal places" or "an exact fraction").

Quick check

1. A particle of mass 0.8 kg moves to the left at 5 m s^{-1}. Taking right as positive, find its momentum and its kinetic energy.
2. Particles P and Q, of masses 1.5 kg and 0.5 kg, move towards each other on a smooth horizontal plane with speeds 4 m s^{-1} and 6 m s^{-1}. They collide and coalesce. Find the velocity of the combined particle and the kinetic energy lost.
3. A particle A of mass 0.2 kg moves at 5 m s^{-1} towards a particle B of mass 0.6 kg, which is at rest. After the collision the particles move in opposite directions with equal speeds. Find this speed.
4. A particle A of mass 0.2 kg moves at 6 m s^{-1} towards a particle B of mass 0.4 kg, which is at rest. A rebounds with speed 2 m s^{-1}. B then hits a wall at right angles to its path and rebounds with a fraction f of its speed. Find the largest value of f for which A and B do not collide again.

Answers

1. Momentum $= 0.8 \times (-5) = -4$ N s. KE $= \frac{1}{2}(0.8)(5^2) = 10$ J (positive: KE has no direction).
2. Take P's direction as positive: $1.5 \times 4 + 0.5 \times (-6) = 2v$, so $3 = 2v$ and $v = 1.5$ m s^{-1} in P's original direction. KE before $= \frac{1}{2}(1.5)(16) + \frac{1}{2}(0.5)(36) = 12 + 9 = 21$ J. KE after $= \frac{1}{2}(2)(1.5^2) = 2.25$ J. Loss $= 18.75$ J.
3. A rebounds, so its velocity is $-v$ and B's is v: $0.2 \times 5 = 0.2(-v) + 0.6v = 0.4v$, so $v = 2.5$ m s^{-1}. (KE before $= 2.5$ J, after $= \frac{1}{2}(0.8)(2.5^2) = 2.5$ J, so no KE is lost: allowed.)
4. $0.2 \times 6 = 0.2 \times (-2) + 0.4v_B$, so $1.2 = -0.4 + 0.4v_B$ and $v_B = 4$ m s^{-1}. After the wall, B moves back towards A at $4f$, while A moves away at 2. No further collision needs $4f \leqslant 2$, so the largest value is $f = \frac{1}{2}$.

Past questions to try

- **9709/43/M/J/24 Q1**: two possible speeds after a collision.
- **9709/42/M/J/25 Q2**: particles that coalesce, then the kinetic energy lost.
- **9709/41/O/N/23 Q4**: friction before and after a collision, and the kinetic energy lost.
- **9709/42/M/J/24 Q6**: two collisions in a row, a total energy loss, then the times between events.