

Cambridge AS &amp; A Level · Mathematics (9709) · Notes

# Newton's laws of motion

AS

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Read first: [Forces and equilibrium](#), [Kinematics of motion in a straight line](#)

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## What you need to know

By the end of this topic you should be able to:

1. **Use Newton's laws of motion** for a particle of constant mass moving in a straight line under constant forces. The forces can include friction, the tension in a light inextensible string and the thrust (push) in a connecting rod. Air resistance or other resistances only appear when the question mentions them.
2. **Use weight**  $W = mg$ , taking  $g = 10 \text{ m s}^{-2}$ . Answers in this part of the course are mainly numerical.
3. **Solve problems about one particle moving with constant acceleration**, either vertically or on an inclined plane. This includes a particle on a rough plane, where the acceleration going up the plane is different from the acceleration coming down.
4. **Solve problems about connected particles**, for example two particles joined by a light inextensible string over a smooth pulley, or a car towing a trailer with a light rope or a light rigid tow-bar.

## Understand it

### The three laws

**Newton's first law.** An object keeps moving at a constant velocity (or stays at rest) unless a resultant force acts on it. So "moving at constant speed in a straight line" means the forces balance, exactly as in equilibrium.

**Newton's second law.** When the forces do not balance, the resultant force makes the object accelerate in the direction of that resultant:

$$\text{resultant force} = \text{mass} \times \text{acceleration}, \quad F = ma.$$

Force is in newtons (N), mass in kilograms (kg) and acceleration in  $\text{m s}^{-2}$ . A resultant of 12 N on a 4 kg box gives it an acceleration of  $3 \text{ m s}^{-2}$ .

**Newton's third law.** If  $A$  pushes or pulls on  $B$ , then  $B$  pushes or pulls on  $A$  with a force of the same size in the opposite direction. The floor pushes up on you as hard as you push down on it; a tow-bar pulls the trailer forward and pulls the car backward with the same force.

## Using $F = ma$ in practice

Almost every question in this topic is solved in the same way:

1. **Draw a force diagram** for each object, with every force on it and the direction of its acceleration.
2. **Choose the direction of motion as positive.** Resolve every force along that direction.
3. **Write "forces forwards – forces backwards =  $ma$ ".**
4. **At right angles to the motion there is no acceleration**, so the forces there balance. That is how you find the normal reaction  $R$ .
5. **Link to the motion** with the constant acceleration formulae ( $v = u + at$ ,  $s = ut + \frac{1}{2}at^2$ ,  $v^2 = u^2 + 2as$ ,  $s = \frac{1}{2}(u + v)t$ ).

The acceleration is the bridge. Sometimes you find it from the forces and then use it in the formulae; sometimes you find it from the motion ("it travels 2 m from rest in 4 s") and then use it in  $F = ma$  to find a force or  $\mu$ .

## Weight and the normal reaction

A mass of  $m$  kg has weight  $mg$  newtons, straight down. Mass is how much stuff there is; weight is a force. In  $F = ma$  the " $m$ " on the right is always a mass, never a weight.

On a horizontal surface with no other vertical forces,  $R = mg$ . As soon as another force pulls upwards at an angle, or the surface slopes,  $R$  changes, so always resolve at right angles to the surface to find it.

## Friction when something moves

While an object slides, friction is **limiting**:  $F = \mu R$ , and it acts **against the direction of motion**. On a rough slope this means friction points down the slope while the object goes up, and up the slope while it comes down. So the two accelerations are different.

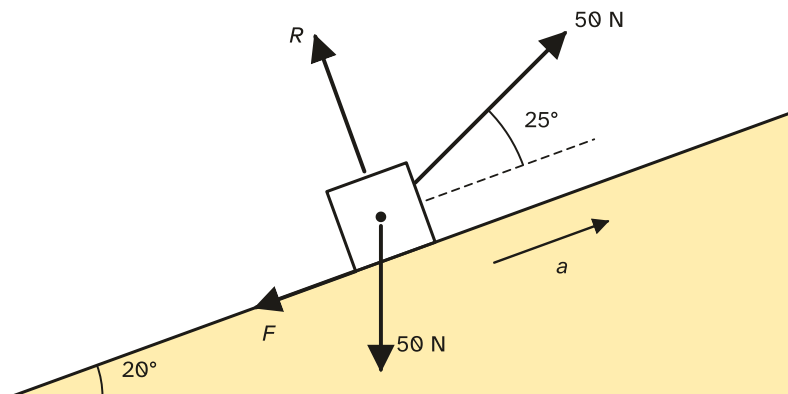
For a particle on a rough plane at angle  $\alpha$  with  $\sin \alpha = 0.6$ ,  $\cos \alpha = 0.8$  and  $\mu = 0.5$  (per kilogram,  $R = 8$  and friction = 4):

- going **up**, weight component and friction both act down the slope:  $a = -(6 + 4) = -10 \text{ m s}^{-2}$ ;
- coming **down**, friction acts up the slope:  $a = 6 - 4 = 2 \text{ m s}^{-2}$ .

A particle that stops on a slope only slides back if the weight component down the slope is bigger than the greatest friction,  $mg \sin \alpha > \mu mg \cos \alpha$ , that is if  $\mu < \tan \alpha$ . Here  $\mu = 0.5 < \tan \alpha = 0.75$ , so it slides back.

## A force pulling at an angle

Moving up the slope, so friction acts down it.  
The pull at  $25^\circ$  reduces  $R$ .

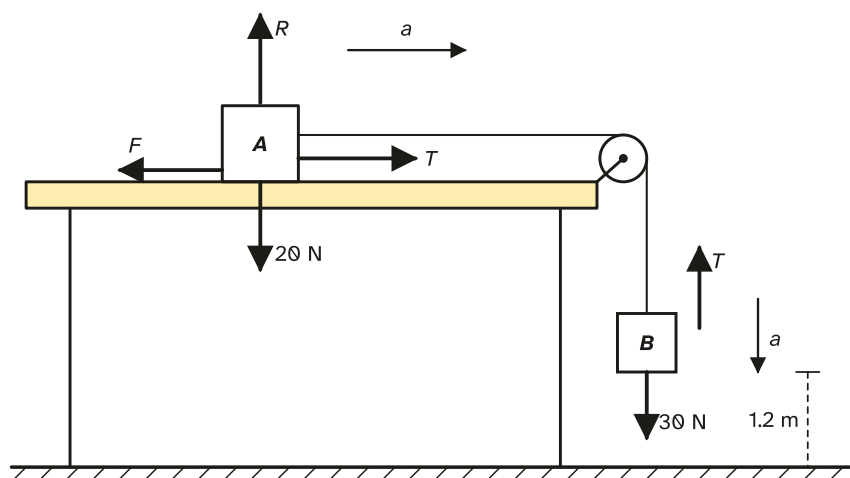


A pull at angle  $\theta$  above the direction of motion does two jobs:  $P \cos \theta$  drives the object along, and  $P \sin \theta$  lifts it slightly off the surface, which **reduces**  $R$  and so reduces friction. A push at angle  $\theta$  below the direction of motion **increases**  $R$ . Forgetting this term in  $R$  is one of the most common errors in the topic.

## Connected particles

Two particles joined by a **light inextensible** string move together, so they have the **same speed and the same size of acceleration**. "Light" means the string has no mass, so the tension is the same all along it; a **smooth** pulley does not change the tension either. So one letter  $T$  does for the whole string.

The same tension  $T$  acts on A and B, and they have the same size of acceleration  $a$ .



You can write  $F = ma$  for each particle separately, each in its own direction of motion, or for the **whole system** at once. In the system equation the tension does not appear (it pulls both ways inside the system and cancels) and the mass is the total mass. For two particles of 5 kg and 3 kg hanging over a smooth pulley:

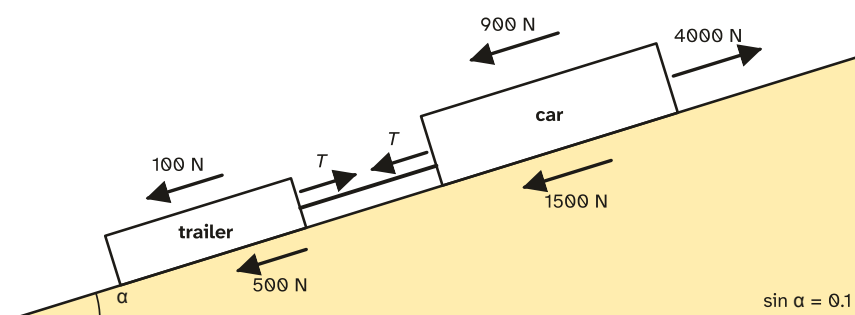
$$5g - T = 5a, \quad T - 3g = 3a \Rightarrow 2g = 8a, \quad a = 2.5 \text{ m s}^{-2}, \quad T = 3(10 + 2.5) = 37.5 \text{ N}.$$

Adding the two particle equations gave the system equation. You need two equations to find two unknowns, and at least one of them must be for a single particle if you want  $T$ .

## Towing: tension or thrust

A light rigid **tow-bar** can pull (tension) or push (thrust); a rope or cable can only pull. When a car brakes, the trailer may run into the tow-bar, which then pushes it back: the force is a thrust. If you take the force as a tension  $T$  and get a negative answer, it is a thrust of that size. A rope in the same situation would go **slack** (tension 0).

Weight components along the road: 1500 N on the car, 500 N on the trailer.  
Normal reactions and the rest of the weights are not shown.



## Vertical motion

For a particle moving vertically, the only forces are usually its weight and a tension or contact force. In a lift accelerating upwards at  $a$ , a person of mass  $m$  feels a floor reaction bigger than their weight:  $R - mg = ma$ . A 70 kg person in a lift accelerating upwards at  $0.5 \text{ m s}^{-2}$  has  $R = 70 \times 10.5 = 735 \text{ N}$ . If the lift accelerates downwards,  $R$  is less than  $mg$ .

A particle falling or thrown with no other force on it has acceleration  $g = 10 \text{ m s}^{-2}$  downwards. That is what happens to a hanging particle once its string goes slack.

## Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it.

| Formula                                                                                            | Status   |
|----------------------------------------------------------------------------------------------------|----------|
| $v = u + at, \quad s = \frac{1}{2}(u + v)t, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$ | Given    |
| Newton's second law: resultant force = $ma$                                                        | Learn it |
| Weight $W = mg$ , with $g = 10 \text{ m s}^{-2}$                                                   | Learn it |

| Formula                                                                                                                                                | Status   |
|--------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| On a slope at angle $\alpha$ : weight gives $mg \sin \alpha$ down the slope and $mg \cos \alpha$ into it                                               | Learn it |
| Sliding friction: $F = \mu R$ , against the direction of motion                                                                                        | Learn it |
| Rough slope: $a = g \sin \alpha - \mu g \cos \alpha$ sliding down; $a = -(g \sin \alpha + \mu g \cos \alpha)$ sliding up, with no other force along it | Learn it |
| Starts to slide or slides back only if $\mu < \tan \alpha$                                                                                             | Learn it |
| Driving force $= \frac{P}{v}$ , from power $P = Fv$                                                                                                    | Learn it |
| Constant speed (or greatest speed): resultant force $= 0$                                                                                              | Learn it |
| Connected by a light inextensible string: same tension throughout, same size of acceleration                                                           | Learn it |
| Newton's third law: equal and opposite forces on the two objects                                                                                       | Learn it |

The driving-force formula  $P = Fv$  belongs to the topic "Energy, work and power", but a third of the questions in this topic use it: power  $P$  in watts, speed  $v$  in  $\text{m s}^{-1}$ , driving force  $F$  in newtons.

## How to do it

In the Paper 4 questions we have tagged for this topic (76 questions, 2021–2025), the types came up this often. 16 of the 76 questions mix two types, so the counts add up to more than 76.

| Question type                                              | Questions |
|------------------------------------------------------------|-----------|
| Vehicle with driving force, resistance and power           | 24        |
| Particles connected over a pulley                          | 21        |
| Towing, and bodies joined in a line                        | 15        |
| One particle on an inclined plane                          | 14        |
| Motion after a string breaks or a particle hits the ground | 9         |
| A force at an angle on a horizontal surface                | 7         |
| Vertical motion: lifts and hanging particles               | 2         |

Every type uses the method from "Understand it": force diagram, resolve at right angles to find  $R$ ,  $F = ma$  along the motion, then the constant acceleration formulae. Keep full calculator accuracy and give answers to 3 significant figures.

## 1. Vehicle with driving force, resistance and power (24 questions)

- Driving force.** If the power  $P$  is given, the driving force at speed  $v$  is  $\frac{P}{v}$ . Change kW to W first (24 kW = 24 000 W).
- Newton's second law along the road:** driving force – resistance –  $mg \sin \alpha = ma$  going uphill; use  $+ mg \sin \alpha$  going downhill; no weight term on a level road.
- Constant (steady, greatest) speed** means  $a = 0$ , so driving force = resistance ( $+ mg \sin \alpha$  uphill).

For example, a car of mass 1200 kg works at 24 kW against a resistance of 800 N. At  $20 \text{ m s}^{-1}$  the driving force is  $\frac{24000}{20} = 1200 \text{ N}$ , so  $1200 - 800 = 1200a$  and  $a = \frac{1}{3} = 0.333 \text{ m s}^{-2}$ .

### Variations you will meet:

- Two situations, two unknowns** (for example the same power  $P$  and resistance  $R$  on the level at one speed and up a hill at another). Write one equation for each, then solve them together.
- Resistance that depends on speed**, such as  $kv$  or  $(340 + 4v) \text{ N}$ . At a steady speed  $\frac{P}{v} = kv$ , which gives  $P = kv^2$ ; with a sum you get a quadratic in  $v$ . Reject the negative root.
- "The power is increased by 9 kW"** means add 9000 W to the old power; **"increased to 40 kW"** means the new power is 40 000 W. Read which.
- "Instantaneous" acceleration** just after the power changes: the speed has not changed yet, so use the old speed in  $\frac{P}{v}$ .
- A driving force that is given directly** (no power): use it as it is.
- Find the angle of the hill:** get  $\sin \alpha$  from the equation, then  $\alpha$  in degrees.

## 2. Particles connected over a pulley (21 questions)

- Decide which way the system moves** (the heavier hanging side usually wins; if not obvious, see the variation below). Take that direction as positive for each particle.
- Write  $F = ma$  for each particle** along its own line of motion, with the same  $T$  and the same  $a$  in both.
  - hanging particle going down:  $m_1g - T = m_1a$ ;
  - particle on a rough table moving towards the pulley:  $T - \mu m_2g = m_2a$ ;
  - particle going up a rough slope:  $T - \mu m_2g \cos \alpha - m_2g \sin \alpha = m_2a$ .
- Add the equations** (the tensions cancel) to find  $a$ , then substitute back to find  $T$ .
- Use the constant acceleration formulae** for speeds, distances and times.

### Variations you will meet:

- Two strings and three particles** (a particle on a table or slope between two hanging ones). Each string has its own tension; all three have the same size of acceleration. Write one equation per particle.
- Two slopes meeting at the pulley:** each particle has its own weight component  $mg \sin \alpha$  and, if rough, its own friction.
- "Determine whether the particles move."** Assume they stay at rest, find the force trying to move them, and compare it with the greatest possible friction  $\mu R$ . If the pull is bigger, they move.

- **Given the tension or the acceleration, find a mass or  $\mu$ :** the same equations, solved for a different unknown.
- **Limiting equilibrium** ("on the point of moving"):  $a = 0$  and  $F = \mu R$ . This is the equilibrium topic, but it often opens a pulley question.
- **"Use an energy method":** the same set-up, but you must use work and energy (a later topic) rather than  $F = ma$ .

### 3. Towing, and bodies joined in a line (15 questions)

1. **System equation** (car and trailer together): driving force — all resistances — all weight components = (total mass)  $a$ . The tow-bar force does **not** appear.
2. **Trailer equation** (the easier one, as it has no driving force):  $T$  — trailer's resistance — trailer's weight component = (trailer's mass)  $a$ .
3. Solve for  $a$  and  $T$ . You can check with the car's own equation: driving force —  $T$  — car's resistance — car's weight component = (car's mass)  $a$ .

For example, a van of 2000 kg tows a trailer of 500 kg on level ground with a driving force of 3000 N and resistances 400 N and 100 N:  $3000 - 500 = 2500a$  gives  $a = 1$ , and the trailer gives  $T - 100 = 500$ , so  $T = 600$  N.

#### Variations you will meet:

- **$a$  and  $T$  are given, find a mass or a force.** Each single-body equation then has only one unknown, so solve them one at a time; there is no need for simultaneous equations.
- **Braking or no driving force.** The acceleration is negative. If the tow-bar force comes out negative, it is a thrust; say so and give its size.
- **"Least braking force for the rope to become slack."** The rope goes slack when  $T = 0$ , that is when the van decelerates at least as fast as the trailer would on its own. Find the trailer's deceleration with  $T = 0$ , then find the braking force that gives the van that deceleration.
- **Constant speed:**  $a = 0$  in every equation, so the tension just balances the trailer's resistance (and weight component).
- **Blocks joined by a rope on a slope, pulled by a force at an angle:** resolve the pull along the slope ( $P \cos \theta$ ) for the system; the rope tension comes from the other block's equation.
- **A string at an angle between two bodies:** only the component along the motion,  $T \cos \theta$ , accelerates the body; the other component changes the normal reaction.

### 4. One particle on an inclined plane (14 questions)

1. **At right angles to the plane:**  $R = mg \cos \alpha$ , minus the perpendicular part of any pull away from the plane (or plus a push into it).
2. **Along the plane:** forces in the direction of motion — forces against it =  $ma$ . Friction  $\mu R$  acts against the motion.
3. **Constant acceleration formulae** for the time, distance or speed asked for.

### Variations you will meet:

- **Motion given, find  $\mu$ .** Find  $a$  from the motion first (for example  $v^2 = u^2 + 2as$ ), then put it in  $F = ma$  to get  $F$ , and  $\mu = \frac{F}{R}$ .
- **Projected up, then comes back.** Two stages with different accelerations (see "Understand it"). Check whether it slides back at all by comparing  $mg \sin \alpha$  with  $\mu mg \cos \alpha$ .
- **A force down the slope, or at an angle to it:** resolve it into both directions, and remember it changes  $R$ .
- **Part of the slope is rough and part smooth:** find the speed at the change-over point, then start a new stage with the new acceleration.
- **Two particles on the same slope that collide:** find each one's acceleration and set up the positions with a common time  $t$ ; after the collision use momentum (a later topic).
- **A bead on a sloping wire** behaves exactly like a particle on a slope.

## 5. Motion after a string breaks or a particle hits the ground (9 questions)

1. **Stage 1 (connected):** find  $a$  as in type 2, then the speed  $v$  at the moment the string breaks or a particle hits the ground, using the distance or time given.
2. **Stage 2 (string slack):** each particle now moves **on its own**, so **find a new acceleration**:
  - a particle hanging in the air:  $g = 10 \text{ m s}^{-2}$  downwards;
  - a particle on a rough table: deceleration  $\mu g$ ;
  - a particle going up a slope: deceleration  $g \sin \alpha$  (smooth) or  $g \sin \alpha + \mu g \cos \alpha$  (rough).
3. **Use**  $v^2 = u^2 + 2as$  with final speed 0 for the extra distance before it stops.
4. **Add the stages.** "Greatest height above the ground" = starting height + distance risen while connected + extra distance risen freely.

### Variations you will meet:

- **A particle projected or given a push at the start:** stage 1 then begins with  $u \neq 0$ , and a particle may move one way, stop and come back. Put the displacement as a signed number in  $s = ut + \frac{1}{2}at^2$ .
- **"Find the time for which  $B$  is in motion"** or "the time until  $A$  is at rest": use  $v = u + at$  or  $s = ut + \frac{1}{2}at^2$  in each stage and add.
- **Time for which a particle is above a certain height:** solve  $s = ut + \frac{1}{2}at^2$  for the two times it passes that height and subtract.

## 6. A force at an angle on a horizontal surface (7 questions)

1. **Vertically:**  $R + P \sin \theta = mg$  for a pull at  $\theta$  above the horizontal (so  $R = mg - P \sin \theta$ ); for a push below the horizontal,  $R = mg + P \sin \theta$ .
2. **Horizontally:**  $P \cos \theta - \mu R = ma$ .
3. Constant acceleration formulae as needed.

### Variations you will meet:

- **A ring threaded on a rough horizontal rod** is the same set-up; if  $P \sin \theta$  is bigger than  $mg$  the rod pushes down on the ring, so  $R = P \sin \theta - mg$ .
- **Several forces at a point** (from the equilibrium topic): find the resultant's horizontal and vertical parts first, then use them as above.
- **"Distance travelled in the third second"**:  $s(3) - s(2)$ , from  $s = \frac{1}{2}at^2$  with  $u = 0$ .
- **Motion given, find  $\mu$  or  $P$** : find  $a$  from the motion first, then solve.

## 7. Vertical motion: lifts and hanging particles (2 questions)

1. Take the direction of acceleration as positive.
2. **Lift**: cable tension — total weight = total mass  $\times a$  for the lift and its load together; for something standing on the floor, reaction — its weight = its mass  $\times a$ .
3. **Two particles hanging one below the other, pulled up by a force**: use the system to find  $a$ , then the lower particle alone to find the tension between them.

A deceleration while moving up is an acceleration **downwards**, so use a negative  $a$  (or take down as positive).

## Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

### Example 1

A car of mass 1000 kg has an engine that works at a constant power of 18 kW. There is a constant resistance to motion of 600 N.

- (a) Find the greatest steady speed of the car on a straight horizontal road. **[2]**
- (b) Find the acceleration of the car on this road when its speed is  $20 \text{ m s}^{-1}$ . **[3]**
- (c) The car now goes up a straight hill inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = 0.05$ . The power and resistance are unchanged. Find the greatest steady speed up the hill. **[3]**

**(a)** At the greatest steady speed  $a = 0$ , so the driving force equals the resistance:  $\frac{18\,000}{v} = 600$ . **M1**

$v = 30 \text{ m s}^{-1}$ . **A1**

$$(b) \text{ Driving force} = \frac{18\,000}{20} = 900 \text{ N. B1}$$

$$900 - 600 = 1000a \text{ M1 (Newton's second law with three terms), so } a = 0.3 \text{ m s}^{-2}. \text{ A1}$$

$$(c) \text{ Weight component down the hill} = 1000 \times 10 \times 0.05 = 500 \text{ N. B1}$$

$$\frac{18\,000}{v} - 600 - 500 = 0 \text{ M1, so } v = \frac{18\,000}{1100} = 16.4 \text{ m s}^{-1} \text{ (3 s.f.). A1}$$

## Example 2

A car of mass 1500 kg tows a trailer of mass 500 kg up a straight hill inclined at an angle  $\alpha$  to the horizontal, where  $\sin \alpha = 0.1$ . They are joined by a light rigid tow-bar parallel to the road. The driving force of the car's engine is 4000 N, and there are resistances of 900 N on the car and 100 N on the trailer.

(a) Find the acceleration of the car and the tension in the tow-bar. [5]

(b) The driving force is removed. Find the force in the tow-bar at this instant, and state whether it is a tension or a thrust. [3]

The third diagram in "Understand it" shows these forces.

(a) Weight components down the hill: car  $1500 \times 10 \times 0.1 = 1500 \text{ N}$ , trailer 500 N.

$$\text{System: } 4000 - 900 - 100 - 1500 - 500 = 2000a \text{ M1 A1, so } a = \frac{1000}{2000} = 0.5 \text{ m s}^{-2}. \text{ A1}$$

$$\text{Trailer: } T - 100 - 500 = 500 \times 0.5 \text{ M1, so } T = 850 \text{ N. A1}$$

(Check with the car:  $4000 - 850 - 900 - 1500 = 750 = 1500 \times 0.5$ .)

(b) System:  $-900 - 100 - 1500 - 500 = 2000a$ , so  $a = -1.5 \text{ m s}^{-2}. \text{ B1}$

$$\text{Trailer: } T - 100 - 500 = 500 \times (-1.5) \text{ M1, so } T = -150.$$

The negative sign means the tow-bar pushes the trailer: it is a **thrust of 150 N. A1**

## Example 3

A block of mass 5 kg is pulled up a line of greatest slope of a rough plane inclined at  $20^\circ$  to the horizontal by a force of magnitude 50 N acting at  $25^\circ$  above the line of greatest slope. The coefficient of friction between the block and the plane is 0.4.

(a) Find the acceleration of the block. [6]

(b) The block starts from rest. Find its speed when it has moved 2 m up the plane. [2]

(c) When the block has moved 2 m, the force is removed. Find the further distance the block moves up the plane, and show that it then stays at rest. [4]

$$(a) \text{ At right angles to the plane: } R + 50 \sin 25^\circ = 5 \times 10 \cos 20^\circ. \text{ M1}$$

$$R = 46.985 - 21.131 = 25.854 \text{ N A1, so } F = 0.4 \times 25.854 = 10.341 \text{ N. B1}$$

$$\text{Along the plane: } 50 \cos 25^\circ - 5 \times 10 \sin 20^\circ - F = 5a \text{ M1 A1}$$

$$45.315 - 17.101 - 10.341 = 5a, \text{ so } a = 3.57 \text{ m s}^{-2} \text{ (3 s.f.). A1}$$

$$\text{(b) } v^2 = 0 + 2 \times 3.5746 \times 2 = 14.298 \text{ M1, so } v = 3.78 \text{ m s}^{-1}. \text{ A1}$$

**(c)** Without the pull,  $R = 50 \cos 20^\circ = 46.985 \text{ N}$ , so friction is now  $0.4 \times 46.985 = 18.794 \text{ N}$ , acting down the slope while the block still moves up. **B1**

$$-17.101 - 18.794 = 5a, \text{ so } a = -7.179 \text{ m s}^{-2}. \text{ M1}$$

$$0 = 14.298 + 2(-7.179)s \text{ gives } s = 0.996 \text{ m (3 s.f.). A1}$$

At rest, the pull down the slope is  $17.1 \text{ N}$ , less than the greatest friction  $18.8 \text{ N}$ , so the block stays at rest. **B1**

## Example 4

Particle  $A$  of mass  $2 \text{ kg}$  lies on a rough horizontal table. It is attached to a light inextensible string that passes over a small smooth pulley at the edge of the table. Particle  $B$ , of mass  $3 \text{ kg}$ , hangs from the other end of the string,  $1.2 \text{ m}$  above the floor. The coefficient of friction between  $A$  and the table is  $0.25$ . The system is released from rest with the string taut.

(a) Find the acceleration of the particles and the tension in the string. **[5]**

(b)  $B$  hits the floor and does not rebound. Find the total distance moved by  $A$  before it comes to rest, assuming it does not reach the pulley. **[5]**

The second diagram in "Understand it" shows this system.

**(a)** For  $A$ :  $R = 20 \text{ N}$ , so friction  $= 0.25 \times 20 = 5 \text{ N. B1}$

$$A: T - 5 = 2a. \quad B: 30 - T = 3a. \text{ M1 A1}$$

$$\text{Adding: } 25 = 5a, \text{ so } a = 5 \text{ m s}^{-2} \text{ A1, and } T = 30 - 15 = 15 \text{ N. A1}$$

**(b)** When  $B$  hits the floor:  $v^2 = 0 + 2 \times 5 \times 1.2 = 12. \text{ M1 A1}$

Now the string is slack, and friction alone slows  $A$ :  $-5 = 2a$ , so  $a = -2.5 \text{ m s}^{-2}. \text{ M1}$

$$0 = 12 + 2(-2.5)s \text{ gives } s = 2.4 \text{ m. M1}$$

$$\text{Total distance moved by } A = 1.2 + 2.4 = 3.6 \text{ m. A1}$$

## Common mistakes

- **Using the old acceleration after a string goes slack.** Once a particle hits the ground or a string breaks, every particle needs a new acceleration from its own forces. A hanging particle then has  $g$ ; a particle on a rough table or slope does not. Examiners see the stage-1 acceleration carried on, or  $g$  used for a particle

on a table, every year.

- **Putting the tension into the system equation.** Inside the car-and-trailer or two-particle system, the tension cancels. Only external forces belong in the system equation, and the mass is the total mass.
- **Using the driving force in the trailer's equation.** The engine pushes the car only. The trailer is pulled by the tow-bar.
- **Forgetting the angled force when finding  $R$ .** A pull at an angle above the surface reduces  $R$ ; writing  $R = mg$  (or  $mg \cos \alpha$  on a slope) anyway is a very common error, and it spoils the friction.
- **Friction in the wrong direction.** It always opposes the motion, so it changes direction when a particle on a slope stops and comes back.
- **Weight in place of mass.** In  $F = ma$ , the mass is in kg. Writing  $30a$  for a 3 kg particle (or the weight in the system's total mass) loses the method mark.
- **Mixing up  $\sin$  and  $\cos$  on a slope.** Down the slope is  $mg \sin \alpha$ ; into the slope is  $mg \cos \alpha$ .
- **"Increased by" read as "increased to"** for power, or the kW-to-W conversion missed.
- **Missing the direction of the answer.** If you get a negative acceleration, say "deceleration" or give its magnitude as the question asks; a negative tow-bar tension must be stated as a thrust.
- **Stopping too soon on "greatest height".** Add every stage and the starting height, and give the height above the ground if that is what is asked.
- **Rounding too early, or to 2 significant figures.** Keep full accuracy through each stage and give 3 significant figures at the end.

## How to get the marks

- **Each Newton's second law equation earns a method mark** if it has the right number of terms, is dimensionally correct (forces on each side, mass times acceleration) and uses the right mass. Write the equation with the numbers in, before simplifying.
- **Write two separate equations** for connected particles (or one particle and the system), and label which body each is for.
- **Find  $R$  by resolving** and show it: it usually earns its own mark before  $F = \mu R$  is used.
- **"Show that"** means the answer is given. Show every force term and do not work backwards from the answer.
- **"Find the tension and the acceleration"** means both. "The magnitude of the acceleration" means a positive number.
- **Later parts use your earlier answers.** A wrong acceleration used correctly in the constant acceleration formulae still earns the method marks, so keep going.
- **"Use an energy method"** means an answer by  $F = ma$  gets few or no marks, even if it is right.
- **Accuracy:** give answers to 3 significant figures (angles to 1 decimal place) and use  $g = 10$ . Values like  $\sin \alpha = 0.05$  are exact, so use them rather than an angle rounded in degrees.
- **"State"** (for example the tension when moving at constant speed) needs only the answer, but it must follow from the forces, such as 0 when nothing resists the trailer.

## Quick check

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1. Two particles of masses 0.7 kg and 0.3 kg are attached to the ends of a light inextensible string that passes over a smooth fixed pulley. The particles hang freely and are released from rest. Find the acceleration of the particles and the tension in the string.
2. A particle is projected up a line of greatest slope of a smooth plane inclined at  $30^\circ$  to the horizontal, with speed  $6 \text{ m s}^{-1}$ . Find the distance it travels up the plane and the time until it returns to its starting point.
3. A ring of mass 0.5 kg is threaded on a rough horizontal rod. The coefficient of friction is 0.4. A force of 6 N acts on the ring at  $30^\circ$  above the horizontal, in the vertical plane containing the rod. Find the acceleration of the ring.
4. A lift of mass 800 kg carries a woman of mass 60 kg. The tension in the lift cable is 9460 N. Find the acceleration of the lift and the force exerted on the woman by the floor.
5. A van of mass 2000 kg tows a trailer of mass 500 kg along a horizontal road with a light horizontal rope. The resistances are 400 N on the van and 150 N on the trailer. The engine is switched off and the driver brakes. Find the least braking force that makes the rope go slack.

## Answers

1.  $0.7g - T = 0.7a$  and  $T - 0.3g = 0.3a$ . Adding:  $4 = 1.0a$ , so  $a = 4 \text{ m s}^{-2}$ , and  $T = 0.3(10 + 4) = 4.2 \text{ N}$ .
2. Up the plane  $a = -10 \sin 30^\circ = -5 \text{ m s}^{-2}$ .  $0 = 36 - 10s$ , so  $s = 3.6 \text{ m}$ . It stops after  $\frac{6}{5} = 1.2 \text{ s}$ ; the plane is smooth, so it comes back with acceleration  $5 \text{ m s}^{-2}$  and takes another  $1.2 \text{ s}$ :  $2.4 \text{ s}$  in total.
3. Vertically:  $R = 0.5 \times 10 - 6 \sin 30^\circ = 2 \text{ N}$ , so friction  $= 0.8 \text{ N}$ . Horizontally:  $6 \cos 30^\circ - 0.8 = 0.5a$ , so  $a = 8.79 \text{ m s}^{-2}$ .
4. Lift and woman together:  $9460 - 860 \times 10 = 860a$ , so  $a = 1 \text{ m s}^{-2}$  upwards. Woman:  $R - 600 = 60 \times 1$ , so  $R = 660 \text{ N}$ .
5. With the rope slack ( $T = 0$ ), the trailer decelerates at  $\frac{150}{500} = 0.3 \text{ m s}^{-2}$ . The rope goes slack if the van decelerates at least this fast:  $B + 400 \geq 2000 \times 0.3 = 600$ , so the least braking force is  $200 \text{ N}$ .

## Past questions to try

- [9709/43/M/J/25 Q2](#): a van and trailer with a tow-bar, finding a resistance and the tension.
- [9709/41/M/J/25 Q1](#): a block pulled up a rough slope.
- [9709/42/F/M/25 Q4](#): particles over a pulley, then one reaches the ground.
- [9709/45/O/N/25 Q5](#): three particles, two strings and a rough slope.