Cambridge AS & A Level · Mathematics (9709) · Notes

Permutations and combinations AS

Read online at pastopic.com/cambridge/mathematics-9709/notes/permutations-and-combinations

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What you need to know

By the end of this topic you should be able to:

1. **Tell a permutation from a combination.** A permutation is an arrangement, where the order matters. A combination is a selection, where only who or what is chosen matters. Use this to count simple selections, such as the number of ways to choose a committee.
2. **Count arrangements of objects in a line**, including people standing in two or more rows. This includes:
   - **objects that repeat**, such as the letters of a word with two Es or three Ss;
   - **restrictions**, such as two people who must, or must not, stand next to each other, a letter that must go at each end, or a set number of letters between two others.

Arrangements in a circle are **not** in the syllabus. Probability questions in this topic just divide one count by another, which uses nothing beyond this note.

Understand it

Order matters, or it doesn't

Choosing a captain and a vice-captain from a team is a **permutation**: Ana as captain with Ben as vice-captain is different from Ben as captain with Ana as vice-captain. Choosing two players to carry the kit is a **combination**: "Ana and Ben" is the same pair whichever way round you say it.

Before any calculation, ask: **if I swapped two of the chosen things around, would I have something different?** Yes means arrangements, no means selections.

Arranging everything: factorials

Put 5 different books on a shelf. The first place can take any of the 5, the second any of the 4 left, and so on:

$$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120.$$

So n different objects can be arranged in a line in $n!$ ways. By definition $0! = 1$.

The same "multiply the choices for each place" idea is behind everything else in this topic.

Arranging some of them: permutations

Put 5 of 8 different books in a row: $8 \times 7 \times 6 \times 5 \times 4 = 6720$ ways. You stop after 5 places. This is written ${}^8P_5$, and

$${}^nP_r = n(n-1)\cdots(n-r+1) = \frac{n!}{(n-r)!}.$$

Choosing some of them: combinations

Now just **choose** 3 of the 8 books to take on holiday. Each set of 3 books can be arranged in $3! = 6$ orders, so the $8 \times 7 \times 6 = 336$ ordered choices count every set 6 times. The number of sets is $336 \div 6 = 56$. This is ${}^8C_3$, also written $\binom{8}{3}$:

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}.$$

Choosing 3 to take is the same as choosing 5 to leave behind, so ${}^8C_3 = {}^8C_5$. Your calculator has nPr and nCr keys.

Objects that repeat

The letters of BANANA can be arranged in fewer than $6!$ ways, because swapping two As makes no visible difference. If the three As and two Ns were all different, there would be $6! = 720$ arrangements. Each visible arrangement appears $3!$ times (the As swapped among themselves) $\times\ 2!$ times (the Ns swapped), so

$$\frac{6!}{3!\,2!} = \frac{720}{12} = 60.$$

In general, n objects with p of one kind alike, q of another alike, and so on, can be arranged in $\dfrac{n!}{p!\,q!\cdots}$ ways. **Divide by a factorial for every letter that repeats**, and only those.

Restrictions: things that must be together

Six people, including Ana and Ben, stand in a line, and Ana and Ben must stand next to each other. **Glue them into one block.** Now there are 5 items (the block and 4 others): $5!$ arrangements. Inside the block they can stand in $2!$ orders. Total $5! \times 2! = 240$.

If the block is made of **identical** letters (two Ss together), it has only one inside order, so don't multiply by $2!$.

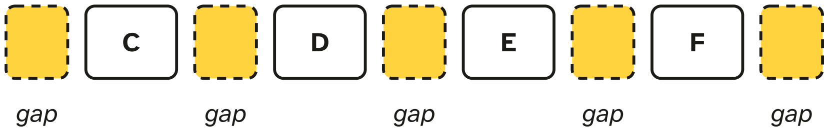
Restrictions: things that must be apart

There are two ways to count "Ana and Ben are **not** next to each other".

Subtract: all arrangements minus those with them together: $6! - 240 = 720 - 240 = 480$.

Gaps: arrange the other 4 people first, then put Ana and Ben into different gaps between or around them.

Step 1: arrange the other 4 people (4! = 24 ways)



Step 2: put A and B into 2 of the 5 gaps,
one person per gap (5 × 4 = 20 ways)

A and B are never next to each other: 24 × 20 = 480

$4! \times 5 \times 4 = 24 \times 20 = 480$. Same answer. If the two items put into gaps are **identical** (two Ls), choose the gaps with ${}^5C_2$ instead of ${}^5P_2$.

The gap method is the one to use when **several** items must all be kept apart ("no two girls next to each other"): arrange the others, then place each one in a different gap.

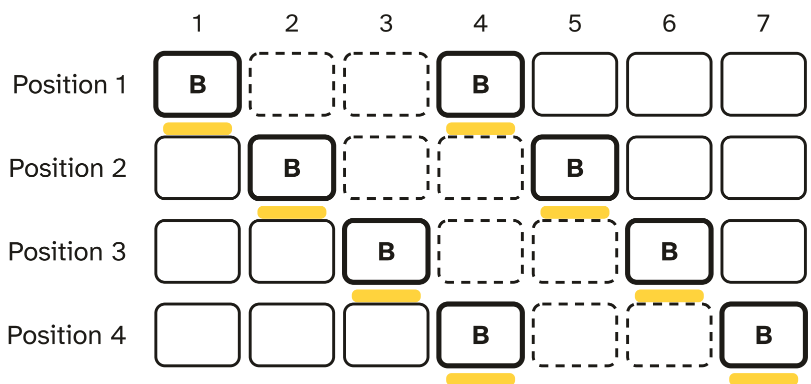
Restrictions: fixed places

"A vowel at each end", "Raman in the middle", "an R at the beginning and an R at the end": **fill the restricted places first**, then arrange whatever is left in the remaining places. If the letters at the ends are fixed (an S at each end of a word with two Ss), there is only 1 way to fill the ends, and you arrange the middle letters as a smaller word.

Rows work the same way. Ten people in a back row of 6 and a front row of 4 is just 10 places, so with no restrictions it is $10!$. If two people must be next to each other in the front row, count their positions in that row first (a row of 4 has 3 pairs of neighbouring places, and 2 orders in each), then fill the other 8 places: $3 \times 2 \times 8!$.

Restrictions: a set number of places apart

"Exactly 2 letters between the two Bs" in a 7-letter arrangement: the Bs and the two places between them form a block $\mathrm{B}\,\_\,\_\,\mathrm{B}$ that is 4 places long. Slide it along the row.



7 places − 2 between − 1 = 4 positions for the pair of Bs.
The other 5 letters fill the 5 empty places in every case.

With n places and exactly k between, there are $n-k-1$ positions for the pair. Then **arrange everything else in all the remaining places**, including the ones between. If the two items are different people, multiply by 2 for their order.

Selections with conditions

A committee of 4 from 5 men and 6 women with **at least** 3 **women**. Split it into **scenarios**, one for each allowed make-up:

Scenario	Ways
3 women, 1 man	${}^6C_3 \times {}^5C_1 = 20 \times 5 = 100$
4 women, 0 men	${}^6C_4 = 15$
Total	115

Within a scenario, the choices happen together, so **multiply**. **Across** scenarios, they are different outcomes, so **add**.

When "at least one" would need many scenarios, use the **complement**: all selections minus those with none. And when a particular person **must** be included, put them in and choose the rest from those left.

Selecting letters from a word

The letters of a word are selected, not arranged, so order doesn't matter, and identical letters can't be told apart. Choosing an O from COCONUT is the same selection whichever O you take. So split into scenarios by **how many of each repeated letter** you take, and use ${}^nC_r$ only for the letters that are all different.

Dividing into groups

Split 6 people into a group of 4 and a group of 2: choose the 4 (${}^6C_4 = 15$ ways) and the rest form the other group. With sizes 5, 4 and 3 from 12: ${}^{12}C_5 \times {}^7C_4 \times {}^3C_3$.

Groups of the **same size** need care. Splitting 6 people into two groups of 3: ${}^6C_3 = 20$ counts $\{A, B, C\}$ with $\{D, E, F\}$ once when you choose ABC and once when you choose DEF. The groups aren't labelled, so divide by $2!$: 10 ways. But if the two groups are **labelled** (team X and team Y, or teams sent to different towns), the 20 really are different, so don't divide.

Probability by counting

When every outcome is equally likely,

$$\mathrm{P(event)} = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes}}.$$

Count the top and bottom **in the same way**: both as arrangements, or both as selections, and with repeated letters treated the same way in both.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it.

Formula	Status
${}^nC_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$	Given
n different objects in a line: $n!$ ways; $0! = 1$	Learn it
${}^nP_r = \dfrac{n!}{(n-r)!}$, arrangements of r from n different objects	Learn it
n objects, p alike, q alike, ...: $\dfrac{n!}{p!\,q!\cdots}$ arrangements	Learn it
${}^nC_r = {}^nC_{n-r}$, and ${}^nP_r = {}^nC_r \times r!$	Learn it
Together: glue into a block, then $\times$ the block's inside orders	Learn it
Not together: total $-$ together; or arrange the rest, then use the $m+1$ gaps around m objects	Learn it
Exactly k between two items in n places: $n-k-1$ positions for the pair	Learn it
Scenarios: multiply within a scenario, add across scenarios	Learn it
Equal-sized unlabelled groups: divide by $r!$ for r groups of the same size	Learn it

How to do it

In the Paper 5 questions we have tagged for this topic (46 questions, 2021–2025), the types came up this often. Almost every question has several parts, and 36 of the 46 mix two or more types, so the counts add up to more than 46.

Question type	Questions
Arrangements with items together or not together	32
Selections with conditions	27
Probability by counting	21
Arrangements with fixed places (ends, middle, rows)	19
Items a set number of places apart	16
All arrangements of letters with repeats	15
Dividing into groups or teams	11
Codes and ordered choices	1

1. Arrangements with items together or not together (32 questions)

1. **Draw the line of places** with the restriction marked. Simple sketches like OOO _ _ _ keep you from missing a factor.
2. **"Together":** make the group one block. Count the items (block + the rest) and arrange them, dividing by factorials for letters that still repeat outside the block. Multiply by the block's inside orders: $r!$ for r different items, 1 for identical ones.
3. **"Not together", two items:** total arrangements minus arrangements with them together. Both counts must include every other condition in the question.
4. **"No two ... together"** (three or more items kept apart, or letters that must alternate): gap method. Arrange the other items, count the gaps (m items give $m+1$ gaps), and place the kept-apart items in different gaps: ${}^gP_r$ ways if they are different, ${}^gC_r$ if identical.

For example, KITTEN with the Ts not together: $\frac{6!}{2!} - 5! = 360 - 120 = 240$. By gaps: $4! \times {}^5C_2 = 24 \times 10 = 240$.

Variations you will meet:

- **Two conditions at once**, such as "the Es together and the Ds not together": treat the Es as a block **first**, then do "not together" for the Ds among the items you now have. Subtraction: (Es together) $-$ (Es together **and** Ds together).
- **"Not all together"** is the complement of "all together": two of the three may be next to each other. So it is total $-$ all three together. It is **not** the same as "no two together", which uses the gap method.
- **Several blocks**: all the men together and all the women together is $2!$ (order of the two blocks) $\times$ men's orders $\times$ women's orders. Each couple standing together: n blocks gives $n! \times 2^n$.
- **Letters in a set order** ("L, E, D together in that order"): the block has only 1 inside order.
- **Alternating** (consonant, vowel, consonant, ...): count the patterns that fit (with 5 and 5, the line can start with either kind, so 2), then arrange each kind in its own places.

2. Selections with conditions (27 questions)

1. **List every scenario** that meets the conditions, in a table with one column for each kind (for example S, H, T). Work systematically, for example from the most of one kind to the least, so none is missed and none is impossible.
2. **Count each scenario** as a product of combinations, one for each kind: ${}^8C_3 \times {}^5C_2 \times {}^{12}C_1$. Write the product before evaluating it.
3. **Add** the scenarios.

Variations you will meet:

- **One kind is fixed** ("exactly 1 drummer"): include ${}^5C_1$ in every scenario.
- **"At least one of each"** or "not both": complement. For example, two sisters who can't both be chosen: all groups minus groups containing both.

- **A particular person must be in**: include them and choose the rest from those left. "Exactly one of Mr and Mrs Lan": ${}^2C_1 \times$ (the rest chosen from the others).
- **More men than women** on a committee of 5: scenarios 5–0, 4–1, 3–2.
- **Letters from a word**: scenarios by how many of each **repeated** letter are taken; the letters that are all different are chosen with ${}^nC_r$. Letters that must be in every selection are put in first; don't multiply by a combination for them. For example, 4 letters from COCONUT containing both Cs: the other 2 come from O, O, N, U, T. Either both Os (1 way) or two different letters from O, N, U, T (${}^4C_2 = 6$), so 7 selections.

3. Probability by counting (21 questions)

1. **Total:** count all equally likely outcomes. For a random selection it is ${}^nC_r$; for a random arrangement it is the total number of arrangements.
2. **Favourable:** count the outcomes in the event, with the same model.
3. **Divide** and simplify. An exact fraction is fine; otherwise give 3 significant figures.

Variations you will meet:

- **Selecting letters at random**: treat every letter as different (the two Os are two separate tiles), so the total is ${}^nC_r$ for r letters from n. For example, 2 letters from 9 tiles where 4 are E and 2 are R (the rest all different): $\mathrm{P(same)} = \dfrac{{}^4C_2 + {}^2C_2}{{}^9C_2} = \dfrac{7}{36}$, so $\mathrm{P(different)} = \dfrac{29}{36}$.
- **A chain of fractions** works too: $\mathrm{P(first\ two\ are\ both\ O)} = \frac{3}{10} \times \frac{2}{9}$. Multiply by the number of orders when the chosen items are of different kinds.
- **Who ends up in the same group**: count the divisions where the named people share a group (one scenario per group they could share), and divide by all divisions. A quick check for two named people among n: add, over the groups, $\dfrac{\text{group size}}{n} \times \dfrac{\text{places left in that group}}{n-1}$ (the chance the first is in that group, times the chance the second joins her). For groups of 4, 3 and 2 from 9: $\frac{4}{9} \times \frac{3}{8} + \frac{3}{9} \times \frac{2}{8} + \frac{2}{9} \times \frac{1}{8} = \frac{5}{18}$. Only when all the groups are the same size does this reduce to $\dfrac{\text{places left in her group}}{n-1}$.
- **Conditional**, "given that the Cs are together": the total is only the arrangements with the Cs together.
- **Percentage**: the probability $\times 100$.

4. Arrangements with fixed places: ends, middle, rows (19 questions)

1. **Fill the fixed places first.** Count the ways to fill them (often 1, if the letters are identical or a named person is placed).
2. **Arrange the rest** in the places left, as a smaller problem with its own repeated letters and restrictions.

Variations you will meet:

- **A letter at each end and another restriction**: fix the ends, then solve "together / not together" for the middle letters, for example $\dfrac{8!}{2!\,3!} - \dfrac{6!}{2!}$ for the middle of SEYCHELLES with the three Es not all together.
- **"First and last letters different"**: total minus the arrangements with the same letter at both ends, one term for each repeated letter.

- **An adult at each end**, with 5 adults: ${}^5P_2 = 20$ ways to fill the ends. The two ends are different places, so use P, not C.
- **Two rows**: count the restricted row first (positions and orders), then arrange everyone else in all the other places.
- **Tables and seats**: treat each seat as a place. Choose who sits at each table, then arrange each table.

5. Items a set number of places apart (16 questions)

1. **Build the block**: the two items with k places between them.
2. **Count its positions**: $n - k - 1$ in a row of n.
3. **Arrange everything else in the remaining $n - 2$ places.** Multiply by 2 if the two items are different.

Variations you will meet:

- **A named letter must be between them** ("two letters between the Ss and one of them is C"): the C takes one of the 2 inside places ($\times 2$), then the rest fill the other places.
- **"At least k between"** or **"no more than k between"**: add the cases k, $k+1$, ... one by one, or subtract the cases you don't want from the total.
- **Another block as well** ("the two Ps together and exactly two letters between the Ss"): split into cases, such as the PP block inside or outside the S block, and add.
- **Three items spaced out** ("at least two people between any two hurdlers"): list the possible sets of places, for example $\{1, 4, 7\}$, $\{1, 4, 8\}$, $\{1, 5, 8\}$, $\{2, 5, 8\}$ in a line of 8; then multiply by the orders of the spaced items and of everyone else.

6. All arrangements of letters with repeats (15 questions)

1. **Count each letter.** Write the tally, for example HAPPINESS: H, A, P$\times 2$, I, N, E, S$\times 2$ (9 letters).
2. **Divide by a factorial for each repeat**: $\dfrac{9!}{2!\,2!} = 90\,720$.

This is usually part (a), worth 1 mark, and its answer is often reused later as a total or a denominator. Give the exact value.

7. Dividing into groups or teams (11 questions)

1. **Different sizes:** choose each group in turn from those left: ${}^{11}C_6 \times {}^5C_3 \times {}^2C_2$. Don't divide.
2. **Equal sizes, not labelled:** divide by $r!$ for r groups of the same size.
3. **Labelled groups** (named cars, different towns, table X and table Y): don't divide, even if sizes are equal.
4. **Conditions**: place the named people first, then fill up each group. For "each team has at least one man and one woman", list how the women can be spread over the teams (for example $1, 1, 3$ or $1, 2, 2$), count one spread, and multiply by the number of ways to assign it to the labelled teams (3 for each of these spreads).

8. Codes and ordered choices (1 question)

A code of 3 different letters from $\{A, \ldots, F\}$ then 2 different digits from 0 to 9: ${}^6P_3 \times {}^{10}P_2 = 120 \times 90 = 10\,800$, since order matters. "Codes containing A": use the complement: $10\,800 - {}^5P_3 \times 90 = 10\,800 - 5400 = 5400$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

The 7 letters of the word CABBAGE are arranged in a line.

(a) Find the number of different arrangements of the 7 letters. **[1]**

(b) Find the number of different arrangements in which the two Bs are next to each other. **[2]**

(c) Find the number of different arrangements that have an A at each end and in which the two Bs are not next to each other. **[3]**

(d) Find the number of different arrangements in which there are exactly 2 letters between the two Bs. **[3]**

(a) Two As and two Bs: $\dfrac{7!}{2!\,2!} = 1260$. **B1**

(b) Glue BB into one block: 6 items with two As. $\dfrac{6!}{2!} = 360$. **M1** for $6!$ divided by $2!$, **A1**.

(c) With an A at each end, the middle 5 places hold C, B, B, G, E.

All of them: $\dfrac{5!}{2!} = 60$. With the Bs together: $4! = 24$. **B1** for $4!$ with the ends fixed. **M1** for subtracting.

$60 - 24 = 36$. **A1**

(By gaps: arrange C, G, E in $3! = 6$ ways, then choose 2 of the 4 gaps for the identical Bs: $6 \times {}^4C_2 = 36$.)

(d) $\mathrm{B}\,\_\,\_\,\mathrm{B}$ has $7 - 2 - 1 = 4$ positions. **B1**

The other 5 letters C, A, A, G, E fill the remaining places in $\dfrac{5!}{2!} = 60$ ways. **M1** for $\times 4$.

$4 \times 60 = 240$. **A1**

Example 2

A choir has 7 sopranos, 5 altos and 4 tenors. A group of 6 singers is chosen.

(a) Find the number of ways the group can be chosen if it must contain at least 3 sopranos, at least 1 alto and at most 1 tenor. **[4]**

(b) Two of the sopranos, Ana and Bea, refuse to be in the group together. Find the number of ways a group of 6 can be chosen from all 16 singers with no other restrictions. **[2]**

(a)

Sopranos	Altos	Tenors	Ways
3	3	0	${}^7C_3 \times {}^5C_3 = 35 \times 10 = 350$
4	2	0	${}^7C_4 \times {}^5C_2 = 35 \times 10 = 350$
5	1	0	${}^7C_5 \times {}^5C_1 = 21 \times 5 = 105$
3	2	1	${}^7C_3 \times {}^5C_2 \times {}^4C_1 = 35 \times 10 \times 4 = 1400$
4	1	1	${}^7C_4 \times {}^5C_1 \times {}^4C_1 = 35 \times 5 \times 4 = 700$

M1 for a product of combinations for one correct scenario, **B1** for two correct values, **M1** for adding exactly these 5 scenarios.

Total $= 2905$. **A1**

(b) All groups: ${}^{16}C_6 = 8008$. Groups with both Ana and Bea: put them in and choose 4 more from 14: ${}^{14}C_4 = 1001$. **M1**

$8008 - 1001 = 7007$. **A1**

Example 3

5 boys and 4 girls stand for a photograph.

(a) They stand in a line. Find the number of arrangements in which the 4 girls stand together. **[2]**

(b) They stand in a line. Find the number of arrangements in which no two girls stand next to each other. **[3]**

(c) They now stand in two rows, 5 in the back row and 4 in the front row. Two of the girls, Hana and Iris, must stand next to each other in the front row. Find the number of arrangements. **[3]**

(a) The girls form one block: 6 items, $6!$ orders, and $4!$ orders inside the block. $6! \times 4! = 17\,280$. **M1 A1**

(b) Arrange the boys: $5! = 120$. **B1**

This makes 6 gaps. The 4 girls go into 4 different gaps, in order: ${}^6P_4 = 360$. **M1**

$120 \times 360 = 43\,200$. **A1**

(c) The front row has 4 places, so 3 pairs of neighbouring places, and Hana and Iris can stand either way round: $3 \times 2 = 6$. **B1**

The other 7 people fill the other 7 places: $7! = 5040$. **M1**

$6 \times 5040 = 30\,240$. **A1**

Example 4

Nine students, including Kim and Lee, are divided into groups.

(a) Find the number of ways in which the 9 students can be divided into three groups of 3. **[2]**

(b) The 9 students are divided at random into three groups of 3. Find the probability that Kim and Lee are in the same group. **[3]**

(c) The 9 students are instead divided at random into a group of 4, a group of 3 and a group of 2. Find the probability that Kim and Lee are in the same group. **[4]**

(a) ${}^9C_3 \times {}^6C_3 \times {}^3C_3 = 84 \times 20 = 1680$ counts each division $3! = 6$ times, since the groups are the same size and not labelled. **M1**

$1680 \div 6 = 280$. **A1**

(b) Kim and Lee's group needs 1 more student from the other 7: 7 ways. The remaining 6 make two groups of 3: ${}^6C_3 \div 2! = 10$ ways. **M1** for the count, **M1** for dividing by 280.

$$\mathrm{P} = \frac{7 \times 10}{280} = \frac{70}{280} = \frac{1}{4}.$$ **A1**

(Check: Lee must take one of the 2 free places in Kim's group, out of 8 places: $\frac{2}{8} = \frac{1}{4}$.)

(c) All divisions: ${}^9C_4 \times {}^5C_3 = 126 \times 10 = 1260$. **B1**

Kim and Lee together:

- in the group of 4: ${}^7C_2 \times {}^5C_3 = 21 \times 10 = 210$;
- in the group of 3: ${}^7C_1 \times {}^6C_4 = 7 \times 15 = 105$;
- in the group of 2: ${}^7C_4 = 35$ (the rest fill the other two groups).

M1 for one correct scenario, **M1** for adding all three.

$$\mathrm{P} = \frac{210 + 105 + 35}{1260} = \frac{350}{1260} = \frac{5}{18}.$$ **A1**

Common mistakes

- **Not dividing for repeated letters.** Every letter that repeats needs its own factorial underneath, including in the smaller arrangements after you fix the ends or glue a block. Examiners also see a repeated letter missed from the tally, so write the tally first.

- **Treating identical letters as different.** Two Ls can't swap to give something new, so don't multiply by $2!$ for a block of identical letters or use ${}^5P_2$ to place them in gaps. Extra factors of 2 are a common way to lose the accuracy mark.
- **Missing or impossible scenarios.** List scenarios in a fixed order. Check that each one meets every condition and can actually happen (a word with two Ds can't give three).
- **Multiplying by combinations for letters that must be there.** If every selection contains a D and an E, put them in and count only the choices for the other places.
- **Forgetting a factor inside a scenario**, such as the ${}^5C_1$ for "exactly one adult", or adding it when you should multiply.
- **Mixing up "not all together" and "no two together".** "The three Es are not all together" is total minus all three together. "No two Es together" needs the gap method.
- **Using C for places that are different**, such as choosing two adults for the two ends. The left end and the right end are different places, so it is ${}^5P_2$ (or 5×4).
- **Dividing groups when you shouldn't, or not dividing when you should.** Divide only for groups of the **same size** that are **not labelled**. Teams sent to different towns are labelled.
- **Counting the top and bottom of a probability differently**, for example the top as arrangements with repeats removed and the bottom as $9!$. Pick one model and use it for both.
- **Only answering part of the condition.** In "the Os together and the Ls not together", both counts in the subtraction must have the Os together.
- **Rounding a probability you could leave exact.** A fraction such as $\frac{7}{55}$ is a full answer; if you give a decimal, use 3 significant figures.

How to get the marks

- **Show the unsimplified expression** for every count, such as $\dfrac{8!}{2!\,3!} - \dfrac{6!}{2!}$ or ${}^8C_3 \times {}^5C_2 \times {}^{12}C_1$. Method marks are given for the right structure, even if a number later goes wrong.
- **Label every scenario** ("3S 2H 1T") next to its calculation. Mark schemes award marks for "correct identified scenarios", and a bare list of numbers may not show which scenario each one is.
- **Short diagrams** such as $\mathrm{D}\,\_\,\_\,\_\,\_\,\mathrm{D}\,\_\,\_\,\_$ or $\wedge\,\mathrm{C}\,\wedge\,\mathrm{G}\,\wedge\,\mathrm{E}\,\wedge$ show your method and help you avoid missing a factor.
- **Give exact integers** for counts. A 1-mark part (a) usually needs the exact value, for example $90\,720$, not $90\,700$.
- **Probabilities**: give an exact fraction or 3 significant figures. The denominator you use (such as ${}^9C_5$) should be visible.
- **"Find the number of different arrangements"** means you must allow for repeats and identical items; "selections" means order doesn't matter.
- **Reuse earlier answers** when the question builds on them (the total from part (a) is often the denominator later), but check that the earlier condition doesn't carry over unless the question says so.

Quick check

1. Find the number of different arrangements of the 9 letters of the word STATISTIC.
2. A committee of 5 is chosen from 6 men and 5 women. Find the number of committees with at least 3 women.
3. Eight people stand in a line. Find the number of arrangements in which Ali and Bo are not next to each other.
4. Find the number of arrangements of the 8 letters of the word PARALLEL in which there are exactly 3 letters between the two As.
5. Four of the 8 letters of PARALLEL are chosen at random. Find the probability that exactly one A is chosen.

Answers

1. S$\times 2$, T$\times 3$, $\mathrm{I}\times 2$, A, C: $\dfrac{9!}{2!\,3!\,2!} = \dfrac{362\,880}{24} = 15\,120$.
2. 3W 2M: ${}^5C_3 \times {}^6C_2 = 10 \times 15 = 150$. 4W 1M: ${}^5C_4 \times {}^6C_1 = 5 \times 6 = 30$. 5W: ${}^5C_5 = 1$. Total 181.
3. All: $8! = 40\,320$. Together: $7! \times 2! = 10\,080$. Not together: $40\,320 - 10\,080 = 30\,240$.
4. $\mathrm{A}\_\_\_\mathrm{A}$ has $8 - 3 - 1 = 4$ positions. The other letters P, R, L, L, E, L fill 6 places in $\dfrac{6!}{3!} = 120$ ways. Total $4 \times 120 = 480$.
5. Treat the 8 letters as different tiles: ${}^8C_4 = 70$ selections. Exactly one A: ${}^2C_1 \times {}^6C_3 = 2 \times 20 = 40$. $\mathrm{P} = \dfrac{40}{70} = \dfrac{4}{7}$.

Past questions to try

- **9709/53/O/N/25 Q2**: a committee with "at least" conditions, by scenarios.
- **9709/52/M/J/25 Q6**: letters with repeats, fixed ends, letters between, and letter selections.
- **9709/52/F/M/24 Q6**: a committee, blocks of men and women, and two rows.
- **9709/51/M/J/25 Q5**: selections by scenarios, then bands chosen in order.