Cambridge AS & A Level · Mathematics (9709) · Notes

Quadratics AS

Read online at pastopic.com/cambridge/mathematics-9709/notes/quadratics

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What you need to know

By the end of this topic you should be able to:

1. **Rewrite a quadratic as a number times a squared bracket plus a number** (for example $2x^2 - 8x + 5$ as $2(x-2)^2 - 3$), and read things from that form: where the turning point of the graph is, and how to draw the curve.
2. **Find the discriminant** $b^2 - 4ac$ and use it, for example to decide how many real roots $ax^2 + bx + c = 0$ has. You should know that a **repeated root** means the two roots are equal.
3. **Find the values of x that make a quadratic equal to zero, and the ranges of x that make it positive or negative.** You should be able to do this three ways: with factors, with the completed square, and with the quadratic formula.
4. **Find where a straight line and a quadratic curve cross**, by rearranging the line's equation and putting it into the curve's equation.
5. **Recognise an equation that hides a quadratic**, where one part is the square of another. Examples are $x^4 - 13x^2 + 36 = 0$ (think of x^2 as the unknown), $2x - 9\sqrt{x} + 4 = 0$ (think of $\sqrt{x}$) and $2\sin^2 x + \sin x = 1$ (think of $\sin x$). Solve for the hidden unknown, then work back to x.

Everything else about functions (domain, range, inverses, transformations of graphs) is in the Functions topic, but many of those questions start with completing the square from this topic.

Understand it

What a quadratic graph looks like

A quadratic is anything of the form $ax^2 + bx + c$ with $a \neq 0$. Its graph $y = ax^2 + bx + c$ is a smooth curve called a parabola.

- If $a > 0$ it is a U shape, with a lowest point.
- If $a < 0$ it is an upside-down U, with a highest point.

That turning point is the **vertex**. The graph is symmetrical about the vertical line through it, the **line of symmetry**. The graph crosses the y-axis at $y = c$ (put $x = 0$), and crosses the x-axis where $ax^2 + bx + c = 0$, which may happen twice, once or not at all.

Completing the square

The idea: any quadratic can be rewritten as "a number times a perfect square, plus a number". Start from a perfect square you know:

$$(x-2)^2 = x^2 - 4x + 4.$$

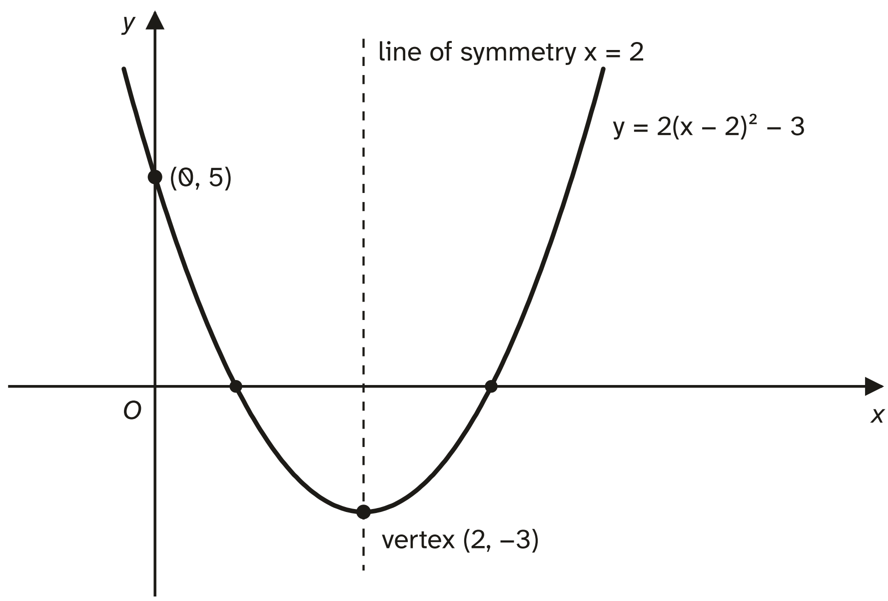
The number inside the bracket is **half the coefficient of** x, and squaring the bracket always makes an extra constant, (half)^2. So to write $x^2 - 4x$ as a square, use $(x-2)^2$ and take the extra 4 away:

$$x^2 - 4x = (x-2)^2 - 4.$$

When the x^2 term has a coefficient, take it out of the x terms first. For $2x^2 - 8x + 5$:

$$2x^2 - 8x + 5 = 2\left[x^2 - 4x\right] + 5 = 2\left[(x-2)^2 - 4\right] + 5 = 2(x-2)^2 - 3.$$

Why this is useful: $(x-2)^2$ is a square, so it is **never negative**, and it equals 0 only when $x = 2$. So $2(x-2)^2 - 3$ is smallest when $x = 2$, and its smallest value is -3. The vertex is $(2, -3)$, and the line of symmetry is $x = 2$.



In general, $y = a(x+p)^2 + q$ has its vertex at $(-p,\ q)$. Watch the sign: $(x-2)^2$ gives $x = +2$. If $a > 0$ the vertex is a minimum and q is the least value of y; if $a < 0$ it is a maximum and q is the greatest value.

The same form also solves equations. $2(x-2)^2 - 3 = 0$ gives $(x-2)^2 = \frac{3}{2}$, so $x - 2 = \pm\sqrt{\frac{3}{2}}$: square-root **both** signs. Completing the square on $ax^2 + bx + c = 0$ in general is exactly how the quadratic formula is made.

The discriminant: how many roots?

The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

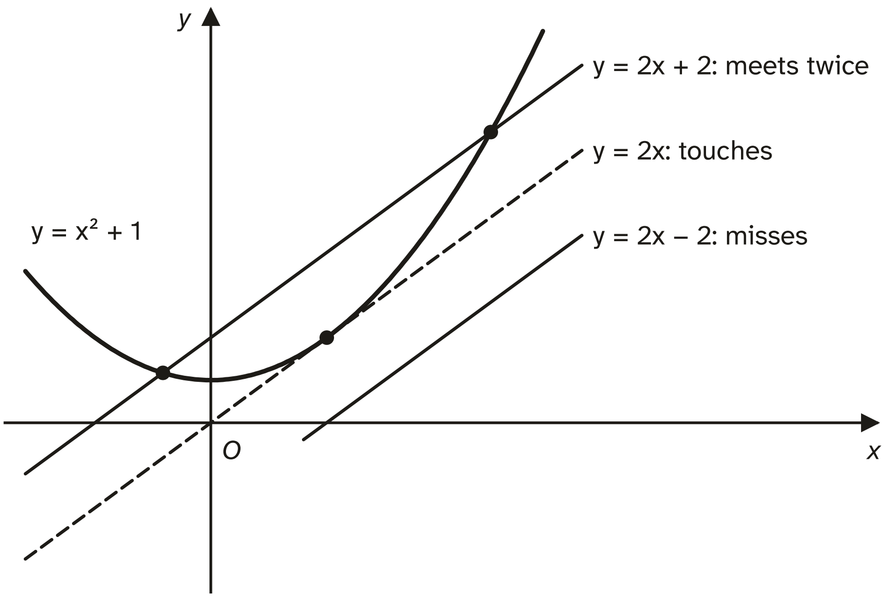
Everything depends on the part under the square root, $b^2 - 4ac$, called the **discriminant**:

- $b^2 - 4ac > 0$: the $\pm$ gives two different answers, so there are **two distinct real roots**. The graph cuts the x-axis twice.
- $b^2 - 4ac = 0$: the $\pm$ adds and takes away 0, so both answers are the same: **one repeated root** (the question may say "exactly one root" or "equal roots"). The graph just touches the x-axis at its vertex.
- $b^2 - 4ac < 0$: you cannot square-root a negative number, so there are **no real roots**. The graph misses the x-axis completely.

"Real roots" (with no word "distinct") means two distinct roots **or** a repeated root, so the condition is $b^2 - 4ac \geqslant 0$.

Where a line meets a curve

To find where the line $y = mx + c$ meets a curve, set the two expressions for y equal. When the curve is a quadratic, this gives a quadratic equation in x, and each root is the x-coordinate of a meeting point. So the discriminant of **that new equation** tells you how many times the line meets the curve:

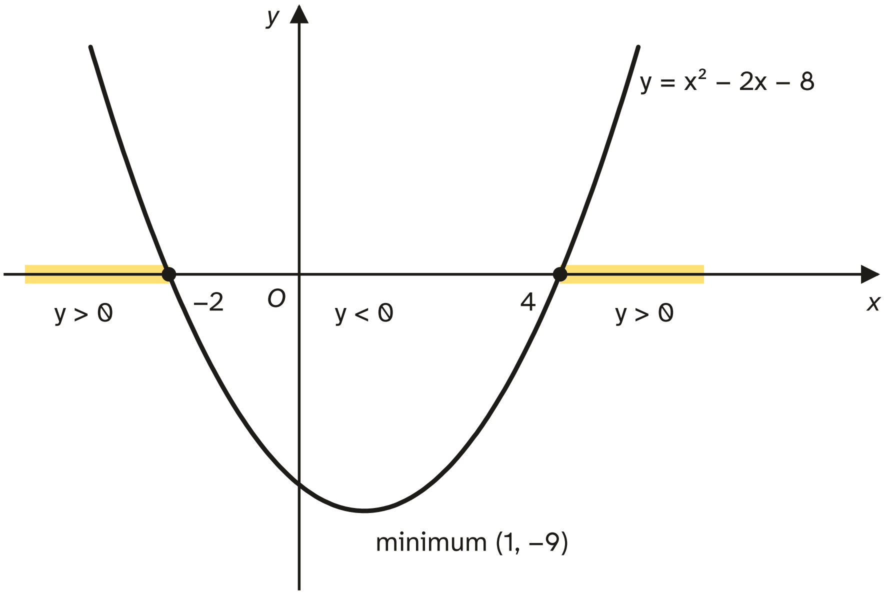


- Two points of intersection: $b^2 - 4ac > 0$.
- The line is a **tangent** (it touches the curve at one point): $b^2 - 4ac = 0$.
- The line and curve **do not meet**: $b^2 - 4ac < 0$.

Because the discriminant is usually an expression in an unknown constant (like k), these questions end with an equation or inequality in k, which is itself often a quadratic.

Quadratic inequalities

To solve $x^2 - 2x - 8 > 0$, think about the graph $y = x^2 - 2x - 8 = (x + 2)(x - 4)$. It is a U shape crossing the x-axis at -2 and 4 (the **critical values**). The curve is **above** the axis outside the roots and **below** it between them.



So $x^2 - 2x - 8 > 0$ when $x < -2$ or $x > 4$ (two separate pieces), and $x^2 - 2x - 8 < 0$ when $-2 < x < 4$ (one piece). Solving the equation only finds the critical values; the sketch tells you which side you want.

Equations in disguise

$x^4 - 13x^2 + 36 = 0$ doesn't look quadratic, but $x^4 = (x^2)^2$. Write $u = x^2$ and it becomes $u^2 - 13u + 36 = 0$, an ordinary quadratic. Solve for u, then go back: $x^2 = u$. The same trick works for anything where one power is double the other: x^6 and x^3, x and $\sqrt{x}$ (since $x = (\sqrt{x})^2$), $\cos^2\theta$ and $\cos\theta$. The last step always matters: some values of u are impossible, such as $x^2 = -3$, $\sqrt{x} = -1$ or $\cos\theta = 3$.

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it.

Formula	Status
Roots of $ax^2 + bx + c = 0$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Given
Discriminant $b^2 - 4ac$: > 0 two distinct real roots, $= 0$ one repeated root, < 0 no real roots	Learn it
Real roots (distinct or repeated): $b^2 - 4ac \geqslant 0$	Learn it
$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$	Learn it
$y = a(x+p)^2 + q$ has vertex $(-p,\ q)$ and line of symmetry $x = -p$; a minimum if $a > 0$, a maximum if $a < 0$	Learn it
Line of symmetry of $y = ax^2 + bx + c$: $x = -\dfrac{b}{2a}$	Learn it

Formula	Status
Line and curve: two points $\Leftrightarrow b^2 - 4ac > 0$; tangent $\Leftrightarrow b^2 - 4ac = 0$; do not meet $\Leftrightarrow b^2 - 4ac < 0$ (for the equation made by eliminating y)	Learn it
For $\alpha < \beta$: $(x-\alpha)(x-\beta) < 0 \Leftrightarrow \alpha < x < \beta$, and $(x-\alpha)(x-\beta) > 0 \Leftrightarrow x < \alpha$ or $x > \beta$	Learn it

How to do it

In the Paper 1 questions we have tagged for this topic (57 questions, 2021–2025), the types came up this often. Many questions mix two types.

Question type	Questions
Complete the square and use it	27
Use the discriminant for a line and a curve	19
Solve an equation that is quadratic in something else	11
Use the discriminant or the completed square for the roots of one quadratic	7
Solve a linear and a quadratic equation simultaneously	5
Solve a quadratic inequality	2
Find unknown constants from given roots	1

1. Complete the square and use it (27 questions)

To write $ax^2 + bx + c$ as $a(x+p)^2 + q$:

1. Take a out of the x^2 and x terms only: $a\left[x^2 + \frac{b}{a}x\right] + c$.
2. Inside the bracket, halve the coefficient of x and write $(x + \text{half})^2 - \text{half}^2$.
3. Multiply out the outer bracket and collect the constants.
4. Check by expanding your answer back.

If a is negative, take out the negative: $5 + 4x - x^2 = -\left[x^2 - 4x\right] + 5 = -\left[(x-2)^2 - 4\right] + 5 = 9 - (x-2)^2$.

If the form asked for is $(2x+a)^2 + b$, the bracket squared gives $4x^2$, so match the x term: for $4x^2 + 20x + 19$, $(2x+5)^2 = 4x^2 + 20x + 25$, so $4x^2 + 20x + 19 = (2x+5)^2 - 6$. Another safe way for any form: expand the form and **compare coefficients**.

Then use it. The later parts are usually one of these:

- **Vertex, minimum or maximum value:** read them off. $a(x+p)^2 + q$ has vertex $(-p, q)$.

- **Sketch:** a U (or upside-down U) with the vertex labelled, and the y-intercept $(0, c)$ if asked. See the first diagram in "Understand it".
- **Solve $ax^2 + bx + c = d$ "hence":** set the completed square form equal to d, rearrange to $(x+p)^2 = \text{number}$, square-root with $\pm$, then solve. Leave surds for "exact".
- **"$ax^2 + bx + c = k$ has no real roots" or "the line $y = k$ meets the curve once":** the curve's lowest point is at height q (when $a > 0$). A horizontal line $y = k$ misses it when $k < q$, touches it once when $k = q$, and crosses twice when $k > q$. For $a < 0$ it's the other way round.
- **Show an expression is always positive**, for example that a derivative $3x^2 - 12x + 20 = 3(x-2)^2 + 8$ is positive for all x (so the function is increasing): a square is never negative, so $3(x-2)^2 + 8 \geqslant 8 > 0$. Say that in words.
- **Range, inverse function, transformations:** the vertex gives the least or greatest value of the range, and the form $a(x+p)^2 + q$ shows the translation and stretch. The details of these are in the Functions topic.

2. Use the discriminant for a line and a curve (19 questions)

You are told whether a line and a curve meet twice, touch (tangent) or don't meet, or asked to show they always meet.

1. **Eliminate y:** set the curve's expression equal to the line's. If the curve has y mixed in (like $x^2 + xy = 6$), substitute the line's y into it instead.
2. **Tidy it into one quadratic with every term on one side:** $Ax^2 + Bx + C = 0$. Collect terms so that A, B and C are clear; they usually contain the unknown constant, for example $B = (3 - k)$.
3. **Apply the right condition to $B^2 - 4AC$:** > 0 for two points, $= 0$ for a tangent, < 0 for no meeting.
4. **Simplify** to an equation or inequality in the constant, and **solve it** by factorising, the formula or completing the square, showing the method.
5. **For an inequality**, find the critical values, then pick the right region from a sketch (between the roots for < 0, outside for > 0). Write it as $-1 < k < 7$ or as "$k < -3$ or $k > 5$".

Variations you will meet:

- **Tangent, then find the point.** Put each value of the constant back into your quadratic in x. It will be a perfect square (a repeated root), such as $(x-2)^2 = 0$. Find x, then y from the line.
- **"Show that they meet for all values of k".** Simplify the discriminant and show it can never be negative: for example $(k-1)^2 \geqslant 0$, or $4k^2 + 9 > 0$ because a square is never negative. End with a sentence: "so the line and curve meet for all values of k".
- **Choose which statement is true** ("intersect for all values / some values / no values of c"): the same method; decide from the sign of the discriminant.
- **The curve has x in a denominator**, such as $y = \frac{k}{x}$: multiply every term by x first, which gives a quadratic.
- **A constant rejected by the question.** If the question says k is positive (or the curve must still be a curve), reject a value such as $k = 0$ and say why.

3. Solve an equation that is quadratic in something else (11 questions)

1. **Spot the pair of powers,** one double the other: x^4 and x^2; x^6 and x^3; x and $\sqrt{x}$; $\cos^2\theta$ and $\cos\theta$.
2. **Clear fractions first** if there are any: multiply every term by the denominator, for example $x^2 - 3 - \frac{4}{x^2} = 0$ times x^2 gives $x^4 - 3x^2 - 4 = 0$. Then check you have three terms on one side.
3. **Substitute** a new letter, such as $u = x^3$ (not x again, which causes confusion).
4. **Solve the quadratic in** u by factorising or the formula, showing the working.
5. **Go back to x:** $x^3 = u$, $x^2 = u$ (both $\pm$ roots) or $\sqrt{x} = u$ (square it). **Reject impossible values**: x^2 can't be negative, $\sqrt{x}$ can't be negative, $\cos\theta$ and $\sin\theta$ must be between -1 and 1. A cube root of a negative number is fine: $x^3 = -8$ gives $x = -2$.

For a trigonometric equation, once you have the values of $\cos\theta$ or $\tan\theta$, finding the angles is the Trigonometry topic.

Also here: an equation like $2x - 1 = \frac{3}{x+1}$. Multiply both sides by $(x+1)$, expand fully, gather into a three-term quadratic and solve.

4. Use the discriminant or the completed square for the roots of one quadratic (7 questions)

- **"Find the set of values of k for which $2x^2 + kx + 8 = 0$ has two distinct real roots"**: write down $b^2 - 4ac$ with the coefficients as they are ($a = 2$, $b = k$, $c = 8$), apply > 0, and solve the inequality in k. When the equation looks like $\ldots = k$, first move k across so the right-hand side is 0: the constant term becomes $c - k$.
- **"Exactly one root"**: $b^2 - 4ac = 0$. If you have the completed square $a(x+p)^2 + q$, then $ax^2 + bx + c = k$ has one root when $k = q$, and that root is $x = -p$ (the vertex).
- **"$f(x) > 2$ for all x"**: complete the square, $f(x) = (x+p)^2 + q$. Its least value is q, so you need $q > 2$. (Or: $f(x) - 2 = 0$ has no real roots, so its discriminant is < 0.)

5. Solve a linear and a quadratic equation simultaneously (5 questions)

1. **Rearrange the linear equation** to make x or y the subject. Choose whichever avoids fractions.
2. **Substitute** into the quadratic equation, expand the brackets carefully (square the whole bracket: $(5 - x)^2 = 25 - 10x + x^2$), and collect into a three-term quadratic $= 0$.
3. **Solve** by factorising or the formula, showing the method.
4. **Find the other coordinate** for **each** root, using the linear equation.
5. **Write the answers in pairs**: $(2, 3)$ and $(3, 2)$, not $x = 2, 3$ and $y = 2, 3$.

Known meeting points. If you are told the x-coordinates (or a whole point) where a line meets a curve, and the equations contain unknown constants, substitute each known point into the equations to get one equation for each unknown. Two unknowns need two equations. Don't use the discriminant: it's for "how many points", not "which points".

6. Solve a quadratic inequality (2 questions)

1. Rearrange so one side is 0: for example $2a^2 - a - 6 \geqslant 0$.
2. Solve the equation $= 0$ to find the critical values, showing the method.
3. Sketch the U-shaped graph through the critical values, or remember: > 0 (or $\geqslant 0$) is **outside** the roots; < 0 (or $\leqslant 0$) is **between** them.
4. Keep the same kind of inequality sign as the question: strict $<$ stays strict, $\leqslant$ includes the critical values.
5. Apply any other condition the question gives, such as $a > 0$: then only the positive part of the answer is kept.

The discriminant questions in types 2 and 4 all end with this step.

7. Find unknown constants from given roots (1 question)

You are told the roots of a quadratic such as $ax^2 + 6x + c = 0$ (possibly in terms of another letter, like $2m$ and $-3m$) and must find the constants.

1. Substitute each root into the equation. That gives one equation per root.
2. Solve the equations simultaneously, for example by eliminating one constant. Dismiss an impossible value such as $m = 0$ if it would make the two roots equal.

Another way: a quadratic with roots α and β is $p(x - \alpha)(x - \beta)$. Expand it and compare coefficients with the given equation.

Worked examples

Example 1

(a) Express $2x^2 - 12x + 23$ in the form $a(x + b)^2 + c$, where a, b and c are constants. **[2]**

(b) State the coordinates of the vertex of the graph of $y = 2x^2 - 12x + 23$. **[1]**

(c) Find the set of values of the constant k for which the equation $2x^2 - 12x + 23 = k$ has no real roots. **[1]**

(d) Hence find the exact solutions of the equation $2x^2 - 12x + 23 = 11$. **[2]**

(a) Take the 2 out of the x terms and complete the square inside:

$$2x^2 - 12x + 23 = 2\left[x^2 - 6x\right] + 23 = 2\left[(x - 3)^2 - 9\right] + 23 = 2(x - 3)^2 + 5$$

B1 for $2(x - 3)^2$, **B1** for $+5$. Check: $2(x^2 - 6x + 9) + 5 = 2x^2 - 12x + 23$. ✓

(b) $(x - 3)^2$ is least (0) when $x = 3$, so the vertex is $(3, 5)$. **B1**

(c) The lowest point of the curve is at height 5, so the horizontal line $y = k$ misses the curve when $k < 5$. **B1**

(d) Use part (a): $2(x-3)^2+5=11$, so $(x-3)^2=3$. **M1** for using the completed square form as far as $(x-3)^2=\ldots$

$$x-3=\pm\sqrt{3}, \qquad x=3\pm\sqrt{3}$$

A1, both roots, exact.

Example 2

A curve has equation $y=x^2+3x+3$ and a line has equation $y=kx-1$, where k is a constant. Find the set of values of k for which the curve and the line do not meet. **[4]**

Eliminate y: $x^2+3x+3=kx-1$, so

$$x^2+(3-k)x+4=0.$$

B1 for a correct three-term quadratic with all terms on one side.

The curve and line do not meet when this has no real roots: $b^2-4ac<0$, with $a=1$, $b=3-k$, $c=4$:

$$(3-k)^2-16<0$$

M1 for using the discriminant with these coefficients.

$$k^2-6k-7<0, \qquad (k-7)(k+1)<0$$

The critical values are $k=-1$ and $k=7$. **A1**

The graph of $(k-7)(k+1)$ is a U shape, below the axis between its roots:

$$-1<k<7$$

A1, one interval with strict signs. (Writing "$k<-1$ or $k>7$" would be the region where the line crosses the curve twice.)

Example 3

Solve the equation $2\sqrt{x}=3+\dfrac{20}{\sqrt{x}}$. **[4]**

Clear the fraction: multiply every term by $\sqrt{x}$ (which is not zero, since it is in a denominator).

$$2x=3\sqrt{x}+20, \qquad 2x-3\sqrt{x}-20=0$$

M1 for multiplying through and reaching three terms on one side.

This is a quadratic in $\sqrt{x}$, because $x=(\sqrt{x})^2$. Let $u=\sqrt{x}$:

$$2u^2-3u-20=0$$

M1 for recognising the quadratic in $\sqrt{x}$.

$$(2u+5)(u-4)=0, \qquad u=4 \text{ or } u=-\frac{5}{2}$$

M1 for solving the quadratic by factorising (or the formula).

$\sqrt{x}$ can't be negative, so $u = -\frac{5}{2}$ gives no solution. From $\sqrt{x} = 4$:

$$x = 16$$

A1, the one value only, with $u = -\frac{5}{2}$ rejected.

Check: $2\sqrt{16} = 8$ and $3 + \frac{20}{4} = 8$. ✓

Example 4

The line $y = mx + 2$ is a tangent to the curve $y = x^2 - 2x + 6$. Find the possible values of the constant m and the coordinates of the corresponding points where the line touches the curve. **[6]**

Eliminate y: $x^2 - 2x + 6 = mx + 2$, so

$$x^2 - (2 + m)x + 4 = 0.$$

M1 for equating and collecting terms on one side.

The line is a tangent, so this has a repeated root: $b^2 - 4ac = 0$.

$$(2 + m)^2 - 16 = 0$$

DM1 for using the discriminant $= 0$ with these coefficients.

$$2 + m = \pm 4, \qquad m = 2 \text{ or } m = -6$$

A1

When $m = 2$: $x^2 - 4x + 4 = 0$, so $(x - 2)^2 = 0$ and $x = 2$. Then $y = 2(2) + 2 = 6$. **DM1** for substituting a value of m back and solving.

When $m = -6$: $x^2 + 4x + 4 = 0$, so $(x + 2)^2 = 0$ and $x = -2$. Then $y = -6(-2) + 2 = 14$.

The points are $(2, 6)$ when $m = 2$ and $(-2, 14)$ when $m = -6$. **A1 A1**

Check with the curve: $(2)^2 - 2(2) + 6 = 6$ ✓ and $(-2)^2 - 2(-2) + 6 = 14$ ✓.

Common mistakes

- **Solving a quadratic on the calculator with no working.** Examiners repeatedly say that answers straight from a calculator's equation solver get no credit for the method. Show the factorisation, the formula with numbers in it, or the completed square, even for easy quadratics.
- **Getting b wrong when completing the square with $a \neq 1$.** For $3x^2 - 18x + 20$, take out the 3 first: $3\left[x^2 - 6x\right] + 20 = 3\left[(x - 3)^2 - 9\right] + 20 = 3(x - 3)^2 - 7$. Forgetting to multiply the -9 by 3 gives $+11$ instead of -7. Expand your answer to check.
- **The vertex sign.** $(x - 3)^2 + 5$ has its vertex at $(3, 5)$, not $(-3, 5)$.

- **Ignoring "hence".** If part (a) gave a completed square, use it in part (b). Examiners report many students restarting with the formula or the discriminant, which takes longer and often loses marks when "hence" is required.
- **Using the discriminant of the wrong equation.** For a line and a curve, the discriminant is of the equation you get **after** eliminating y, with every term on one side. Using the curve's own $b^2 - 4ac$ scores nothing.
- **Bracket and sign errors in b and c.** When $b = (3 - k)$ or $c = (6 - k)$, keep the brackets: $b^2 = (3 - k)^2$, not $9 - k^2$. Collect all terms on one side before reading off a, b and c.
- **The wrong condition.** "Do not meet" is < 0, not > 0; "tangent" or "exactly one root" is $= 0$. For "$f(x) > 2$ for all x" using the discriminant, it is $f(x) - 2$ that has no real roots, so its discriminant is < 0.
- **Stopping at the critical values.** "Find the set of values" wants an inequality, not just $k = -1$ and $k = 7$. Examiners note that some students leave the interval out entirely.
- **Choosing the wrong region.** Sketch the U-shaped graph of the expression in k. "Less than zero" is between the roots. Writing "$k < -1$ or $k > 7$" for a "less than" condition, or only considering positive k, loses the mark.
- **Writing the interval wrongly.** $-1 < k < 7$ is one interval, so don't write it as "$k > -1$ or $k < 7$". $k < -3$ or $k > 5$ is two separate pieces, so don't join them as $5 < k < -3$.
- **Using $\leqslant$ or $\geqslant$ when the question needs strict signs.** "Two distinct points" is > 0 strictly; "real roots" allows $= 0$.
- **Losing one of the $\pm$ roots.** $(x - 2)^2 = 9$ gives $x - 2 = 3$ or $x - 2 = -3$. Both count.
- **Squaring terms one at a time** when substituting: $x^2 + (5 - x)^2$ needs $(5 - x)^2 = 25 - 10x + x^2$, not $25 + x^2$ or $25 - x^2$.
- **Forgetting to go back to x** in a disguised quadratic, or keeping impossible values. Finding $u = 8$ and $u = -1$ from $u = x^3$ isn't the answer: $x = 2$ and $x = -1$ are. Reject $x^2 = -2$ or $\cos\theta = 3$, but don't wrongly reject a negative value of x^3.
- **Using x as the substitution letter.** Writing "let $x = x^3$" causes confusion. Use u or another letter.
- **Not rejecting an impossible constant.** If the question says k is positive, $k = 0$ must be rejected; check every value you find makes sense.
- **Using earlier results after "given instead".** "Given instead" means start again with the new information; the values from the earlier part no longer apply.

How to get the marks

- **Show the method for every quadratic you solve**: factors that expand back to your quadratic, the formula with values substituted, or the completed square. A correct answer with no working can score zero.
- **"Express in the form $a(x + b)^2 + c$"** gets a mark for each correct part. Give the whole expression, not only the values of a, b and c. Fractions are fine if they are exact, such as $2\left(x - \frac{3}{4}\right)^2 + \frac{7}{8}$.
- **"Hence"** means use the previous part. "Hence or otherwise" allows another method, but "hence" is usually quicker.

- **"Exact"** means surds or fractions, such as $3 \pm \sqrt{3}$, not decimals. Otherwise give non-exact answers to 3 significant figures unless the question asks for something else.
- **Discriminant questions** earn a method mark for a correct three-term quadratic (all terms on one side), another for using $b^2 - 4ac$ with its coefficients, and more for solving and stating the right region. Show each step so the method marks are safe even if an arithmetic slip creeps in.
- **"Find the set of values"** wants inequalities in the constant: $-1 < k < 7$, or $k < -3,\ k > 5$. Don't write the final answer in terms of x if the question is about k.
- **"Show that"** means the answer is given. Every step must be there, and the last line must match exactly. For "show that the line and curve meet for all values of k", finish with a clear conclusion in words.
- **"Determine" or "decide which statement is correct"** needs working that leads to the decision, and the decision stated.
- **Coordinates** should be given as pairs, $(2, 6)$ and $(-2, 14)$, with each x matched to its own y.
- **"State"** (for example "state the value of k" or "state the coordinates of the vertex") means you can write the answer down from earlier work without new working.
- **"Sketch"** means a clear shape, the right way up, with the points the question asks for labelled (such as the vertex). It doesn't need to be to scale.
- **Method marks follow through.** If your completed square was wrong, using it correctly in the next part can still earn marks. Keep going.

Quick check

1. Solve the inequality $3x^2 + 5x - 2 < 0$.
2. Express $3x^2 + 12x + 7$ in the form $a(x + b)^2 + c$, and state the coordinates of the vertex of the graph of $y = 3x^2 + 12x + 7$.
3. Find the values of the constant k for which the equation $x^2 + (k + 1)x + 4 = 0$ has a repeated root.
4. Solve the simultaneous equations $x + y = 5$ and $x^2 + y^2 = 13$.
5. Show that the line $y = kx + 1$ meets the curve $y = x^2 + 3x - 2$ for all values of the constant k.

Answers

1. $(3x - 1)(x + 2) < 0$. The critical values are -2 and $\frac{1}{3}$, and "less than" is between them: $-2 < x < \frac{1}{3}$.
2. $3x^2 + 12x + 7 = 3\left[x^2 + 4x\right] + 7 = 3\left[(x + 2)^2 - 4\right] + 7 = 3(x + 2)^2 - 5$. The vertex is $(-2, -5)$, a minimum.
3. A repeated root needs $b^2 - 4ac = 0$: $(k + 1)^2 - 16 = 0$, so $k + 1 = \pm 4$, giving $k = 3$ or $k = -5$.
4. $y = 5 - x$, so $x^2 + (5 - x)^2 = 13$, $2x^2 - 10x + 12 = 0$, $x^2 - 5x + 6 = 0$, $(x - 2)(x - 3) = 0$. The solutions are $x = 2, y = 3$ and $x = 3, y = 2$.
5. $x^2 + 3x - 2 = kx + 1$ gives $x^2 + (3 - k)x - 3 = 0$. The discriminant is $(3 - k)^2 + 12$. A square is never negative, so this is at least 12, which is positive. So the equation always has two real roots, and the line meets the curve for all values of k.

Past questions to try

- **9709/12/M/J/25 Q2**: where a line meets a curve with an xy term.
- **9709/13/O/N/23 Q6**: a line that is a tangent, with an unknown in the curve.
- **9709/12/M/J/23 Q4**: an equation that is quadratic in x^3.
- **9709/13/M/J/23 Q2**: a quadratic that must stay above a value for all x.