Cambridge AS & A Level · Mathematics (9709) · Notes

Series AS

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Read first: Quadratics

What you need to know

By the end of this topic you should be able to:

1. **Expand $(a+b)^n$ when n is a positive whole number**, using the binomial expansion. Use the notations $n!$ and $\binom{n}{r}$, find any single term or coefficient, and handle expansions such as $(2-3x)^5$ or $\left(x^2-\frac{2}{x}\right)^6$. You do **not** need the greatest term or special properties of the coefficients.
2. **Recognise arithmetic and geometric progressions**: a list of numbers that goes up or down by the same amount each time (arithmetic), or is multiplied by the same number each time (geometric).
3. **Use the formulas for the nth term and for the sum of the first n terms** of an arithmetic or a geometric progression to solve problems. This includes knowing that three numbers a, b, c are in arithmetic progression when $2b = a + c$, and in geometric progression when $b^2 = ac$. A question may involve two progressions at once.
4. **Know when a geometric progression converges** (when $-1 < r < 1$) and use the formula for its **sum to infinity**.

Expansions of $(1+x)^n$ when n is a fraction or negative are **not** in this topic (they are in Paper 3).

Understand it

Progressions

A *sequence* is a list of numbers in order: $u_1, u_2, u_3, \ldots$, where u_n means "the nth term". A *series* is what you get when you add the terms up. S_n means the sum of the first n terms: $S_n = u_1 + u_2 + \cdots + u_n$.

Two kinds of sequence come up in this topic.

- In an **arithmetic progression** (AP) you **add** the same number every time. That number is the *common difference* d. For example $3, 5, 7, 9, \ldots$ has $d = 2$, and $40, 37, 34, \ldots$ has $d = -3$.
- In a **geometric progression** (GP) you **multiply** by the same number every time. That number is the *common ratio* r. For example $5, 15, 45, \ldots$ has $r = 3$, and $48, -24, 12, \ldots$ has $r = -\frac{1}{2}$.

To test which kind you have, subtract neighbouring terms (all the same for an AP) or divide them (all the same for a GP).

The nth term

In an AP starting at a, to get from u_1 to u_n you add d exactly $n-1$ times (not n times: u_1 already exists before you add anything). So

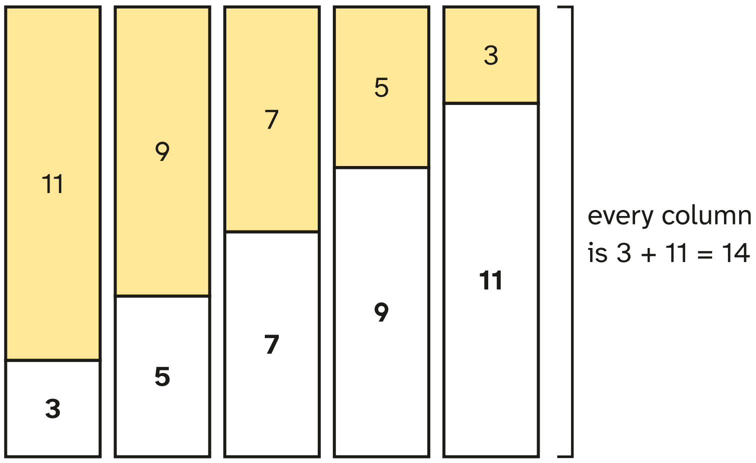
$$u_n = a + (n-1)d.$$

In a GP starting at a, you multiply by r exactly $n-1$ times, so

$$u_n = ar^{n-1}.$$

Adding up an AP

Write the sum forwards and then backwards, and add the two lines term by term. Each pair (first + last, second + second-last, ...) has the same total, $a+l$, where l is the last term. With n pairs, that gives $2S_n = n(a+l)$.



5 columns: 2S = 5 × 14, so S = 35

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}.$$

The second form just replaces l with $a+(n-1)d$. Use it when you know d but not the last term.

Adding up a GP

Write $S_n = a + ar + ar^2 + \cdots + ar^{n-1}$. Multiply every term by r: $rS_n = ar + ar^2 + \cdots + ar^n$. The two lines share almost all their terms, so subtracting leaves only the ends: $S_n - rS_n = a - ar^n$. Dividing by $1-r$,

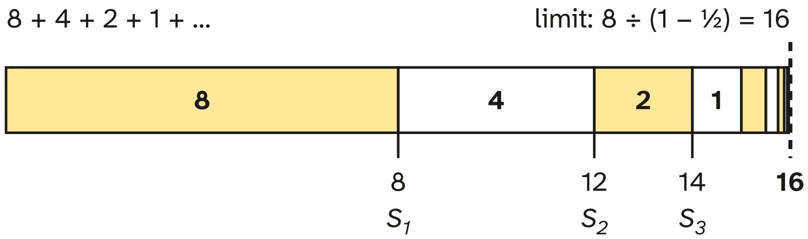
$$S_n = \frac{a(1-r^n)}{1-r} \qquad (r \neq 1).$$

The sum to infinity

When r is between -1 and 1, each term is smaller in size than the one before, and r^n shrinks towards 0 as n grows. The sums $S_1, S_2, S_3, \ldots$ then settle down to a fixed value, the **sum to infinity** S_∞. Put $r^n = 0$ into the sum formula:

$$S_\infty = \frac{a}{1-r} \qquad (-1 < r < 1).$$

For example, $8 + 4 + 2 + 1 + \cdots$ has $a = 8$ and $r = \frac{1}{2}$. Each piece fills half of the gap that is left, so the total creeps up towards 16 but never passes it:



If $r \geqslant 1$ or $r \leqslant -1$, the terms do not shrink, and there is **no** sum to infinity. A GP with a sum to infinity is called *convergent*. So whenever a question mentions a sum to infinity, you know $-1 < r < 1$, and you can use that to reject a value of r.

Three terms in a row

If a, b, c are in an AP, the gaps are equal: $b - a = c - b$, which is $2b = a + c$ (the middle term is the average of the other two).

If a, b, c are in a GP, the ratios are equal: $\frac{b}{a} = \frac{c}{b}$, which is $b^2 = ac$.

These two facts turn "these three expressions are consecutive terms" into an equation, often a quadratic.

The binomial expansion

Multiply out $(a+b)^n$, which is n brackets $(a+b)(a+b)\cdots(a+b)$. Each term of the answer comes from choosing either a or b from every bracket. To get a term with b^r, you choose b from r of the n brackets and a from the rest, so the term is a number times $a^{n-r}b^r$. The number counts how many ways you can choose which r brackets give the b: that is $\binom{n}{r}$, read "n choose r". So

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + b^n.$$

The powers of a go down by one each time while the powers of b go up by one, and in every term the two powers add up to n.

The coefficients are

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!},$$

where $n! = n \times (n-1) \times \cdots \times 2 \times 1$ (and $0! = 1$). Your calculator has a button for them (${}^{n}\mathrm{C}_{r}$). They also form **Pascal's triangle**, where each number is the sum of the two above it:

n	**Coefficients**
2	1, 2, 1
3	1, 3, 3, 1
4	1, 4, 6, 4, 1
5	1, 5, 10, 10, 5, 1
6	1, 6, 15, 20, 15, 6, 1

One term at a time

You rarely need the whole expansion. The term with b^r is

$$\binom{n}{r} a^{n-r} b^r.$$

When a and b are themselves terms like $2x$ or $-\frac{3}{x}$, put the **whole** of each in brackets before you raise it to a power: $(-3x)^2 = 9x^2$, not $-3x^2$. A negative b makes every odd power of b negative, so the signs alternate.

If x appears in both a and b (for example $\left(x^2 + \frac{1}{x}\right)^6$), the power of x in the general term depends on r. Work out that power in terms of r first, then choose r to get the power you want. The *term independent of x* is the term where the power of x is 0 (the constant term).

Key facts and formulas

"Given" means it is in the MF19 list of formulae in the exam. "Learn it" means you must remember it.

Formula	**Status**
AP: $u_n = a + (n-1)d$	Given
AP: $S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$	Given
GP: $u_n = ar^{n-1}$	Given
GP: $S_n = \dfrac{a(1-r^n)}{1-r} \quad (r \neq 1)$	Given
GP: $S_\infty = \dfrac{a}{1-r} \quad (\lvert r\rvert < 1)$	Given
$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + b^n$ (n a positive integer)	Given
$\dbinom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$	Given

Formula	Status
The term in b^r is $\binom{n}{r}a^{n-r}b^r$	Learn it
$d = u_2 - u_1$ and $r = \dfrac{u_2}{u_1}$	Learn it
a, b, c in AP: $2b = a + c$; a, b, c in GP: $b^2 = ac$	Learn it
A GP converges (has a sum to infinity) only when $-1 < r < 1$	Learn it
$u_n = S_n - S_{n-1}$; the sum of the terms after the kth is $S_\infty - S_k$	Learn it

The GP sum can also be written $S_n = \dfrac{a(r^n - 1)}{r - 1}$, which is tidier when $r > 1$. Useful expansions to recognise: $(1 + x)^2 = 1 + 2x + x^2$ and $(1 + x)^3 = 1 + 3x + 3x^2 + x^3$.

How to do it

In the Paper 1 questions we have tagged for this topic (82 questions, 2021–2025), the types came up this often. Six of the progression questions have terms written with $\sin\theta$, $\cos\theta$ or $\tan\theta$, and three are set in a real-life story.

Question type	Questions
Binomial expansion: find terms, coefficients or an unknown constant	36
A problem with both an arithmetic and a geometric progression	17
Geometric progressions	16
Arithmetic progressions	13

1. Binomial expansion (36 questions)

Find the first few terms. "The first three terms in ascending powers of x" means the terms in x^0, x^1, x^2, in that order.

1. Match the bracket to $(a + b)^n$. For $(4 - 3x)^6$: $a = 4$, $b = -3x$, $n = 6$.
2. Write each term as $\binom{n}{r}(a)^{n-r}(b)^r$, with a and b in brackets.
3. Work each one out to a single number times a power of x. Include the constant term.

Find the coefficient of one power in a product, such as x^2 in $(2 - x)(1 + 5x)^7$.

1. Expand the binomial only as far as you need (up to x^2 here).
2. List the pairs of terms, one from each bracket, whose powers add up to the one you want: (constant × x^2 term) + (x term × x term) + ...
3. Multiply each pair's coefficients and add. Don't multiply everything out: it takes longer and invites errors.

Find the term with a given power, or the term independent of x, when x is in both parts, e.g. $\left(3x - \frac{1}{x^2}\right)^6$.

1. Write the general term: $\binom{6}{r}(3x)^{6-r}\left(-x^{-2}\right)^{r}$.
2. Collect the powers of x: $x^{6-r} \times x^{-2r} = x^{6-3r}$.
3. Set that power equal to the one you want (0 for "independent of x") and solve for r. Then work out the term with that r.

Find an unknown constant. The question gives a fact about a coefficient, such as "the coefficient of x^2 is 700", or "the coefficient of x^3 is 4 times the coefficient of x". Find each coefficient in terms of the constant (take care to apply the power to the constant too: $(ax)^2 = a^2x^2$), form the equation and solve. This often gives a quadratic; use the question's condition (for example "a is positive") to choose. Where two unknowns appear, as in $(m+nx)^5$, two given facts give two equations: divide one by the other to find the ratio first.

Approximate a number. To estimate, say, 0.97^8 from the first few terms of $(1-x)^8$, choose x so the bracket equals the number ($1-x=0.97$, so $x=0.03$), and substitute it into your terms.

2. Both an arithmetic and a geometric progression (17 questions)

These come in two forms.

The same terms, two cases. "The first three terms of a progression are $2k$, $k+6$ and k. (a) Given that it is arithmetic... (b) Given instead that it is geometric...". Treat the parts separately:

1. For the AP, use $2b = a + c$ (or equal differences) to find the unknown, then d.
2. For the GP, use $b^2 = ac$ (or equal ratios), which usually gives a quadratic. Then find r, and reject any value that breaks a condition (for example $|r| \geqslant 1$ when a sum to infinity is asked for, or a term that must be positive).
3. "Given instead" means start again: don't reuse the answer from the AP part.

Two linked progressions. "The 3rd, 6th and 12th terms of an AP are the first three terms of a GP."

1. Write each named term with the letters of its own progression: AP terms as $a+(n-1)d$, GP terms as ar^{n-1}.
2. Use the link: here $(a+5d)^2 = (a+2d)(a+11d)$ by the $b^2 = ac$ rule. Or set the matching terms equal, such as "GP 2nd term $=$ AP 4th term".
3. Simplify to an equation in one letter, often a quadratic that factorises. Use "the terms are not all equal" or "$d \neq 0$" to reject the trivial answer.
4. Then answer each part with the correct formula for **that** progression.

3. Geometric progressions (16 questions)

1. **Write the facts as equations in a and r.** For example "the second term is 12" is $ar = 12$, and "the sum to infinity is 64" is $\dfrac{a}{1-r} = 64$.
2. **Eliminate a.** Dividing two term equations removes it: $\dfrac{ar^5}{ar^3} = r^2$. Or rearrange one ($a = \frac{12}{r}$) and substitute into the other.

3. **Solve for r**, often a quadratic, and check each root against the question: "the common ratio is negative", "the progression is convergent" ($|r| < 1$), and so on. Keep every value that works.
4. **Answer what's asked**: a term with ar^{n-1}, a sum with S_n, or S_∞.

Variations you will meet:

- **Terms involving a letter** (e.g. $k-1$, $2k$, $5k+3$): use $b^2 = ac$ to find the letter.
- **Terms given with surds**, such as $4+\sqrt{5}$ and $3-\sqrt{5}$: $r = \frac{u_2}{u_1}$, then rationalise the denominator by multiplying top and bottom by the conjugate.
- **Terms with $\sin\theta$, $\cos\theta$ or $\tan\theta$:** $r = \frac{u_2}{u_1}$ still works (e.g. $\frac{2\sin\theta\cos\theta}{2\sin\theta} = \cos\theta$). Solve the trigonometric equation for θ in the given interval.
- **Sum of the terms after the kth:** either $S_\infty - S_k$, or start a new GP at u_{k+1} with the same r and find its sum to infinity.
- **Every other term** (the 1st, 3rd, 5th, ... terms, or the 2nd, 4th, 6th, ...): these form a new GP with ratio r^2, starting at a or ar.
- **Real-life stories** (savings growing by 3% a year, a swing whose arcs get shorter each time): "increases by 3%" means $r = 1.03$; "the greatest possible total" means S_∞.

4. Arithmetic progressions (13 questions)

1. **Write the facts as equations in a and d**: "the 5th term is 23" is $a + 4d = 23$; "the sum of the first 12 terms is 282" is $6(2a + 11d) = 282$.
2. **Solve the simultaneous equations** for a and d.
3. **Answer what's asked** with u_n or S_n.

Variations you will meet:

- **Find n.** If n is unknown, the sum formula gives a quadratic in n. n must be a positive whole number, so reject any other root.
- **The least n for which a sum passes a value** (e.g. $S_n > 2000$ or $S_n < -300$): solve the equation $S_n = 2000$, then take the next whole number in the right direction. Check it by working out S_n for that n and the one before.
- **The first negative term:** solve $a + (n-1)d < 0$ and take the smallest whole n.
- **The sum of all the terms between two values** (e.g. between 100 and 300): find the first term above the lower value and the last term below the upper value, count the terms from u_n, and use $S = \frac{1}{2}n(\text{first} + \text{last})$. Or subtract two sums from the start: $S_q - S_p$.
- **Sums of different lengths**, such as "$S_{3k} = 4S_k$": write both with the sum formula and solve for k.
- **Sums forming a new progression**, such as "S_1, $S_3 + 2$, S_6 are in AP": write each sum in terms of d, then use $2b = a + c$.
- **Terms with $\sin x$ or $\cos x$:** equal differences give a trigonometric equation. Solve it, then use exact values for the sum.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

(a) Find the first three terms, in ascending powers of x, in the expansion of $(5-2x)^4$. **[3]**

(b) The coefficient of x^2 in the expansion of $(a+x)(5-2x)^4$ is 800. Find the value of the constant a. **[3]**

(a) Here the bracket's first part is 5, its second part is $-2x$ and $n=4$.

$$(5-2x)^4 = 5^4 + \binom{4}{1}5^3(-2x) + \binom{4}{2}5^2(-2x)^2 + \cdots$$

M1 for the correct binomial coefficients with the right powers of 5 and $-2x$.

$$= 625 - 1000x + 600x^2 + \cdots$$

A1 for two correct terms, **A1** for all three.

(b) The x^2 terms come from two products: $a \times 600x^2$ and $x \times (-1000x)$. **M1** for choosing both products and no others.

$$600a - 1000 = 800 \quad \Rightarrow \quad 600a = 1800$$

M1 for forming the equation.

$$a = 3$$

A1

Example 2

In the expansion of $\left(2x^2 - \dfrac{k}{x}\right)^6$, where k is a positive constant, the term independent of x is 960.

(a) Find the value of k. **[4]**

(b) Hence find the coefficient of x^3 in the expansion. **[2]**

(a) The general term is

$$\binom{6}{r}\left(2x^2\right)^{6-r}\left(-kx^{-1}\right)^r.$$

Its power of x is $2(6-r)-r=12-3r$. For the term independent of x, $12-3r=0$, so $r=4$. **M1** for finding the right term.

$$\binom{6}{4}(2x^2)^2(-kx^{-1})^4=15\times 4x^4\times k^4x^{-4}=60k^4$$

A1 for $60k^4$.

$60k^4=960$, so $k^4=16$. **M1** for solving. Since $k>0$, $k=2$. **A1** (reject $k=-2$).

(b) For x^3: $12-3r=3$, so $r=3$.

$$\binom{6}{3}(2x^2)^3(-2x^{-1})^3=20\times 8x^6\times(-8x^{-3})=-1280x^3$$

M1 for the right term with $k=2$. The coefficient is -1280. **A1** (the coefficient is the number, without x^3).

Example 3

The second term of a geometric progression is 12 and its sum to infinity is 64.

(a) Find the two possible values of the common ratio. **[4]**

(b) For the progression with the larger common ratio, find the sum of all the terms after the fourth term. **[3]**

(a) $ar=12$ and $\dfrac{a}{1-r}=64$. **B1** for both equations.

Substitute $a=\dfrac{12}{r}$: $\dfrac{12}{r(1-r)}=64$, so $64r-64r^2=12$. **M1** for eliminating a to get a quadratic in r.

$$16r^2-16r+3=0\quad\Rightarrow\quad(4r-1)(4r-3)=0$$

M1 for solving the quadratic.

$r=\frac{1}{4}$ or $r=\frac{3}{4}$. **A1** Both lie between -1 and 1, so both are valid.

(b) With $r=\frac{3}{4}$, $a=12\div\frac{3}{4}=16$.

The fifth term is $16\times\left(\frac{3}{4}\right)^4=\frac{81}{16}$. The terms from the fifth onwards form a new GP with this first term and the same ratio. **M1** for u_5, or for $S_4=\dfrac{16\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}}=\dfrac{175}{4}$.

$$\text{Sum}=\frac{\frac{81}{16}}{1-\frac{3}{4}}=\frac{81}{4}$$

M1 for a sum to infinity from u_5 (or $S_\infty-S_4=64-\frac{175}{4}$). $=\dfrac{81}{4}$ (or 20.25). **A1**

Example 4

An arithmetic progression has first term 3 and common difference d, where $d \neq 0$. The first, third and ninth terms of the arithmetic progression are, in that order, the first three terms of a geometric progression.

(a) Find the value of d. **[3]**

(b) Find the sum of the first 6 terms of the geometric progression. **[2]**

(c) Find the least value of n for which the sum of the first n terms of the arithmetic progression is greater than 1000. **[4]**

(a) The terms are $u_1 = 3$, $u_3 = 3 + 2d$ and $u_9 = 3 + 8d$. **B1** for the three AP terms.

In a GP, (middle)^2 = (first) × (third):

$$(3 + 2d)^2 = 3(3 + 8d) \quad \Rightarrow \quad 9 + 12d + 4d^2 = 9 + 24d \quad \Rightarrow \quad 4d^2 = 12d$$

M1 for using $b^2 = ac$ with the AP terms. Since $d \neq 0$, divide by $4d$: $d = 3$. **A1**

(b) The GP is $3, 9, 27, \ldots$, so $a = 3$ and $r = 3$.

$$S_6 = \frac{3(3^6 - 1)}{3 - 1} = \frac{3 \times 728}{2} = 1092$$

M1 for the GP sum formula with the GP's own a and r, **A1**.

(c) For the AP, $S_n = \frac{1}{2}n\{6 + 3(n - 1)\} = \frac{3}{2}n(n + 1)$. **M1** for the AP sum formula.

$\frac{3}{2}n(n + 1) > 1000$ gives $3n^2 + 3n - 2000 > 0$. **M1** for a three-term quadratic in n.

The positive root of $3n^2 + 3n - 2000 = 0$ is $n = \dfrac{-3 + \sqrt{24009}}{6} = 25.3$ (3 s.f.). **A1**

n must be a whole number greater than 25.3, so $n = 26$. **A1**

Check: $S_{25} = 975$, which is too small, and $S_{26} = 1053$. ✓

Common mistakes

- **Using the wrong kind of progression.** Every year some students use AP formulas on a GP question, or the other way round, and score nothing. Before you start, write "AP: add d" or "GP: multiply by r" at the top of your working.
- **Treating two progressions as one.** When a question has an AP and a GP, they have different letters: the AP's a and d, the GP's own first term and r. Don't assume they share a first term unless the question says so.
- **Getting r upside down.** $r = \dfrac{\text{later term}}{\text{earlier term}}$. For $12, 9, \ldots$, $r = \frac{9}{12} = \frac{3}{4}$, not $\frac{12}{9}$ (which would have no sum to infinity).
- **Using n instead of $n - 1$.** The nth term is $a + (n - 1)d$ or ar^{n-1}. The 40th term adds 39 lots of d.

- **Using the nth term formula when the question asks for a sum**, or the other way round. "Sum of the first 8 terms" is S_8, not u_8.
- **Rejecting the wrong value of r.** For a sum to infinity, reject r only if $|r| \geqslant 1$. A negative r like $-\frac{1}{2}$ is fine.
- **Losing negative signs**, especially in $(-2x)^3 = -8x^3$ and in terms like ar^9 with negative r.
- **Applying the power to only part of a term.** $(2x^2)^3 = 8x^6$, not $2x^6$; $(kx)^2 = k^2x^2$, so the coefficient of x^2 in $(1+kx)^8$ is $28k^2$, not $28k$.
- **Multiplying out every bracket.** Writing $(4-3x)^6$ as six brackets and expanding them one by one is slow and error-prone. Use the binomial formula and find only the terms you need.
- **Giving a term when a coefficient is asked for.** The coefficient of x^3 in $-1280x^3$ is -1280.
- **Missing a product.** The coefficient of x^2 in $(1+4x-x^2)(\ldots)$ needs three products: $1 \times (x^2 \text{ term})$, $4x \times (x \text{ term})$, $-x^2 \times (\text{constant})$.
- **Forgetting the constant term**, or mixing up "ascending" (lowest power first) and "descending" powers.
- **Reusing earlier answers after "given instead".** The new condition replaces the old one; start that part afresh.
- **Factors that don't multiply back.** When you factorise a quadratic in r, d or k, expand your brackets in your head to check they give the original.

How to get the marks

- **Write the equations in symbols first**, such as $ar = 12$ and $\dfrac{a}{1-r} = 64$. Method marks are given for forming correct equations, even if the algebra later slips.
- **Show the binomial terms** in the form $\binom{4}{2}5^2(-2x)^2$ before simplifying. Listing coefficients from a calculator with no working risks the method mark if one is wrong.
- **Simplify each term** to a single number times a power of x. The final coefficient should be a number, not an unsimplified product.
- **Justify rejected values.** Say why: "$r = -2$ is rejected because a convergent GP needs $|r| < 1$", or "$k > 0$, so $k = 2$".
- **"Show that"** means the answer is given, so every step must be visible. Expand brackets rather than jumping to the result.
- **"Hence"** means use the part before: for example, use the expansion from (a) to find the coefficient in (b).
- **"Exact"** means fractions, surds or π, no decimals. Otherwise give non-exact answers to 3 significant figures (angles to 1 decimal place in degrees) unless the question asks for a different accuracy, such as "4 significant figures". Don't round in the middle.
- **Least or greatest n:** show the equation or inequality, its root, and your conclusion as a whole number. Checking the sums either side makes your answer convincing.
- **Money and measurement stories:** state which progression it is, and write the answer with units.

Quick check

1. Find the coefficient of x^3 in the expansion of $(1-5x)^6$.
2. The fourth term of an arithmetic progression is 17 and the tenth term is 47. Find the sum of the first 20 terms.
3. A geometric progression has first term 45, third term 5 and a negative common ratio. Find its sum to infinity.
4. The first three terms of a geometric progression are $k+3$, k and $10-k$, where k is a positive constant. Find the sum to infinity of the progression.
5. Find the term independent of x in the expansion of $\left(x^2 - \dfrac{2}{x}\right)^9$.

Answers

1. $\binom{6}{3}(-5x)^3 = 20 \times (-125x^3)$, so the coefficient is -2500.
2. $a + 3d = 17$ and $a + 9d = 47$ give $6d = 30$, so $d = 5$ and $a = 2$. $S_{20} = 10(2 \times 2 + 19 \times 5) = 990$.
3. $45r^2 = 5$, so $r^2 = \frac{1}{9}$ and $r = -\frac{1}{3}$ (negative). $S_\infty = \dfrac{45}{1 + \frac{1}{3}} = \dfrac{135}{4}$ $(= 33.75)$.
4. $k^2 = (k + 3)(10 - k)$ gives $2k^2 - 7k - 30 = 0$, $(2k + 5)(k - 6) = 0$, so $k = 6$. The terms are $9, 6, 4$, with $r = \frac{2}{3}$, and $S_\infty = \dfrac{9}{1 - \frac{2}{3}} = 27$.
5. The general term is $\binom{9}{r}(x^2)^{9-r}(-2x^{-1})^r$, with power $18 - 2r - r = 18 - 3r = 0$, so $r = 6$: $\binom{9}{6} \times (-2)^6 = 84 \times 64 = 5376$.

Past questions to try

- **9709/12/O/N/25 Q2**: the term independent of x.
- **9709/11/M/J/25 Q3**: a geometric progression with a negative common ratio.
- **9709/12/F/M/25 Q5**: the sum of the terms of an arithmetic progression between two values.
- **9709/12/O/N/25 Q8**: an arithmetic and a geometric progression sharing their first two terms.