Cambridge AS & A Level · Mathematics 9709 · Notes

The Poisson distribution A2

Read online at pastopic.com/cambridge/mathematics-9709/notes/the-poisson-distribution

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Read first: Discrete random variables, The normal distribution

What you need to know

By the end of this topic you should be able to:

1. **Work out Poisson probabilities.** If $X \sim Po(\lambda)$, find probabilities such as $P(X=3)$, $P(X \leqslant 2)$ or $P(X \geqslant 4)$ from the formula $P(X=r) = e^{-\lambda}\frac{\lambda^r}{r!}$.
2. **Use the fact that the mean and the variance of $Po(\lambda)$ are both λ.** You don't need to prove it.
3. **Know when a Poisson distribution is a sensible model.** It counts events that happen at random, one at a time, independently of each other, at a constant average rate. Use it as a model, including for a different length of time or space from the one you were given.
4. **Approximate a binomial distribution by a Poisson one** when n is large and p is small, roughly $n > 50$ and $np < 5$. Know these conditions and use them to justify the approximation.
5. **Approximate a Poisson distribution by a normal one**, with a continuity correction, when λ is large, roughly $\lambda > 15$. Know this condition too.

Two other ideas turn up in most Poisson questions, so this note shows how to use them: adding independent Poisson variables (from topic 6.2, Linear combinations of random variables) and the Poisson calculations inside a hypothesis test (the test itself is topic 6.5, Hypothesis tests).

Understand it

Counting random events

Some things happen at random moments: calls to a help desk, cars passing a point, typing errors on a page, flaws along a length of cable. If you count how many happen in a fixed interval of time (or length, area or volume), the count X can be $0, 1, 2, 3, \ldots$ with no upper limit. The **Poisson distribution** models such a count when the events

- happen **singly** (two can't happen at exactly the same moment),
- happen **independently** of each other (one event doesn't make another more or less likely),
- happen **at random**,
- at a **constant average rate** (the mean number per hour, or per page, doesn't change).

The only number needed to describe it is λ (lambda), the **mean number of events in the interval**. We write $X \sim Po(\lambda)$.

The formula

$$P(X = r) = e^{-\lambda}\,\frac{\lambda^r}{r!}, \qquad r = 0, 1, 2, \ldots$$

For calls to a help desk at an average of 3 per hour, $X \sim Po(3)$:

r	0	1	2	3	4
$P(X=r)$	$e^{-3} = 0.0498$	$3e^{-3} = 0.1494$	$\frac{3^2}{2}e^{-3} = 0.2240$	$\frac{3^3}{6}e^{-3} = 0.2240$	$\frac{3^4}{24}e^{-3} = 0.1680$

Remember $0! = 1$ and $\lambda^0 = 1$, so $P(X = 0) = e^{-\lambda}$.

Because there is no largest value, "at least" questions always use the complement:

$$P(X \geqslant 3) = 1 - P(X \leqslant 2) = 1 - e^{-3}\left(1 + 3 + \frac{3^2}{2!}\right) = 1 - 0.4232 = 0.577.$$

Taking $e^{-\lambda}$ out as a common factor, as here, keeps the working short and is exactly what examiners want to see.

Each probability is the one before it multiplied by $\frac{\lambda}{r}$: $P(X = r) = P(X = r-1) \times \frac{\lambda}{r}$. So the probabilities go **up** while $r < \lambda$ and **down** once $r > \lambda$, and the most likely value is close to λ. (When λ is a whole number, as here, $P(X = \lambda - 1) = P(X = \lambda)$: both 2 and 3 have probability 0.2240.)

Mean and variance

For $X \sim Po(\lambda)$:

$$E(X) = \lambda, \qquad \mathrm{Var}(X) = \lambda.$$

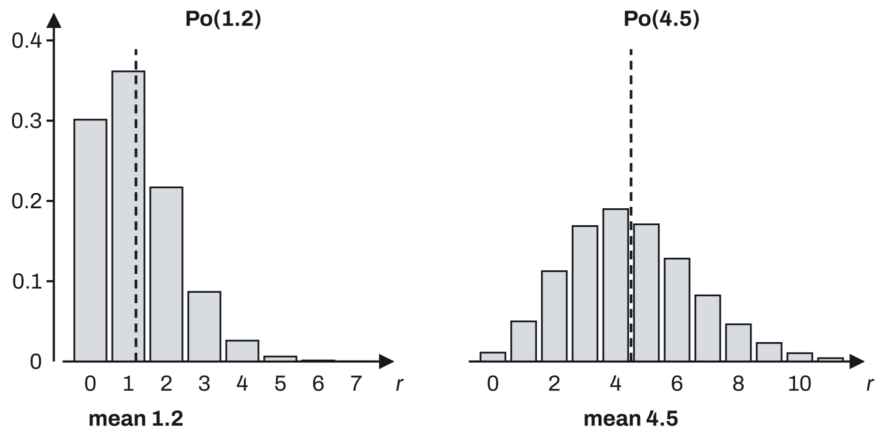
The mean and variance being **equal** is a quick test of whether data could come from a Poisson distribution: if a sample's mean is 4.1 and its variance is 4.3, a Poisson model is reasonable; if the variance is 12, it isn't.

Changing the interval

If calls arrive at an average of 3 per hour, then in half an hour the average is 1.5, and in 4 hours it is 12. The count in any interval is still Poisson, with λ **scaled in proportion to the length of the interval**:

$$\text{rate 3 per hour} \;\Rightarrow\; \text{30 minutes: } Po(1.5), \quad \text{4 hours: } Po(12).$$

Scaling works because the rate is constant. If calls come twice as fast in the daytime as at night, the rate isn't constant over a whole day, and you need a separate Poisson model for each part of the day.



The diagram shows how the shape changes with λ. For small λ the distribution is squashed against 0 and skewed. As λ grows it spreads out (the variance grows with it) and becomes more and more symmetrical.

Adding independent Poisson variables

If $X \sim Po(\lambda)$ and $Y \sim Po(\mu)$ are **independent**, then

$$X + Y \sim Po(\lambda + \mu).$$

This makes sense: if cars arrive at 2 per minute and trucks at 0.5 per minute, vehicles arrive at 2.5 per minute. It works for any number of independent Poisson variables, including several values of the same one: the sum of 4 independent values of $Po(0.5)$ is $Po(2)$. It only works for **sums**: $2X$ is not a Poisson variable, and nor is $X - Y$.

The Poisson approximation to the binomial

Suppose $X \sim B(n, p)$ with n large and p small: 1 in 1000 light bulbs is faulty and you check 2000 of them. Faults are rare events among very many chances, which is exactly the Poisson situation, so

$$B(n, p) \approx Po(np) \qquad \text{when } n > 50 \text{ and } np < 5 \text{ (roughly)}.$$

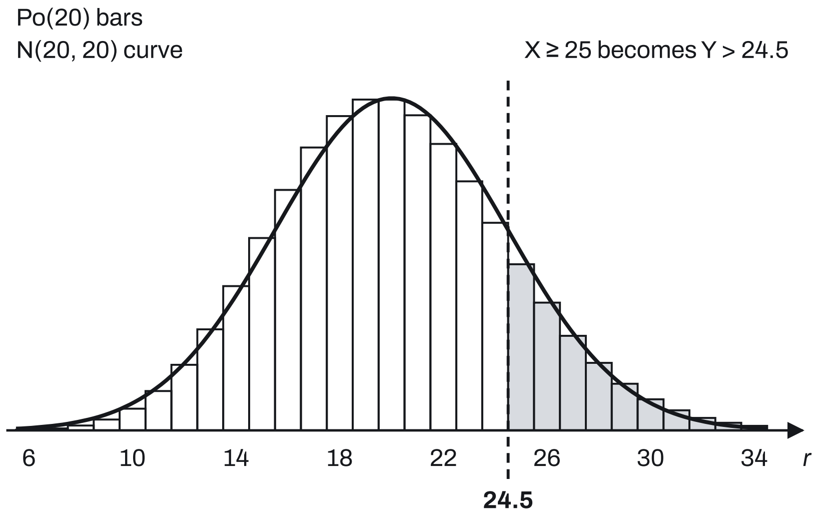
Here $n = 2000 > 50$ and $np = 2 < 5$, so use $Po(2)$. It works because the binomial mean np and variance $np(1-p)$ are almost equal when p is close to 0, just as a Poisson's mean and variance are equal. That is also why p must be small: if p is not close to 0, $np(1-p)$ is noticeably smaller than np.

The normal approximation to the Poisson

When λ is large, adding up many Poisson terms is slow, but the shape is close to a normal curve. Use the normal distribution with the **same mean and variance**:

$$Po(\lambda) \approx N(\lambda,\ \lambda) \qquad \text{when } \lambda > 15 \text{ (roughly)}.$$

The standard deviation is $\sqrt{\lambda}$. A Poisson count is a whole number and the normal variable Y is continuous, so each whole number r becomes the bar from $r - 0.5$ to $r + 0.5$: this is the **continuity correction**, the same as for the normal approximation to the binomial.



For $X \sim Po(20)$: $P(X \geqslant 25) \approx P(Y > 24.5) = 1 - \Phi\left(\frac{24.5-20}{\sqrt{20}}\right) = 1 - \Phi(1.006) = 0.157$, very close to the exact Poisson value 0.1568.

Poisson	**Normal with continuity correction**
$P(X \leqslant 30)$	$P(Y < 30.5)$
$P(X < 30)$, that is $X \leqslant 29$	$P(Y < 29.5)$
$P(X \geqslant 30)$	$P(Y > 29.5)$
$P(X > 30)$, that is $X \geqslant 31$	$P(Y > 30.5)$
$P(20 \leqslant X \leqslant 25)$	$P(19.5 < Y < 25.5)$

Key facts and formulas

"Given" means it is in the MF19 list of formulae and tables in the exam. "Learn it" means you must remember it.

Formula	**Status**
$X \sim Po(\lambda)$: $P(X = r) = e^{-\lambda}\dfrac{\lambda^r}{r!}$	Given
$E(X) = \lambda$, $\mathrm{Var}(X) = \lambda$	Given
Conditions: events occur singly, independently, at random, at a constant mean rate	Learn it
λ is proportional to the length of the interval (rate $\times$ time)	Learn it
$P(X = 0) = e^{-\lambda}$, $P(X \geqslant 1) = 1 - e^{-\lambda}$	Learn it
$X \sim Po(\lambda), Y \sim Po(\mu)$ independent $\Rightarrow X + Y \sim Po(\lambda + \mu)$	Learn it
$B(n, p) \approx Po(np)$ when $n > 50$ and $np < 5$	Learn it
Binomial: $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$, $\mu = np, \sigma^2 = np(1-p)$	Given

Formula	Status
$Po(\lambda) \approx N(\lambda, \lambda)$ when $\lambda > 15$, with a continuity correction	Learn it
Standardising: $z = \dfrac{x - \lambda}{\sqrt{\lambda}}$	Learn it
$\Phi(z) = P(Z \leqslant z)$ in the table, and $\Phi(-z) = 1 - \Phi(z)$	Given

How to do it

In the Paper 6 questions we have tagged for this topic (55 different questions, 2021–2025, some shared by two papers), the types came up this often. 34 of the 55 questions mix types, so the counts add up to more than 55.

Question type	Questions
Normal approximation to a Poisson distribution	25
Poisson probabilities, with the rate scaled to the interval	20
Poisson approximation to a binomial distribution	19
Sums of independent Poisson variables	17
Finding an unknown: λ, n, a time or the most likely value	9
Conditions for the model, mean and variance	9
Poisson probabilities in a hypothesis test	9

1. Normal approximation to a Poisson distribution (25 questions)

1. **Find** λ for the whole interval asked about, for example 4.7 per day for 30 days gives $\lambda = 141$.
2. **State the approximation**: $N(141, 141)$. The mean and the variance are both λ, so the standard deviation is $\sqrt{141}$.
3. **Continuity correction**: write the whole numbers you want, then go 0.5 beyond: "more than 150" is $X \geqslant 151$, so $P(Y > 150.5)$.
4. **Standardise**: $z = \frac{150.5 - 141}{\sqrt{141}} = 0.800$.
5. **Find the area**: $1 - \Phi(0.800) = 1 - 0.7881 = 0.212$, to 3 significant figures.

Variations you will meet:

- **"Justify your approximating distribution"**: "$\lambda = 141 > 15$", with the actual value. Saying "λ is large" is not enough.
- **"State a suitable approximating distribution, giving the values of any parameters"**: write $N(27.5,\ 27.5)$ in full, both numbers.
- **Between two values**: standardise both ends, for example "between 40 and 50", not including either, is $P(40.5 < Y < 49.5)$.

- **The total of many values**: 50 values of $Po(2)$ and 40 values of $Po(1.5)$ add to $Po(100+60) = Po(160)$, so use $N(160, 160)$.
- **Working backwards for n**: if T is the total of n values of $Po(m)$, then $T \approx N(mn,\ mn)$. A given probability gives a z-value, and an equation such as $\frac{c+0.5-mn}{\sqrt{mn}} = z$. Put $s = \sqrt{mn}$ to get the quadratic $s^2 + zs - (c+0.5) = 0$, keep the positive root, then $n = \frac{s^2}{m}$.
- **Comparing two totals**, such as "fewer lemon sweets than orange sweets" with $L \approx N(30, 30)$ and $O \approx N(24, 24)$: use $O - L \sim N(-6,\ 30+24)$ (topic 6.2) and find $P(O - L > 0)$, or $P(O - L > 0.5)$ with a continuity correction; both were accepted.

2. Poisson probabilities, with the rate scaled to the interval (20 questions)

1. **Scale** λ to the interval in the question. Convert units first: 10.5 per hour for 20 minutes is $10.5 \times \frac{20}{60} = 3.5$.
2. **Write the whole numbers you want.** "More than 4" is $5, 6, 7, \ldots$, so use $1 - P(X \leqslant 4)$. "Fewer than 3" is $0, 1, 2$. "More than 6 and less than 9" is $7, 8$.
3. **Write the expression** with $e^{-\lambda}$ taken out, for example $1 - e^{-3.5}\left(1 + 3.5 + \frac{3.5^2}{2!} + \frac{3.5^3}{3!} + \frac{3.5^4}{4!}\right)$.
4. **Evaluate** and give 3 significant figures.

Variations you will meet:

- **Two separate counts that must both happen**, such as "at least 4 cars **and** at least 2 trucks" (independent): find each probability, then multiply.
- **Two separate periods**, such as "exactly 1 order in one hour and at least 2 in the other": find $a = P(X = 1)$ and $b = P(X \geqslant 2)$, then the answer is $2ab$, because either hour can be the one with a single order.
- **"Both values are greater than 1"**: $\big(P(X > 1)\big)^2$.
- **More girls late than boys**, when values of 3 or more can be ignored: list the cases $(G, B) = (1, 0), (2, 0), (2, 1)$, multiply the probabilities in each, and add.
- **"Write down an expression in terms of e"**: for example $P(X = 5) = e^{-8}\frac{8^5}{5!}$ for $X \sim Po(8)$; don't evaluate.
- **Expectation and standard deviation of money**: if a shop makes $X \sim Po(20)$ sales in a week and earns 3 dollars a sale, the amount is $3X$, with mean $3 \times 20 = 60$ and variance $3^2 \times 20 = 180$, so standard deviation $\sqrt{180} = 13.4$.

3. Poisson approximation to a binomial distribution (19 questions)

1. **Recognise it**: a count out of a large n with a small probability p ("1 in 5000", "1.6%"), and usually "use a suitable approximating distribution".
2. $\lambda = np$: for example "1 in 4000" with $n = 10\,000$ gives $10\,000 \times \frac{1}{4000} = 2.5$, so $X \approx Po(2.5)$.
3. **Work out the probability** exactly as in type 2.
4. **Justify** (often a separate mark): "$n = 10\,000 > 50$ and $np = 2.5 < 5$". Both values must be stated.

Variations you will meet:

- **Two approximations added**: $W \sim B(600, 0.004) \approx Po(2.4)$; the sum of two independent values of W is approximately $Po(4.8)$.
- **An expression in p**: for $Y \sim B(200, p)$ with small p, $\lambda = 200p$, so $P(Y < 3) \approx e^{-200p}\left(1 + 200p + \frac{(200p)^2}{2}\right)$.
- **"Explain why p must be close to 0"**: the binomial has mean np and variance $np(1-p)$; these are nearly equal (as for a Poisson) only when $1-p$ is close to 1.
- **"Find $E(X)$ and $\mathrm{Var}(X)$ and explain how they suggest the approximation is appropriate"**: np and $np(1-p)$ are almost equal.
- **The proportion comes from the question**: "3120 out of 1 560 000" gives $p = \frac{3120}{1560000} = 0.002$, so a sample of 1500 has $\lambda = 1500 \times 0.002 = 3$.
- **If the question says "use a Poisson distribution"**, a binomial calculation earns little, and a normal approximation almost nothing.

4. Sums of independent Poisson variables (17 questions)

1. **Scale each rate** to the same interval. Two desks with $Po(1.6)$ and $Po(2.2)$ per 10 minutes give, for 5 minutes, $Po(0.8)$ and $Po(1.1)$.
2. **Add** the means: the total is $Po(1.9)$. Say that the variables are independent.
3. **Work out the probability** as in type 2.

Variations you will meet:

- **Several values of one variable**: the sum of 5 independent values of $Po(0.4)$ is $Po(2)$.
- **An exact ratio**, such as the exact value of $\frac{P(S\leqslant 2)}{P(S\leqslant 1)}$ for $S \sim Po(3)$: the $e^{-\lambda}$ factors cancel. Keep fractions: $\frac{1+3+4.5}{1+3} = \frac{8.5}{4} = \frac{17}{8}$.
- **Given the total**: "given that the total is n, find the probability that $X = a$". Use conditional probability:

$$P(X = a \mid X + Y = n) = \frac{P(X = a)\,P(Y = n - a)}{P(X + Y = n)}.$$

  The numerator uses the separate distributions of X and Y; the denominator uses $X + Y \sim Po(\lambda + \mu)$.
- **"$X = 2$ given that $X + Y < 4$"**: the numerator is $P(X = 2) \times P(Y \leqslant 1)$, because Y can be 0 or 1.
- **A sample mean of $X + Y$**: if n pairs are averaged and $X + Y \sim Po(m)$, the mean has approximately $N\left(m, \frac{m}{n}\right)$ (topic 6.4); or use the total, $Po(nm) \approx N(nm, nm)$.

5. Finding an unknown: λ, n, a time or the most likely value (9 questions)

- **λ from $P(X = 0)$**: $e^{-\lambda} = 0.3$ gives $\lambda = -\ln 0.3 = 1.204$.
- **Probabilities that are equal**: $P(X = n) = P(X = n + 1)$ means $e^{-\lambda}\frac{\lambda^n}{n!} = e^{-\lambda}\frac{\lambda^{n+1}}{(n+1)!}$. Cancel $e^{-\lambda}$, λ^n and $n!$ to get $1 = \frac{\lambda}{n+1}$, so $n = \lambda - 1$. For $Po(9)$, $n = 8$.
- **An equation with several terms**, such as $2P(Y = 3) + P(Y = 4) = P(Y = 5)$: divide by $e^{-\lambda}\lambda^3$ and multiply by $5!$ to get $40 + 5\lambda = \lambda^2$, a quadratic. Keep the positive root.

- **Two variables linked**: with $X \sim Po(\lambda)$ and $Y \sim Po(\mu)$, write each fact you are given as an equation in λ and μ, cancel what you can, and solve them together. For example $P(X=1) = P(Y=0)$ gives $\lambda e^{-\lambda} = e^{-\mu}$.
- **Minimum time (or largest sample)**: "at least one customer with probability at least 0.95" at 1.5 per 10 minutes. In t minutes $\lambda = 0.15t$, and $1 - e^{-0.15t} \geqslant 0.95$ gives $e^{-0.15t} \leqslant 0.05$, $t \geqslant \frac{\ln 20}{0.15} = 19.97$, so 20 minutes. Solve with logarithms; say which way the inequality goes when you round.
- **A new value of** p that doubles $P(X=0)$: $e^{-np_{\text{new}}} = 2e^{-np_{\text{old}}}$; take logs.
- **Most likely value**: $P(X=r) < P(X=r+1)$ means $1 < \frac{\lambda}{r+1}$, so $r < \lambda - 1$. For $Po(3.6)$: $r < 2.6$, so $r = 0, 1, 2$, and the probabilities rise up to $r=3$ then fall. The most likely value is 3.

6. Conditions for the model, mean and variance (9 questions)

1. **"State a condition for X to have a Poisson distribution"**: pick one of singly, independently, randomly, constant mean rate, and say it **about the context**: "calls arrive independently of each other", not "events are independent".
2. **"Explain why the model is unsuitable"**: find the condition that fails, for example "the hit rate in the day is twice the rate at night, so the rate is not constant".
3. **Separate models**: split the interval. If the average over a whole day is 0.6 per minute, and the day's two 12-hour halves have daytime twice as busy as night-time, the rates are 0.8 and 0.4 per minute, so $Po(0.8)$ and $Po(0.4)$ per minute.
4. **Mean and variance**: "$E(W) = \mathrm{Var}(W)$" is the relationship to state for a Poisson variable.

7. Poisson probabilities in a hypothesis test (9 questions)

The full method of a test is in the Hypothesis tests topic; the Poisson part is this.

1. **Hypotheses** use λ for the interval of the test: 2.6 per year over 2 years gives $H_0: \lambda = 5.2$ and, for a decrease, $H_1: \lambda < 5.2$.
2. **The probability** is of the observed value **or more extreme**, under H_0: for $X=1$ and a decrease, $P(X \leqslant 1) = e^{-5.2}(1+5.2) = 0.0342$. For an increase, use $P(X \geqslant \text{observed}) = 1 - P(X \leqslant \text{observed} - 1)$.
3. **Compare** with the significance level and conclude in context, not definitely: "there is evidence that the mean number of accidents has decreased".
4. **Critical region**: find the tail probabilities one value at a time until the next one passes the significance level. At the 5% level, $P(X \leqslant 1) = 0.0342 < 0.05$ but $P(X \leqslant 2) = e^{-5.2}(1 + 5.2 + 13.52) = 0.109 > 0.05$, so the critical region is $X \leqslant 1$.
5. **P(Type I error)** $=$ the probability of landing in the critical region when H_0 is true (0.0342 here). **P(Type II error)** $=$ the probability of **not** landing in it with the new λ: if the true mean is 1 for the two years, $P(X \geqslant 2) = 1 - e^{-1}(1+1) = 0.264$.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

Emails arrive in an office inbox at random, at a constant average rate of 3.6 per hour.

(a) State, in context, one other condition needed for the number of emails to be modelled by a Poisson distribution. **[1]**

(b) Find the probability that exactly 2 emails arrive in a randomly chosen 30-minute period. **[2]**

(c) Find the probability that at least 3 emails arrive in a randomly chosen 20-minute period. **[3]**

(a) Emails arrive independently of each other (or: emails arrive one at a time). **B1**

(b) $\lambda = 3.6 \times \frac{30}{60} = 1.8$.

$$P(X = 2) = e^{-1.8}\frac{1.8^2}{2!} = 0.16530 \times 1.62 = 0.268$$

M1 for the Poisson expression with any λ, **A1**.

(c) $\lambda = 3.6 \times \frac{20}{60} = 1.2$. **B1**

$$P(X \geqslant 3) = 1 - e^{-1.2}\left(1 + 1.2 + \frac{1.2^2}{2!}\right) = 1 - 0.30119 \times 2.92$$

M1 for $1 - P(X \leqslant 2)$ with the terms shown.

$= 1 - 0.87949 = 0.121$ **A1**

Example 2

On average, 0.8% of the light bulbs made by a factory are faulty. A box contains 250 bulbs.

(a) Use a suitable approximating distribution to find the probability that there are more than 3 faulty bulbs in the box. **[3]**

(b) Justify your approximating distribution. **[1]**

(c) Find the probability that two boxes contain fewer than 2 faulty bulbs altogether. **[3]**

(a) $X \sim B(250, 0.008)$, approximately $Po(250 \times 0.008) = Po(2)$. **B1**

$$P(X > 3) = 1 - e^{-2}\left(1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!}\right) = 1 - (0.13534 + 0.27067 + 0.27067 + 0.18045)$$

M1, $= 1 - 0.85712 = 0.143$ **A1**. (The exact binomial answer is 0.142, very close.)

(b) $n = 250 > 50$ and $np = 2 < 5$. **B1**

(c) The two boxes are independent, so the total is approximately $Po(2+2) = Po(4)$. **B1**

$$P(T < 2) = e^{-4}(1+4) = 5e^{-4}$$

M1, $= 0.0916$ **A1**

Example 3

Calls come in to two offices, A and B, independently and at random. The numbers of calls in a 10-minute period have the distributions $Po(1.4)$ at A and $Po(2.1)$ at B.

(a) Find the probability that fewer than 5 calls altogether arrive at the two offices in a randomly chosen 20-minute period. **[3]**

(b) In a particular 10-minute period exactly 3 calls arrive altogether. Find the probability that all 3 were to office A. **[4]**

(a) In 20 minutes: $A \sim Po(2.8)$, $B \sim Po(4.2)$, so the total $T \sim Po(7)$. **B1**

$$P(T \leqslant 4) = e^{-7}\left(1 + 7 + \frac{7^2}{2!} + \frac{7^3}{3!} + \frac{7^4}{4!}\right) = e^{-7}(1 + 7 + 24.5 + 57.1667 + 100.0417)$$

M1, $= 0.000911882 \times 189.7083 = 0.173$ **A1**

(b) In 10 minutes the total is $Po(3.5)$. **B1**

$$P(A = 3 \mid A + B = 3) = \frac{P(A=3)\,P(B=0)}{P(A+B=3)} = \frac{e^{-1.4}\frac{1.4^3}{3!} \times e^{-2.1}}{e^{-3.5}\frac{3.5^3}{3!}}$$

M1 for the product of the two separate probabilities, **M1** for dividing by $P(A+B=3)$.

The e terms cancel ($e^{-1.4} \times e^{-2.1} = e^{-3.5}$), leaving $\left(\frac{1.4}{3.5}\right)^3 = 0.4^3 = 0.064$. **A1** (In decimals: $\frac{0.013810}{0.21579} = 0.0640$.)

Example 4

Faults occur at random along a telephone cable at a constant average rate of 0.6 per kilometre.

(a) Use a suitable approximating distribution to find the probability that a 50 km length of cable has at most 25 faults. **[4]**

(b) Justify the use of your approximating distribution. **[1]**

(c) Find the greatest length of cable, in metres, for which the probability of no faults is more than 0.9. **[3]**

(a) $\lambda = 0.6 \times 50 = 30$, so use $N(30, 30)$. **B1**

"At most 25" is $X \leqslant 25$, so find $P(Y < 25.5)$.

$$P(Y < 25.5) = P\left(Z < \frac{25.5 - 30}{\sqrt{30}}\right) = P(Z < -0.822)$$

M1 for standardising with $\sqrt{30}$, **M1** for the continuity correction.

$= 1 - \Phi(0.822) = 1 - 0.7944$ **M1** for the correct area, $= 0.206$ **A1**

(b) $\lambda = 30 > 15$. **B1**

(c) For L km, $\lambda = 0.6L$ and $P(X = 0) = e^{-0.6L}$. **B1**

$e^{-0.6L} > 0.9 \Rightarrow -0.6L > \ln 0.9 \Rightarrow L < \frac{-\ln 0.9}{0.6} = 0.17560$ **M1** for taking logs correctly.

The greatest length is 175 m (rounded **down**, since 176 m would make the probability less than 0.9). **A1**

Common mistakes

- **Using the wrong λ.** The rate is given per hour but the question asks about 20 minutes, or two variables must be added first. Examiners see a wrong λ every year. Write "$\lambda = \ldots$" as the first line.
- **One term too many or too few.** "Fewer than 3" is $0, 1, 2$; "more than 6 and less than 9" is just $7, 8$. List the values before writing the terms.
- **Forgetting "$1-$"** in an "at least" or "more than" question, or subtracting from 1 when it isn't needed.
- **Not showing the Poisson expression.** An answer from the calculator alone earns much less. Write the terms (or $e^{-\lambda}(\ldots)$) before the value.
- **Using the variance as the standard deviation** in the normal approximation. For $N(40, 40)$ divide by $\sqrt{40}$, not 40.
- **Missing or wrong continuity correction**, for example 50 or 49.5 for "more than 50" instead of 50.5.
- **Using a normal distribution or the binomial when the question asks for a Poisson approximation.** Read the question: "a suitable approximating distribution" for $B(3000, 0.001)$ means $Po(3)$.
- **Justifying with words only.** "n is large and p is small" earns nothing; write "$n = 300 > 50$ and $np = 2.4 < 5$" with the numbers. For the normal approximation write "$\lambda = 120 > 15$".
- **Conditions not in context.** "Events are independent" loses the mark; "customers arrive independently" gets it.
- **Adding rates for different intervals.** Scale each rate to the same interval before adding. And two independent values $X_1 + X_2$ have $Po(2\lambda)$, but $2X$ (one value doubled) is not Poisson.
- **Rounding too early.** Keep λ exact and terms to at least 4 significant figures until the end.

How to get the marks

- **Show λ.** It usually earns a **B1** on its own, and the method mark allows any λ, so a wrong one still gets method credit if the expression is right.
- **Show the full expression**: $1 - e^{-4.5}\left(1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!}\right)$ or the list of terms. Mark schemes accept one end error for the method mark but not for the answer.
- **Approximations**: state the distribution with its parameters ($Po(3.5)$, $N(90, 90)$), then the continuity-corrected value in the standardisation, then the area. For a normal approximation that is **B1 M1 M1 A1**.
- **"Justify"**: the condition with the numbers, $n > 50$ and $np < 5$, or $\lambda > 15$.
- **"State a condition" or "explain"**: one sentence in the context of the question.
- **"Show that the exact value"**: keep fractions and cancel $e^{-\lambda}$; no rounded decimals.
- **"Hence"**: use the previous part's answer, for example the λ or probability just found.
- **Accuracy**: 3 significant figures unless told otherwise; a time "to the nearest minute" means round in the direction that keeps the condition true.
- **Method marks follow through.** A wrong λ in part (a) used correctly in part (b) still earns the later method marks.

Quick check

1. $X \sim Po(2.5)$. Find $P(X = 3)$.
2. $X \sim Po(\lambda)$ and $P(X = 0) = 0.2$. Find λ, and hence $P(X = 1)$.
3. $X \sim B(80, 0.03)$. Explain why a Poisson approximation is suitable, and use it to estimate $P(X = 0)$.
4. $X \sim Po(3)$ and $Y \sim Po(2.2)$ are independent. Find $P(X + Y = 2)$.
5. $X \sim Po(36)$. Use a suitable approximation to find $P(X \geqslant 42)$.

Answers

1. $P(X=3)=e^{-2.5}\dfrac{2.5^3}{3!}=0.082085\times 2.6042=0.214.$
2. $e^{-\lambda}=0.2$, so $\lambda=\ln 5=1.609$. $P(X=1)=e^{-\lambda}\lambda=0.2\times 1.6094=0.322.$
3. $n=80>50$ and $np=2.4<5$, so use $Po(2.4)$. $P(X=0)\approx e^{-2.4}=0.0907.$
4. $X+Y\sim Po(5.2)$. $P(X+Y=2)=e^{-5.2}\dfrac{5.2^2}{2!}=0.0746.$
5. $\lambda=36>15$, so use $N(36,36)$. $P(X\geqslant 42)\approx P(Y>41.5)=1-\Phi\left(\dfrac{41.5-36}{6}\right)=1-\Phi(0.917)=1-0.8204=0.180.$

Past questions to try

- **9709/62/O/N/23 Q7**: Poisson probabilities, two independent values, and the most likely value.
- **9709/63/O/N/24 Q6**: scaling the rate, adding two desks, and the least waiting time.
- **9709/62/M/J/24 Q5**: goals in two halves: a sum, then a probability given the total.
- **9709/61/M/J/23 Q1**: a Poisson approximation from a population count, and justifying it.