Cambridge AS & A Level · Physics 9702 · Notes

Capacitance A2

Read online at **pastopic.com/cambridge/physics-9702/notes/capacitance**

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Read first: **Electricity**, **D.C. circuits**, **Electric fields**

What you need to know

By the end of this topic you should be able to:

1. **Define capacitance**, both for an isolated charged sphere and for a parallel plate capacitor: the charge stored per unit potential (difference).
2. **Recall and use** $C = \dfrac{Q}{V}$**.**
3. **Derive**, starting from $C = \dfrac{Q}{V}$, the formulas for the combined capacitance of capacitors **in series** and **in parallel**.
4. **Use** the series and parallel formulas, including for networks that mix the two.
5. **Find the energy stored** in a capacitor from the area under a graph of potential difference against charge.
6. **Recall and use** $W = \frac{1}{2}QV = \frac{1}{2}CV^2$**.**
7. **Analyse graphs** of p.d., charge and current against time for a capacitor discharging through a resistor.
8. **Recall and use** $\tau = RC$, the time constant of the discharge.
9. **Use** $x = x_0\, e^{-t/RC}$, where x is the current, the charge or the p.d. during the discharge.

Charge, current and p.d. ($Q = It, V = IR$) come from Electricity and D.C. circuits; the potential of a charged sphere comes from Electric fields. Equations for **charging** a capacitor through a resistor are not in the syllabus.

Understand it

What a capacitor is

A capacitor is two conductors (usually two metal plates) separated by an insulator. Connect it to a supply and electrons are pushed off one plate and onto the other, through the circuit, not across the gap. One plate ends up with charge $+Q$ and the other with $-Q$, and there is a potential difference V between them. The flow stops when this p.d. equals the supply's.

The **charge stored** by a capacitor, Q, means the charge on **one** plate. (The total over both plates is zero.) The separated charges make an electric field in the gap, and that field stores energy.

Capacitance

Double the p.d. and the charge doubles: $Q \propto V$. The constant ratio is the **capacitance**:

$$C = \frac{Q}{V}$$

- **Parallel plate capacitor**: capacitance is the charge on one plate divided by the potential difference between the plates.
- **Isolated sphere**: capacitance is the charge on the sphere divided by its potential (measured from zero at infinity).

The unit is the **farad**, F: one coulomb per volt. A farad is huge, so values are usually in μF (10^{-6}), nF (10^{-9}) or pF (10^{-12}). A 100 μF capacitor at 20 V holds $100 \times 10^{-6} \times 20 = 2.0 \times 10^{-3}$ C on each plate.

On a graph of Q against V the line is straight through the origin and its **gradient is** C. (On V against Q the gradient is $\frac{1}{C}$.)

For a sphere of radius r carrying charge Q, the potential at the surface is $V = \dfrac{Q}{4\pi\varepsilon_0 r}$, so

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 r$$

A sphere of radius 0.20 m has $C = 4\pi \times 8.85 \times 10^{-12} \times 0.20 = 2.22 \times 10^{-11}$ F, about 22 pF: a bigger sphere holds more charge at the same potential.

Capacitors in parallel

In parallel, every capacitor is connected across the same two points, so each has the **same p.d.** V. Each stores its own charge, and the total charge from the supply is the **sum**:

$$Q = Q_1 + Q_2 \;\Rightarrow\; CV = C_1V + C_2V \;\Rightarrow\; C = C_1 + C_2$$

Adding capacitors in parallel always **increases** the capacitance: it is like making the plates bigger. For example, 2.2 μF and 3.3 μF in parallel give 5.5 μF.

Capacitors in series

In series, the supply moves charge $+Q$ onto the first plate and takes Q off the last plate. The two inner plates joined by a wire are cut off from the supply, so their total charge stays zero: $+Q$ on one means $-Q$ on the other. So **every capacitor in series has the same charge** Q. The p.d.s across them add up to the supply's:

$$V = V_1 + V_2 \;\Rightarrow\; \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} \;\Rightarrow\; \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

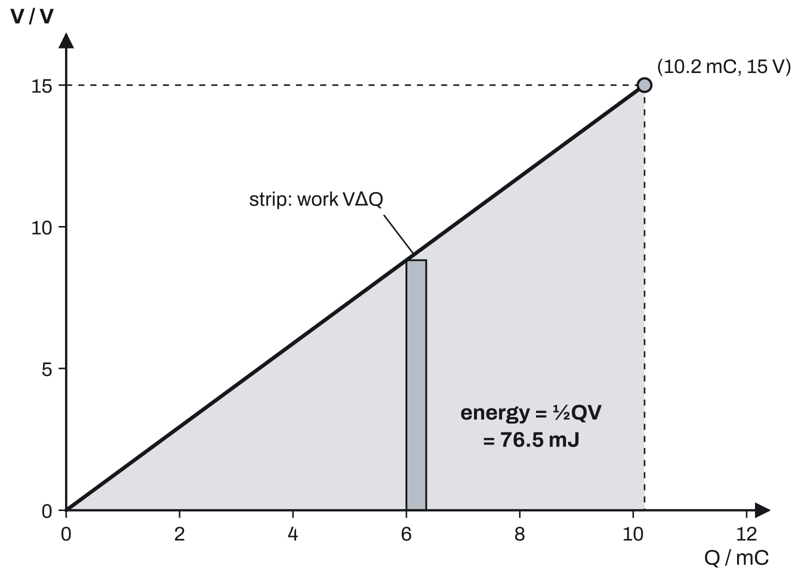
Capacitors in series have a **smaller** capacitance than the smallest one. For example, 2.2 μF and 3.3 μF in series give $\left(\frac{1}{2.2} + \frac{1}{3.3}\right)^{-1} = 1.32$ μF. The smaller capacitor takes the **bigger** share of the p.d., because $V = \frac{Q}{C}$ with the same Q.

These rules are the opposite way round from resistors: capacitors add **directly in parallel** and **as reciprocals in series**.

Energy stored

Charging a capacitor means pushing each extra bit of charge ΔQ onto a plate against the p.d. already there. When the p.d. is V, that takes work $V\Delta Q$: the area of a thin strip under a graph of V against Q. Adding up all the strips from zero to Q gives the **area under the graph**, a triangle:

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$



It is $\frac{1}{2}QV$, not QV, because the p.d. starts at zero and only reaches V at the end: on average it is $\frac{1}{2}V$. The supply that charges the capacitor does work QV; the other half turns into thermal energy in the resistance of the circuit.

The three forms suit different data. At a fixed p.d., $W = \frac{1}{2}CV^2$ is proportional to C. For an isolated capacitor, whose charge cannot change, $W = \frac{Q^2}{2C}$ is the handy one. And because $W \propto V^2$, doubling the p.d. stores **four** times the energy.

Discharging through a resistor

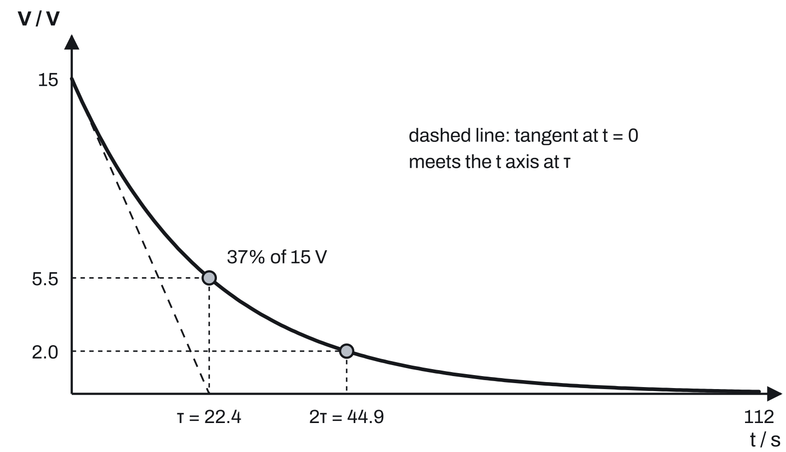
Charge a capacitor to V_0, then connect it across a resistor R. The p.d. across the resistor is the p.d. across the capacitor, so the current starts at $I_0 = \dfrac{V_0}{R}$ (not zero). Then:

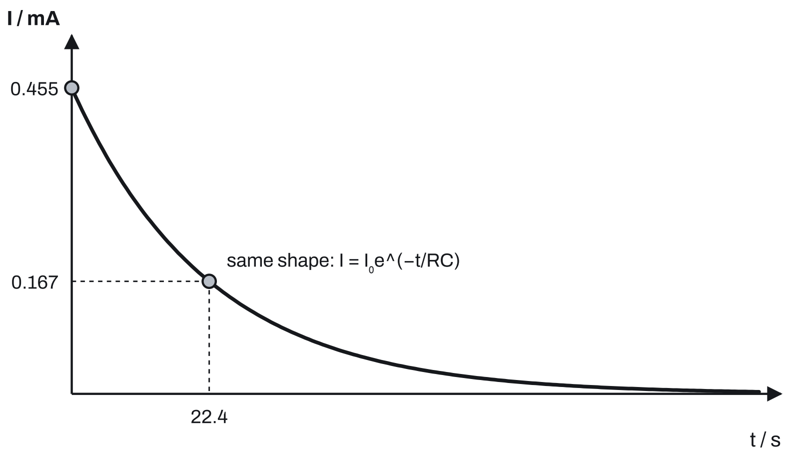
1. charge flows off the plates, so Q falls;
2. $V = \frac{Q}{C}$ falls with it;
3. so the current $I = \frac{V}{R}$ falls too, and the charge leaves more slowly.

The current is the rate at which the charge falls, and the current is proportional to the charge that is left. A quantity that falls at a rate **proportional to itself** decays **exponentially**, like the number of undecayed nuclei in radioactive decay:

$$x = x_0\, e^{-t/RC}$$

where x is the charge, the p.d. or the current. All three fall with the same shape and the same timing.





The time constant

The product RC is the **time constant** τ:

$$\tau = RC$$

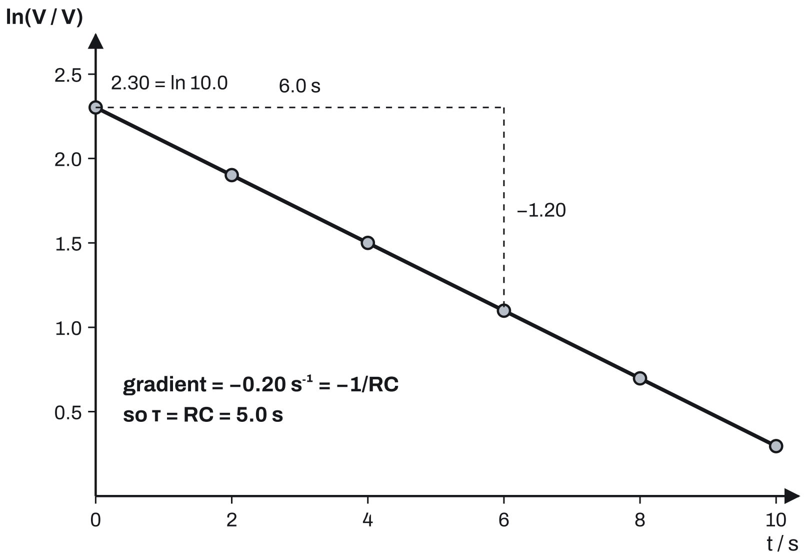
- Its unit is the second: $\Omega \times \mathrm{F} = \frac{\mathrm{V}}{\mathrm{A}} \times \frac{\mathrm{C}}{\mathrm{V}} = \frac{\mathrm{C}}{\mathrm{A}} = \mathrm{s}$.
- After one time constant, $x = x_0 e^{-1} = 0.37x_0$: the charge, p.d. and current have fallen to **37%** of their starting values. After 2τ they are at $0.37^2 = 14\%$; after 5τ, under 1%.
- A **bigger** R (a smaller current) or a **bigger** C (more charge to move) makes a slower discharge. Doubling either doubles τ.
- In equal time intervals the value falls by the **same fraction**. The time to halve is $t_{1/2} = \tau \ln 2 = 0.693\tau$.
- The tangent to the curve at $t = 0$ meets the time axis at $t = \tau$ (the discharge would take one time constant if the starting rate carried on).

Straight-line graph

Taking natural logs of $V = V_0 e^{-t/RC}$ gives

$$\ln V = \ln V_0 - \frac{t}{RC}$$

So a graph of $\ln V$ (or $\ln Q, \ln I$) against t is a straight line with **gradient** $-\dfrac{1}{RC}$ and intercept $\ln V_0$. Take the value of V_0 from the intercept with $e^{\text{intercept}}$; don't put a log value into the exponential equation.



Key facts and formulas

"Given" means it is on the Paper 4 data sheet printed in the exam paper. "Learn it" means you must remember it.

Formula	Status
Capacitance $C = \dfrac{Q}{V}$ (so $Q = CV$)	Learn it
Capacitors in series $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$	Given
Capacitors in parallel $C = C_1 + C_2 + \ldots$	Given
Isolated sphere $C = 4\pi\varepsilon_0 r$ (from $V = \dfrac{Q}{4\pi\varepsilon_0 r}$)	Learn it
Energy stored $W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$	Learn it
Discharge $x = x_0\, e^{-t/RC}$	Given
Time constant $\tau = RC$	Learn it
$\ln x = \ln x_0 - \dfrac{t}{RC}$; $t_{1/2} = RC \ln 2$	Learn it

Formula	Status
$I = \dfrac{Q}{t}, V = IR, I_0 = \dfrac{V_0}{R}$	Learn it
$\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}, $\dfrac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9$ m F^{-1}	Given

Units: 1 μF $= 10^{-6}$ F, 1 nF $= 10^{-9}$ F, 1 pF $= 10^{-12}$ F; 1 mC $= 10^{-3}$ C; 1 kΩ $= 10^3$ Ω, 1 MΩ $= 10^6$ Ω.

How to do it

In the Paper 4 questions we have tagged for this topic (19 different questions, 2021–2025; 5 more appear on two papers), the types came up this often. 18 of the 19 questions mix two or more types, so the counts add up to more than 19.

Question type	Questions
Defining capacitance and using $C = \frac{Q}{V}$	16
Discharge through a resistor: time constant and exponential decay	11
Energy stored in a capacitor	10
Capacitors in series and parallel	8
Charge shared between two capacitors	3
Capacitance of an isolated sphere	2
Repeated charging and discharging: $I = fCV$	2
Changing the plate separation of an isolated capacitor	1

1. Defining capacitance and using $C = \frac{Q}{V}$ (16 questions)

1. **Definition**: capacitance is charge divided by potential (difference). For a **parallel plate capacitor**, say that the charge is the charge on **one plate** and the potential is the **p.d. between the plates**. For an isolated sphere: the charge on the sphere divided by its potential.
2. **Calculate** with $Q = CV$, converting μF, mC and so on to base units first.
3. **From a graph**: read Q and V at one point on the line (or use the gradient of Q against V). Pick a point far from the origin, where readings are more accurate.

Variations you will meet:

- **"Explain how two plates act as a capacitor"**: a p.d. across the plates separates charge (equal and opposite charges on the two plates), and energy is stored in the field between them.
- **Two graphs from the same discharge**, such as Q against V and I against V: the gradient of the first is C, the inverse gradient of the second is R, and then $\tau = RC$.

- **"Show that C is ..."**: say where every number comes from (for example "gradient of the graph $= 4.0 \times 10^{-4}$ C V-1"), then give one more significant figure than the value printed.
- **The charge in terms of other letters**, for example "state Q in terms of C and V": $Q = CV$, and nothing else.

2. Discharge through a resistor: time constant and exponential decay (11 questions)

1. **Time constant**: $\tau = RC$, with R in Ω and C in F, giving seconds. C is the capacitance **discharging**, so for a network use the combined capacitance, and for several resistors use the combined resistance.
2. **Starting values**: $Q_0 = CV_0$ and $I_0 = \dfrac{V_0}{R}$.
3. **A value after time t**: $x = x_0 e^{-t/RC}$. For example, 100 μF charged to 20 V discharging through 39 kΩ has $\tau = 3.9$ s; after 6.0 s the p.d. is $20e^{-6.0/3.9} = 4.29$ V.
4. **The time to reach a value**: take logs, $t = RC \ln \dfrac{x_0}{x}$. The same capacitor takes $3.9 \ln 4 = 5.41$ s to fall to a quarter of its p.d.

Variations you will meet:

- **Time constant from a curve.** Any of these works:
  - read x_0, find $0.37x_0$, and read the time at which the curve reaches it;
  - read two points and use $\dfrac{x_2}{x_1} = e^{-(t_2-t_1)/\tau}$: 16 V at 2.0 s and 10 V at 7.0 s give $\tau = \dfrac{5.0}{\ln 1.6} = 10.6$ s;
  - read the half-life and use $\tau = \dfrac{t_{1/2}}{\ln 2}$: a half-life of 4.0 s means $\tau = 5.77$ s;
  - draw the tangent at the start: it meets the time axis at τ.
- **A log graph** ($\ln V$ or $\ln Q$ against t): $\tau = -\dfrac{1}{\text{gradient}}$. Where the axis is $-\ln \dfrac{V}{V_0}$, the value is 1.0 when $t = \tau$. Read the starting charge from the intercept: $Q_0 = e^{\text{intercept}}$ in the unit on the axis label.
- **Something changes the circuit**: a second identical resistor in **parallel** halves R and halves τ (twice as steep a log line, same intercept); in **series** it doubles τ. A second identical capacitor in parallel doubles C and so doubles τ. Say "doubled" or "halved", not just "increased".
- **A network discharging.** 40 μF and 10 μF in series have 8.0 μF; through 250 kΩ, $\tau = 2.0$ s. Each capacitor's p.d. decays with this **same** τ, starting from its own share: charged by 25 V, the 10 μF capacitor starts at 20 V and the 40 μF one at 5.0 V. Relate a capacitor's later p.d. to **its own** starting p.d.
- **Energy against time**: $W \propto V^2$, so $W = W_0 e^{-2t/RC}$. Do not put energy into $x = x_0 e^{-t/RC}$. To find when the energy has fallen to a fraction k, find when V has fallen to $\sqrt{k}$ of V_0. The energy halves in $\frac{1}{2}\tau \ln 2 = 0.347\tau$.
- **"Explain the shape"** of a discharge curve: the p.d. across the resistor equals the p.d. across the capacitor; the current is proportional to that p.d.; the charge is proportional to the p.d.; the current is the rate of loss of charge. So the rate of loss of charge is proportional to the charge left, and it gets smaller as the charge falls: an exponential decay. Give points, not "it is inversely proportional".
- **"State the relationship between V_C and V_R"** during a discharge: they are equal.
- **Sketching**: start at x_0 on the axis (the current is **not** zero at $t = 0$), fall steeply, get less steep, and never reach or touch the time axis. Mark $0.37x_0$ at τ if you know it.

- **Smoothing** (from Alternating currents): the output falls exponentially between peaks. Read two points in **one** falling section and use the **time between them**: 9.0 V falling to 8.1 V in 7.5 ms gives $\tau = \dfrac{7.5}{\ln(9.0/8.1)} = 71.2$ ms.
- **A charged sphere discharged through a resistor to earth**: $\tau = R \times 4\pi\varepsilon_0 r$, then the same equation.

3. Energy stored in a capacitor (10 questions)

1. Choose the form that matches your data: $\frac{1}{2}CV^2$ (know C and V), $\frac{1}{2}QV$ (read off a graph), $\dfrac{Q^2}{2C}$ (charge fixed).
2. Convert units and square only the p.d. (or the charge). 330 μF at 18 V stores $\frac{1}{2} \times 330 \times 10^{-6} \times 18^2 = 53.5$ mJ.

Variations you will meet:

- **Area under the graph.** On a V against Q (or Q against V) straight line, the energy is the triangle's area $\frac{1}{2}QV$. If the p.d. rises from V_1 to V_2, the extra energy is the **trapezium** between them, $\frac{1}{2}C(V_2^2 - V_1^2)$, not $\frac{1}{2}C(V_2 - V_1)^2$: 100 μF raised from 10 V to 15 V gains 6.25 mJ.
- **Sketch Q against V while charging**: a straight line through the origin ending at the final values (V, Q); then "work done $=$ area under the graph $= \frac{1}{2}QV$".
- **Find V or C from an energy**: $V = \sqrt{\dfrac{2W}{C}}$, $C = \dfrac{2W}{V^2}$.
- **Energy against capacitance at a fixed p.d.**: $W = \frac{1}{2}CV^2$ is a straight line in C. If a variable capacitor is in parallel with a fixed one on a fixed supply, the total energy is a straight line that starts at the fixed capacitor's energy, not at zero.
- **Network energy**: find the combined capacitance, then $\frac{1}{2}CV^2$ with the supply p.d. That is quicker than adding each capacitor's energy (it gives the same answer).

4. Capacitors in series and parallel (8 questions)

1. **Parallel**: same p.d., charges add, $C = C_1 + C_2$.
2. **Series**: same charge, p.d.s add, $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$. **Invert** at the end.
3. **Networks**: combine the innermost group first. 4.0 μF and 8.0 μF in parallel make 12 μF; that in series with 6.0 μF gives $\left(\frac{1}{12} + \frac{1}{6}\right)^{-1} = 4.0$ μF.

Variations you will meet:

- **Derive the formula** ("explain your reasoning"): state the physics first, then the algebra.
  - Series: the **charge is the same** on each capacitor; the **p.d.s add**, $V = V_1 + V_2$; substitute $V = \frac{Q}{C}$ and divide by Q.
  - Parallel: the **p.d. is the same** across each; the **charges add**, $Q = Q_1 + Q_2$; substitute $Q = CV$ and divide by V. Keep the same letters as the question.

- **A table of relationships**: series, $Q = Q_1 = Q_2$ and $V = V_1 + V_2$; parallel, $Q = Q_1 + Q_2$ and $V = V_1 = V_2$.
- **The p.d. across one capacitor in series**: find Q from the combined capacitance, then $V_1 = \frac{Q}{C_1}$. The shares are in the **inverse** ratio of the capacitances.
- **Identical capacitors**: n in parallel give nC; n in series give $\frac{C}{n}$. A network of one capacitor in series with two in parallel gives $\left(\frac{1}{C} + \frac{1}{2C}\right)^{-1} = \frac{2}{3}C$; one in parallel with two in series gives $C + \frac{C}{2} = \frac{3}{2}C$.
- **Finding a single capacitance from a network's graph**: the network's gradient is its combined capacitance written in terms of C (such as $\frac{3}{2}C$); set them equal and solve.

5. Charge shared between two capacitors (3 questions)

A charged capacitor C_1 (p.d. V_0, charge $Q_0 = C_1V_0$) is disconnected from the supply and joined to an uncharged capacitor C_2.

1. Charge flows from the first to the second **because their p.d.s are different**, and it stops when the **p.d.s are equal**. The first does **not** fully discharge.
2. **Charge is conserved**: the total is still Q_0. The two act as capacitors in parallel, so the final p.d. is $V = \dfrac{Q_0}{C_1 + C_2}$.
3. Each charge is C_1V and C_2V, in the ratio of the capacitances.
4. **Energy**: final $= \frac{1}{2}(C_1 + C_2)V^2$, always **less** than the start; a fraction $\dfrac{C_1}{C_1 + C_2}$ remains. The rest becomes thermal energy in the connecting wires. For two identical capacitors the p.d. halves and half of the energy is lost.

In a "show that" or "explain your reasoning", write both principles in words: the final p.d.s are equal, and the total charge is unchanged.

6. Capacitance of an isolated sphere (2 questions)

1. $C = \dfrac{Q}{V}$ with $V = \dfrac{Q}{4\pi\varepsilon_0 r}$ gives $C = 4\pi\varepsilon_0 r$. Use it both ways: radius from C, or C from radius.
2. With a charge and a capacitance, get the potential first ($V = \frac{Q}{C}$), then the radius from $r = \dfrac{Q}{4\pi\varepsilon_0 V}$.
3. Field and potential questions on the same sphere use the Electric fields formulas; r is the radius.

7. Repeated charging and discharging: $I = fCV$ (2 questions)

A switch flips f times a second between a supply (charging the capacitor to V) and a sensitive ammeter (discharging it completely).

1. Each discharge sends charge $Q = CV$ through the ammeter.
2. f discharges a second send charge fCV each second, so the **average current** is $I = \dfrac{Q}{t} = \dfrac{CV}{1/f} = fCV$.

3. For example, 150 pF charged to 75 V and switched 400 times a second gives $I = 400 \times 150 \times 10^{-12} \times 75 = 4.5$ μA.

Changes: with f and V fixed, $I \propto C$. A second identical capacitor in **series** halves C, so it halves the charge per discharge and halves the current; in **parallel** it doubles C and the current. Explain through the charge per discharge (or $I \propto C$), not just "the current halves".

8. Changing the plate separation of an isolated capacitor (1 question)

You do not need a formula for the capacitance of plates; the question tells you that C is inversely proportional to the separation.

1. **Disconnected from the supply, the charge is trapped**: Q stays the same.
2. Find the new C from the proportion, then the new p.d. $V = \frac{Q}{C}$ and the new energy $\frac{Q^2}{2C}$.
3. For example, pulling the plates apart to **3 times** the separation makes the capacitance $\frac{C}{3}$, the p.d. $3V$ and the energy $3 \times \frac{1}{2}CV^2$. The energy goes up because work is done pulling the oppositely charged plates apart against their attraction; pushing them closer lets the attraction do work, so the stored energy falls.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A 16 μF capacitor and a 48 μF capacitor are connected in series to a 9.0 V supply.

(a) Calculate the combined capacitance. **[1]**

(b) Calculate the charge on each capacitor and the p.d. across each. **[3]**

(c) The capacitors are now reconnected in parallel to the same supply. Calculate the total energy stored, and compare it with the energy stored in series. **[3]**

Solution

(a) $\dfrac{1}{C} = \dfrac{1}{16} + \dfrac{1}{48} = \dfrac{4}{48}$, so $C = 12$ μF. **A1**

(b) In series each has the same charge: $Q = CV = 12 \times 10^{-6} \times 9.0 = 1.08 \times 10^{-4}$ C. **C1 A1** $V_{16} = \dfrac{1.08 \times 10^{-4}}{16 \times 10^{-6}} = 6.75$ V and $V_{48} = \dfrac{1.08 \times 10^{-4}}{48 \times 10^{-6}} = 2.25$ V (they add to 9.0 V). **A1**

(c) In parallel $C = 16 + 48 = 64$ μF. **C1** $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 64 \times 10^{-6} \times 9.0^2 = 2.59 \times 10^{-3}$ J. **A1** In series, $W = \frac{1}{2} \times 12 \times 10^{-6} \times 9.0^2 = 4.86 \times 10^{-4}$ J, so the parallel arrangement stores $\frac{64}{12} = 5.3$ times as much. **B1**

Example 2

A 680 μF capacitor is charged to 15 V and then, at time $t = 0$, connected across a 33 kΩ resistor.

(a) Calculate the initial charge on the capacitor and the initial current in the resistor. **[2]**

(b) Calculate the time constant of the circuit. **[2]**

(c) Calculate the time taken for the p.d. across the capacitor to fall to 4.0 V. **[2]**

(d) Calculate the time taken for the energy stored to fall to 10% of its initial value. **[3]**

Solution

(a) $Q_0 = CV_0 = 680 \times 10^{-6} \times 15 = 1.02 \times 10^{-2}$ C. **A1** $I_0 = \dfrac{V_0}{R} = \dfrac{15}{33 \times 10^3} = 4.55 \times 10^{-4}$ A. **A1**

(b) $\tau = RC$ **C1** $= 33 \times 10^3 \times 680 \times 10^{-6} = 22.4$ s. **A1**

(c) $4.0 = 15e^{-t/22.44}$ **C1**, so $t = 22.44 \ln \dfrac{15}{4.0} = 29.7$ s. **A1**

(d) $W \propto V^2$, so the energy is 10% when $\left(\dfrac{V}{V_0}\right)^2 = 0.10$, that is $\dfrac{V}{V_0} = \sqrt{0.10} = 0.316$. **C1** $0.316 = e^{-t/22.44}$ **C1**, so $t = 22.44 \times \frac{1}{2} \ln 10 = 25.8$ s. **A1**

The first two diagrams in "Understand it" show this capacitor: the energy stored at the start (the shaded area, 76.5 mJ) and the discharge curves.

Example 3

A capacitor of capacitance 250 μF is charged and then discharged through a resistor of resistance R. The graph of $\ln(V/\mathrm{V})$ against time t is a straight line, with $\ln(V/\mathrm{V}) = 2.30$ at $t = 0$ and 1.10 at $t = 6.0$ s.

(a) Determine the initial p.d. across the capacitor. **[1]**

(b) Determine the time constant, and hence the resistance R. **[4]**

(c) A second resistor of resistance R is connected in series with the first and the experiment is repeated from the same starting p.d. Describe the new line. **[2]**

Solution

(a) $V_0 = e^{2.30} = 10.0$ V. **B1**

(b) Gradient $= \dfrac{1.10 - 2.30}{6.0} = -0.200$ s^{-1}. **C1** Gradient $= -\dfrac{1}{RC}$ **C1**, so $\tau = RC = \dfrac{1}{0.200} = 5.0$ s. **A1** $R = \dfrac{\tau}{C} = \dfrac{5.0}{250 \times 10^{-6}} = 2.0 \times 10^4$ Ω. **A1**

(c) The same intercept, 2.30 (same starting p.d.). **B1** The resistance doubles, so the time constant doubles to 10 s and the gradient **halves**, to -0.100 s^{-1}. **B1**

The third diagram in "Understand it" shows this line.

Example 4

A 40 μF capacitor A is charged to 35 V and disconnected from the supply. It is then connected across an uncharged 60 μF capacitor B.

(a) Explain what happens to the charge on A. **[2]**

(b) Calculate the final p.d. across each capacitor and the final charge on each. **[3]**

(c) Calculate the energy lost, and suggest where it goes. **[3]**

Solution

(a) Some of the charge on A flows to B, because at first the p.d. across A is bigger than across B. **B1** The flow stops when the p.d.s across A and B are equal; A keeps part of its charge. **B1**

(b) Charge is conserved: $Q = 40 \times 10^{-6} \times 35 = 1.4 \times 10^{-3}$ C in total. **C1** The final p.d.s are equal, so the pair acts as 100 μF: $V = \dfrac{1.4 \times 10^{-3}}{100 \times 10^{-6}} = 14$ V across each. **A1** $Q_A = 40 \times 10^{-6} \times 14 = 5.6 \times 10^{-4}$ C and $Q_B = 60 \times 10^{-6} \times 14 = 8.4 \times 10^{-4}$ C. **A1**

(c) Before: $\frac{1}{2} \times 40 \times 10^{-6} \times 35^2 = 24.5$ mJ. After: $\frac{1}{2} \times 100 \times 10^{-6} \times 14^2 = 9.8$ mJ. **C1** Energy lost $=$ $24.5 - 9.8 = 14.7$ mJ. **A1** It becomes thermal energy in the resistance of the connecting wires while the charge flows. **B1**

Common mistakes

- **An incomplete definition.** Reports from 2021 and 2022 found "charge over potential" without saying that the charge is on **one plate** and the potential is the **p.d. between the plates**. Some candidates saw "parallel plate" and wrote the formula for capacitors in parallel instead (June 2022).
- **Not inverting.** Writing $\frac{1}{C} = 15$ μF, or giving $\frac{1}{C}$ as the answer, loses the mark (June 2022). Finish with $C = (\ldots)^{-1}$.
- **The wrong capacitance in $\tau = RC$.** Use the combined capacitance of whatever is discharging, not one capacitor's value (June 2022, March 2025). One report found the current put in place of C (November 2022).
- **Mixing up p.d.s in the exponential.** For one capacitor in a series pair, relate its later p.d. to its **own** starting p.d., or the pair's final p.d. to the supply's; don't mix the two (June 2022).
- **Putting energy into $x = x_0 e^{-t/RC}$.** Energy goes as V^2; work out the p.d. that goes with the energy asked for, then use the exponential (March 2022).
- **A log value used as a charge.** On a $\ln Q$ graph the axis values are logs; convert with $e^{\cdots}$ before using them as charges (March 2023).

- **"The current is zero at the start".** At $t = 0$ the current is at its **largest**, $\frac{V_0}{R}$ (November 2022).
- **Sketches that touch the axis or go flat.** An exponential decay gets closer and closer to zero but never reaches it (November 2022).
- **"Inversely proportional" for the discharge curve.** The explanation needs the rate of loss of charge proportional to the charge (November 2022).
- **Vague changes.** Doubling C doubles τ; a parallel identical resistor halves it. Say the factor (March 2022, March 2023).
- **Charge sharing.** "A loses all its charge" is wrong; charge moves until the p.d.s are equal, and the total charge stays the same (June 2023).
- **Forgetting the charge is trapped.** An isolated capacitor keeps its charge when the plates move; its p.d. changes (November 2021). And the energy has a $\frac{1}{2}$ in it.
- **Smoothing time.** Use the time **between** two readings in one discharge, not a clock reading (June 2022, June 2023).
- **No unit, or the wrong one, for τ.** A time constant is a time, in seconds (March 2025).
- **Using the wrong p.d.** Read the question for the p.d. across the capacitor itself; it may not be the supply's (March 2022).

How to get the marks

- **Definitions**: "charge (on one plate) per unit potential difference (between the plates)"; for a sphere "charge per unit potential".
- **Derivations** ("explain your reasoning"): each physics statement is a mark on its own. Series: "the charge on each capacitor is the same" and "the total p.d. is the sum of the p.d.s". Parallel: "the p.d. across each is the same" and "the total charge is the sum". Then the algebra, in the question's letters.
- **Calculations**: **C1** for the right formula with numbers in (such as $\tau = RC$, $W = \frac{1}{2}CV^2$, or the exponential with values), **A1** for the answer. Show the substitution even if the answer is wrong: later marks follow through from an earlier error.
- **"Show that"**: say where each value comes from (a gradient, a graph reading, an earlier part), substitute everything, and give one more significant figure than the printed value.
- **Graph readings**: quote the values you read, with their units, and choose them from the part of the graph where readings are big and clear.
- **Units**: μF, mC, kΩ and mJ must be converted; answer in the unit asked (μF, kΩ, mJ). "Give a unit" for τ means s; for C, F.
- **"Explain the shape"**: give separate points (p.d. across resistor = p.d. across capacitor; $I \propto V$; $Q \propto V$; I = rate of loss of charge), then the conclusion that the rate of decrease is proportional to what is left.
- **"State and explain"** a change: the change with its factor (doubled, halved) and the reason.
- **Sketches**: exponential decays start at the right value, fall steeply, level off towards the axis without touching; straight lines go through the origin (or the correct starting value) and end at the right point.
- Give answers to 2 or 3 significant figures to match the data.

Quick check

1. An isolated metal sphere in a vacuum has a radius of 0.35 m. Calculate its capacitance, and the charge on it when its potential is 3.0 kV.
2. Two 8.0 μF capacitors are connected in parallel, and this pair is connected in series with a third 8.0 μF capacitor. Calculate the combined capacitance.
3. A capacitor stores 50 mJ when the p.d. across it is 25 V. Calculate its capacitance and its charge.
4. A capacitor discharging through a resistor has its p.d. fall from 9.0 V to 6.0 V in 4.0 s. Calculate the time constant, and the p.d. 4.0 s later.
5. A 330 pF capacitor is charged to 90 V and discharged through a sensitive ammeter 200 times each second. Calculate the average current.

Answers

1. $C = 4\pi\varepsilon_0 r = 4\pi \times 8.85 \times 10^{-12} \times 0.35 = 3.89 \times 10^{-11}$ F (38.9 pF). $Q = CV = 3.89 \times 10^{-11} \times 3000 = 1.17 \times 10^{-7}$ C.
2. The parallel pair is 16 μF. In series with 8.0 μF: $\left(\frac{1}{16} + \frac{1}{8}\right)^{-1} = \frac{16}{3} = 5.3$ μF.
3. $C = \dfrac{2W}{V^2} = \dfrac{2 \times 0.050}{25^2} = 1.6 \times 10^{-4}$ F (160 μF). $Q = \dfrac{2W}{V} = \dfrac{2 \times 0.050}{25} = 4.0 \times 10^{-3}$ C.
4. $\dfrac{6.0}{9.0} = e^{-4.0/\tau}$, so $\tau = \dfrac{4.0}{\ln 1.5} = 9.87$ s. In the next 4.0 s it falls by the same fraction again: $6.0 \times \frac{6.0}{9.0} = 4.0$ V.
5. $I = fCV = 200 \times 330 \times 10^{-12} \times 90 = 5.94 \times 10^{-6}$ A (5.94 μA).

Past questions to try

- **9702/42/M/J/22 Q5**: deriving the parallel formula, then one capacitor of a series pair discharging.
- **9702/42/F/M/23 Q5**: capacitance and resistance from a graph of $\ln Q$ against time.
- **9702/43/O/N/21 Q6**: moving the plates of an isolated capacitor, with expressions for charge, p.d. and energy.
- **9702/43/M/J/21 Q7**: the average current $I = fCV$ from repeated switching, and a second capacitor in series.