Cambridge AS & A Level · Physics (9702) · Notes

Forces, density and pressure AS

Read online at pastopic.com/cambridge/physics-9702/notes/forces-density-and-pressure

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Read first: Physical quantities and units, Dynamics

What you need to know

By the end of this topic you should be able to:

1. **Understand centre of gravity**: the weight of an object can be treated as acting at one point, its centre of gravity.
2. **Define and use the moment of a force**: force × perpendicular distance from the point to the line of action of the force.
3. **Understand that a couple** is a pair of forces that makes something rotate without moving it along.
4. **Define and use the torque of a couple**: one of the forces × the perpendicular distance between their lines of action.
5. **State and apply the principle of moments.**
6. **Understand that a system is in equilibrium** when there is no resultant force and no resultant torque on it.
7. **Use a vector triangle** to represent three coplanar forces in equilibrium.
8. **Define and use density**: mass per unit volume.
9. **Define and use pressure**: force per unit area.
10. **Derive** $\Delta p = \rho g \Delta h$ for the pressure in a fluid, starting from the definitions of pressure and density.
11. **Use** $\Delta p = \rho g \Delta h$**.**
12. **Understand that upthrust** on an object in a fluid comes from the difference in hydrostatic pressure between its top and bottom.
13. **Calculate upthrust with** $F = \rho g V$ (Archimedes' principle).

Resolving a force into components comes from Physical quantities and units, and weight and $F = ma$ come from Dynamics. Stress and strain, and the pressure of a gas from its molecules, come in later topics.

Understand it

Centre of gravity

Every small piece of an object has weight. For working out how the object balances or turns, you can replace all of these by **one force, the total weight, acting at one point**: the **centre of gravity**. It is where the weight *may be taken to act*; gravity still pulls on every part.

- For a **uniform** object (the same material and thickness throughout) the centre of gravity is at the **geometric centre**: the middle of a uniform rod, the centre of a uniform square sheet.
- For a **non-uniform** object it is nearer the heavier part. A bat's centre of gravity is nearer the thick end.
- An object **hanging freely** from a pivot comes to rest with its centre of gravity **directly below the pivot**, because only then does its weight have no moment about the pivot. Hang it from two different points, draw the vertical line below the pivot each time, and the lines cross at the centre of gravity.

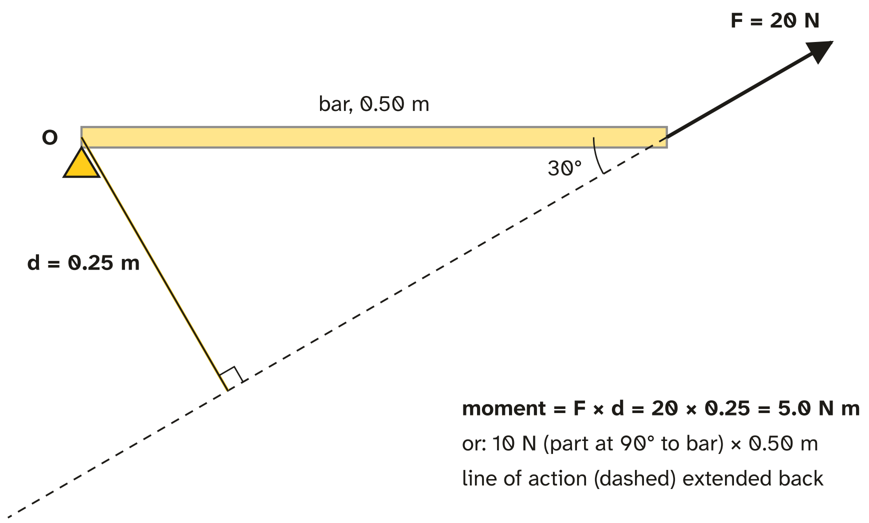
Moment of a force

A force can turn an object about a point. How strongly it turns it is its **moment**:

$$\text{moment} = F \times d$$

where d is the **perpendicular distance from the point to the line of action of the force**. The unit is N m. Pushing a door at the handle, far from the hinge, turns it easily; pushing near the hinge barely moves it.

When the force is at an angle, the distance to use is **not** the distance along the object.



Two ways to get the same moment for this 20 N force at 30° to a 0.50 m bar:

- **Perpendicular distance**: extend the line of action, drop a perpendicular from O: $d = 0.50 \sin 30^\circ = 0.25$ m, so moment $= 20 \times 0.25 = 5.0$ N m.
- **Component**: only the part of the force at right angles to the bar turns it: $20 \sin 30^\circ = 10$ N, times 0.50 m $= 5.0$ N m. The part along the bar points straight at O and has no moment.

The same force at right angles to the bar gives the biggest moment, $20 \times 0.50 = 10$ N m. A force whose line of action passes **through** the point has no moment about it.

The principle of moments

The principle of moments: for an object in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about **the same point**.

A seesaw with its pivot in the middle: a child of weight 300 N sits 1.5 m from the pivot. A second child of weight 450 N balances it at a distance x on the other side:

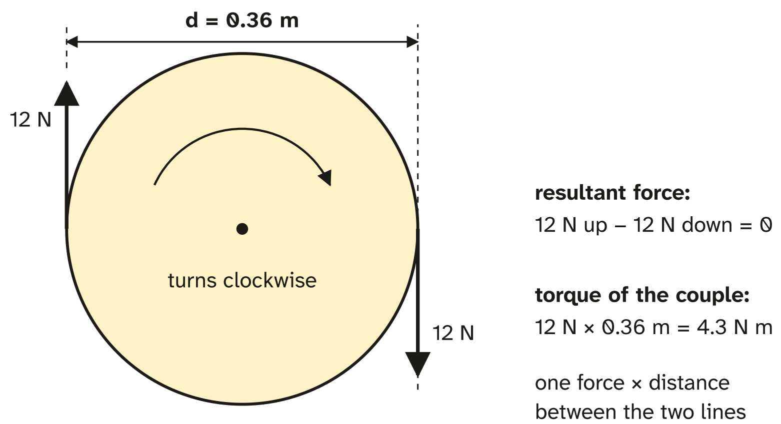
$$450x = 300 \times 1.5 \quad \Rightarrow \quad x = 1.0 \text{ m}.$$

The plank's own weight acts at the pivot, so it has no moment. The pivot pushes up with a force equal to all the weights: for a 200 N plank, $300 + 450 + 200 = 950$ N.

Choosing the point. You may take moments about any point, and you should choose the point where an unknown force acts (a hinge, a support you don't care about): that force then has no moment and drops out of the equation.

Couples and torque

A **couple** is a pair of forces that are **equal in size, opposite in direction and act along different (parallel) lines**. They add up to zero, so there is **no resultant force**: the object doesn't start moving along. But they both turn it the same way, so it **rotates**. A couple produces rotation only.



The turning effect of a couple is its **torque**:

$$\text{torque of a couple} = F \times d$$

where F is **one** of the forces and d is the **perpendicular distance between their lines of action**. For the wheel, $12 \times 0.36 = 4.3$ N m.

It doesn't matter which point you take moments about. About a point 0.10 m from the left-hand line: $12 \times 0.10 + 12 \times 0.26 = 4.32$ N m, the same as 12×0.36. That's why a couple has one torque, not a different moment about each point.

Two things that are **not** couples: two equal forces in the **same** direction (resultant $2F$), and two equal and opposite forces along the **same** line (no turning at all).

"Moment" and "torque" are both turning effects measured in N m. We say the **moment** of a single force about a point and the **torque** of a couple.

Equilibrium

An object is in **equilibrium** when

1. the **resultant force** on it is zero, so it has no acceleration (it stays at rest or moves at a constant velocity), **and**
2. the **resultant torque** (resultant moment) about any point is zero, so it doesn't start rotating (or rotate faster or slower).

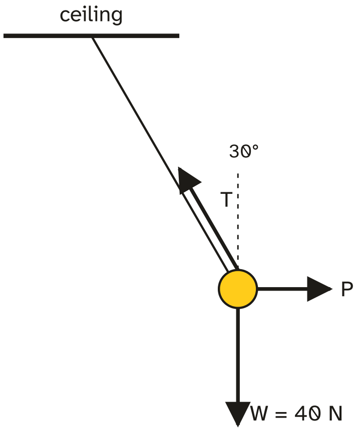
Both must be true. A couple alone gives zero resultant force but a resultant torque, so it is not equilibrium. A car turning a corner at a steady speed is not in equilibrium either: its velocity is changing direction.

Useful results:

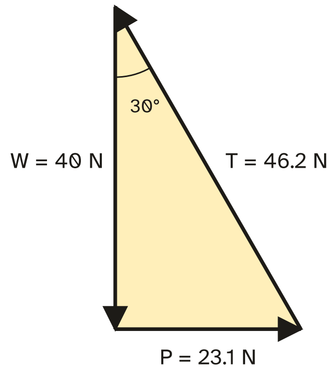
- **Two forces only**: they must be equal, opposite and act along the **same line**.
- **Three forces that are not parallel**: their lines of action must pass through **one point**, or they would have a resultant moment about where two of them cross. This tells you the direction of a hinge force: it points through the point where the other two lines of action meet.
- **Resolving**: in equilibrium, the components of all the forces add to zero in **every** direction, so (up) = (down) and (left) = (right).

Vector triangles

If three forces act on an object in equilibrium, their resultant is zero. So when you draw them **tip to tail**, in any order, they form a **closed triangle**.



forces on the lantern



tip to tail: a closed triangle

For the lantern:

- **By scale drawing**: choose a scale (say 1 cm to 5 N), draw W straight down, draw a horizontal line from its tip, and draw a line at 30° to the vertical back from the start. Where they cross fixes P and T; measure them.
- **By calculation** from the right-angled triangle: $T = \dfrac{W}{\cos 30^\circ} = \dfrac{40}{0.866} = 46.2$ N and $P = W \tan 30^\circ = 23.1$ N.
- **By resolving** gives the same thing: vertically $T \cos 30^\circ = W$, horizontally $T \sin 30^\circ = P$.

The arrows go the same way round the triangle, and each side is parallel to its force. If the triangle doesn't close, there is a resultant force.

Density

Density is mass per unit volume:

$$\rho = \frac{m}{V}$$

The unit is kg m^{-3}. Water is 1000 kg m^{-3}, which is 1.0 g cm^{-3} (multiply g cm^{-3} by 1000 to get kg m^{-3}, because 1 m^3 $= 10^6$ cm^3 and 1 kg $= 1000$ g). An aluminium block 5.0 cm × 4.0 cm × 2.0 cm has volume 40 cm^3; with a mass of 108 g its density is $\frac{108}{40} = 2.7$ g cm^{-3} $= 2700$ kg m^{-3}.

Density belongs to the material: cutting a block in half halves the mass and the volume, and the density stays the same. It doesn't depend on g either, so a rock has the same density on any planet. For a **mixture**, add the masses and add the volumes: $\rho = \dfrac{m_1 + m_2}{V_1 + V_2}$ (not the average of the two densities, unless the volumes are equal).

Pressure

Pressure is the force acting at right angles (normal) to a surface per unit area:

$$p = \frac{F}{A}$$

The unit is the pascal: 1 Pa $= 1$ N m^{-2}. Pressure is a scalar.

The aluminium block above weighs $0.108 \times 9.81 = 1.06$ N. Resting on its biggest face ($5.0 \times 4.0 = 20$ cm² $= 2.0 \times 10^{-3}$ m²) it exerts $\frac{1.06}{2.0\times10^{-3}} = 530$ Pa; standing on its smallest face (8.0×10^{-4} m²) it exerts 1320 Pa. Same force, smaller area, more pressure. (For any block with vertical sides, $p = \frac{\rho A h g}{A} = \rho g h$, where h is its height as it stands.)

Pressure in a liquid: deriving $\Delta p = \rho g \Delta h$

Picture a column of liquid of density ρ, height Δh and cross-sectional area A, resting on a horizontal area A.

1. Volume of the column $= A\Delta h$.
2. Mass $= \rho \times$ volume $= \rho A \Delta h$ (definition of density).
3. Weight $= mg = \rho A \Delta h \, g$.
4. Pressure $= \dfrac{\text{force}}{\text{area}} = \dfrac{\rho A \Delta h \, g}{A}$ (definition of pressure), so

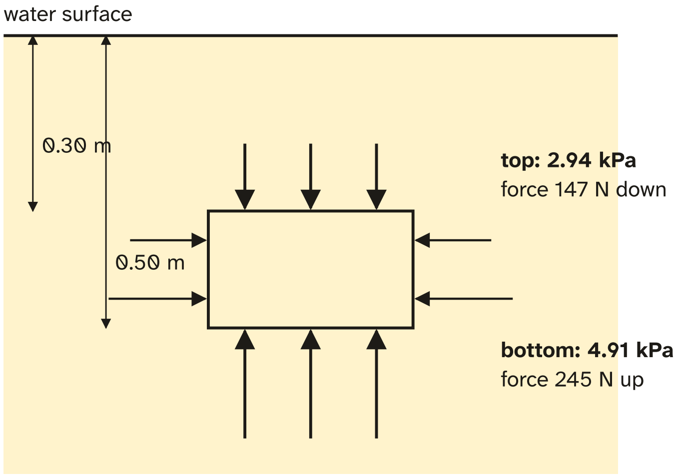
$$\Delta p = \rho g \Delta h.$$

This is the extra pressure from the liquid above; A cancels, so it doesn't depend on the width or shape of the container, only on the depth. At a depth h in water, the **total** pressure is the air pressure on the surface plus $\rho g h$. Every 10 m of water adds $1000 \times 9.81 \times 10 = 98\,100$ Pa $= 98.1$ kPa, nearly one more atmosphere, so a diver 10 m down feels about $101 + 98.1 = 199$ kPa.

Pressure in a fluid acts **in all directions**, and it is the same at every point at the same depth in the same still liquid. That's why, in a U-tube holding one liquid, both surfaces are level; and with two liquids that don't mix, you compare the pressures at the level of the boundary between them.

Upthrust

The pressure on the **bottom** of a submerged object is bigger than on its **top**, because the bottom is deeper. So the liquid pushes up on the bottom harder than it pushes down on the top. The difference is an upward force: the **upthrust**.



upthrust = 245 − 147 = 98 N up (= ρgV, V = 0.010 m³)

For this block in water:

- top face, 0.30 m deep: $p = 1000 \times 9.81 \times 0.30 = 2943$ Pa, force $= 2943 \times 0.050 = 147$ N **down**;
- bottom face, 0.50 m deep: $p = 1000 \times 9.81 \times 0.50 = 4905$ Pa, force $= 4905 \times 0.050 = 245$ N **up**;
- the sideways forces on opposite sides are at the same depths, so they cancel;
- upthrust $= 245 - 147 = 98$ N up.

In symbols: $(p_{\text{bottom}} - p_{\text{top}})A = \rho g(\text{height of block})A = \rho g V$. That is **Archimedes' principle**:

$$F = \rho g V$$

where ρ is the density of the **fluid** and V is the volume of the object **below the surface** (the volume of fluid pushed aside). For the block, $1000 \times 9.81 \times 0.010 = 98.1$ N. The upthrust equals the weight of the fluid displaced.

What follows from $F = \rho g V$:

- Once an object is fully under, the upthrust **doesn't change with depth**: both pressures grow by the same amount. A ball rising from the sea bed has a constant upthrust until it reaches the surface.
- The upthrust doesn't depend on what the object is made of: a steel block and a wooden block of the same volume, both fully under water, get the same upthrust.
- It is proportional to g: on the Moon, where $g = 1.6$ m s^{-2}, the same object in the same liquid gets $\frac{1.6}{9.81} = 0.16$ times the upthrust.
- It works in any fluid, including air. A 0.50 m^3 balloon in air of density 1.2 kg m^{-3} has an upthrust of $1.2 \times 9.81 \times 0.50 = 5.9$ N.

Floating. A floating object is in equilibrium, so upthrust = weight: $\rho_{\text{fluid}}\, g\, V_{\text{under}} = \rho_{\text{object}}\, g\, V$. So the **fraction below the surface** $= \dfrac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$. Ice (920 kg m^{-3}) in sea water (1030 kg m^{-3}) floats with $\frac{920}{1030} = 0.89$, i.e. 89%, under the surface. Anything denser than the fluid sinks.

Weighing under water. A newton meter holding an object in a liquid reads the weight minus the upthrust: reading $= W - \rho g V$.

Key facts and formulas

"Given" means it is on the data sheet in the exam. "Learn it" means you must remember it.

Formula	Status
Moment $= F \times d$, d = perpendicular distance from the point to the line of action (N m)	Learn it
Principle of moments: in equilibrium, sum of clockwise moments = sum of anticlockwise moments about the same point	Learn it
Torque of a couple $= F \times d$, one force × perpendicular distance between the lines of action	Learn it
Equilibrium: resultant force $= 0$ and resultant torque $= 0$	Learn it
Three forces in equilibrium form a closed vector triangle	Learn it
Density $\rho = \dfrac{m}{V}$ (kg m^{-3}; 1 g cm^{-3} $= 1000$ kg m^{-3})	Learn it
Pressure $p = \dfrac{F}{A}$ (Pa $=$ N m^{-2})	Learn it
Hydrostatic pressure $\Delta p = \rho g \Delta h$	Given
Total pressure at depth h = atmospheric pressure $+\rho g h$	Learn it
Upthrust $F = \rho g V$ (ρ of the fluid, V submerged)	Given
Floating: upthrust = weight; fraction submerged $= \rho_{\text{object}}/\rho_{\text{fluid}}$	Learn it
Volume of a sphere $\frac{4}{3}\pi r^3$; of a cylinder $\pi r^2 h$	Learn it
$g = 9.81$ m s^{-2}	Given

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (201 different questions, 2021–2025; 6 more appear on two papers), the types came up this often. 27 of the 201 questions mix two or more types, so the counts add up to more than 201.

Question type	Questions
Moments and the principle of moments	60
Upthrust and floating	45
Equilibrium, resolving forces and vector triangles	36
Couples and torque	29
Density	22
Hydrostatic pressure, $\Delta p = \rho g \Delta h$	18
Pressure from force and area	15
Centre of gravity	8

1. Moments and the principle of moments (60 questions)

1. **Draw the object with every force on it**, including its own weight at its centre of gravity (the middle, if it is uniform). Missing the beam's weight is the most common reason for a wrong answer.
2. **Choose the point** to take moments about: where an unknown force you don't need acts (a hinge, a support).
3. **Find each moment** = force × **perpendicular** distance from that point. For a force at an angle, use the component at right angles to the beam (Example 2).
4. **Write clockwise moments = anticlockwise moments** and solve.
5. **For a second unknown force**, use up = down (resolving), or take moments about a different point.

Variations you will meet:

- **Beam on two supports** (a plank, a bridge, a person standing on it): moments about one support give the other support's force (Example 1). As a person walks towards a support, that support's force increases and the other's decreases. When the beam is about to tip about one support, the force at the other support is zero.
- **Hinged beam held by a cable or string at an angle**: take moments about the hinge; only the vertical component $T \sin\theta$ of a cable at θ to a horizontal beam has a moment about the hinge (Example 2). Then the hinge's force components come from resolving.
- **Metre rule balanced on a pivot**: use the scale readings to get distances from the pivot. The rule's own weight acts at the 50.0 cm mark if it is uniform.
- **Moment of a weight when the object is tilted**: an object hanging at an angle θ to the vertical from a hinge at its top has its centre of gravity at a **horizontal** distance $\frac{L}{2}\sin\theta$ from the hinge, so the moment is $W \times \frac{L}{2}\sin\theta$. For a rod at angle θ to the **horizontal**, the horizontal distance is $\frac{L}{2}\cos\theta$; the moment is greatest when the rod is horizontal and zero when it is vertical.
- **Resultant moment**: find the clockwise total and the anticlockwise total separately and subtract; give the direction. When a string is cut, the resultant moment is the moment that string was providing.

- **Finding a centre of gravity or an unknown weight**: write the principle of moments with the unknown distance or weight as the only unknown.
- **"Which change increases the force?"** (levers): a longer distance from the pivot on the effort side, or a shorter distance on the load side, gives a bigger force on the load.

2. Upthrust and floating (45 questions)

1. **List the forces**: weight (down), upthrust (up), and any tension, extra push or drag.
2. **Upthrust** $= \rho g V$ with the **fluid's** density and the **submerged** volume. Recall $\frac{4}{3}\pi r^3$ for a sphere and Ah for a cylinder or prism.
3. **If it is in equilibrium**, up = down: for example upthrust = weight + tension in a string holding it under, or upthrust + reading on a newton meter = weight.
4. **If it isn't**, the resultant force $=$ upthrust $-$ weight $= ma$ (Dynamics).

Variations you will meet:

- **"Explain why there is an upthrust"**: the pressure increases with depth, so the pressure on the bottom is greater than on the top; the upward force on the bottom is greater than the downward force on the top, so the resultant force from the fluid is upwards. Don't use Newton's third law here.
- **Floating** (a sphere, an iceberg, a cylinder): upthrust = weight, so the submerged fraction $= \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$, and the height above the surface $= H\left(1 - \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}\right)$ for an upright cylinder of height H (Example 3).
- **Held under and released**: the initial acceleration $= \dfrac{U - W}{m}$. As it speeds up, drag grows and the acceleration falls; the upthrust stays the same.
- **Balloons and airships**: upthrust $= \rho_{\text{air}}\, g\, V$. With a rope holding it down, $U = W + T$; released, $a = \frac{U-W}{m}$.
- **Newton meter readings**: reading $= W - U$. In two liquids, $\dfrac{\rho_1}{\rho_2} = \dfrac{W - R_1}{W - R_2}$, the ratio of the upthrusts.
- **Graph of upthrust against depth** for an object being lowered in: it rises steadily while more of the object goes under, then stays constant once it is fully submerged. A newton meter reading does the opposite.
- **Object falling through a liquid at terminal velocity**: weight = drag + upthrust.
- **"Which change doubles the upthrust?"**: only the fluid's density, the submerged volume or g matter; the object's density, orientation and depth (once fully under) don't.
- **Density of the object from floating**: $\rho_{\text{object}} = \rho_{\text{fluid}} \times$ fraction submerged.

3. Equilibrium, resolving forces and vector triangles (36 questions)

1. **Draw every force** on the one object (a knot, a lantern, a pulley, a block on a slope).
2. **Either resolve**: choose two directions at right angles (usually horizontal and vertical, or along and across a slope) and write "sum in each direction = 0";
3. **or draw a vector triangle**: arrows tip to tail, each parallel to its force, forming a closed triangle; then use trigonometry (right-angled: $\sin$, $\cos$, $\tan$) or measure a scale drawing.

Variations you will meet:

- **State the conditions for equilibrium**: resultant force zero **and** resultant torque (moment) zero. "The forces are balanced" is not enough.
- **"Which diagram shows the vector triangle?"**: the arrows must follow each other round the triangle in one direction, with no arrow pointing head to head.
- **A mass on a string pulled sideways** by a horizontal force: $T\cos\theta = W$ and $T\sin\theta = F$, so $F = W\tan\theta$ and $T^2 = F^2 + W^2$.
- **Object hanging from two strings**: resolve horizontally (the horizontal components are equal) and vertically (the vertical components add to the weight). The closer the strings are to horizontal (for example a shorter loop over a nail), the bigger the tension needed, though the total force on the nail still equals the weight.
- **An elastic cord or string pulled into a V** with tension T in each half: the resultant along the middle line is $2T\cos\alpha$, where α is each half's angle to that line. Work out $\cos\alpha$ from the lengths given.
- **Block on a slope in equilibrium**: along the slope, friction $= W\sin\theta$; at right angles, the normal contact force $= W\cos\theta$.
- **Moving at constant velocity is equilibrium**: an aircraft climbing steadily at angle θ has lift $= W\cos\theta$ and thrust $=$ drag $+W\sin\theta$. Anything changing direction (even at a constant speed) is not in equilibrium.
- **Direction of a hinge force**: for three non-parallel forces the lines of action meet at one point, so the hinge force points towards where the weight's line crosses the cable's line.
- **Two forces only**: equal, opposite, along the same line through the object.

4. Couples and torque (29 questions)

1. **Check it is a couple**: two forces, equal in size, opposite in direction, along parallel but different lines. The resultant force is zero.
2. **Find the perpendicular distance between the two lines of action**, not the distance from each to the centre.
3. **Torque** $= F \times d$, with F **one** of the forces.

Variations you will meet:

- **Forces at opposite edges of a wheel, lid or tap**: d is the diameter, so torque $= F \times 2r$. For a lid that needs a torque τ, each force is $F = \frac{\tau}{2r}$.
- **Forces not at right angles to the line joining them**: find the perpendicular distance between the lines of action, or multiply the perpendicular component of each force by its distance from the centre and add.
- **Thrusters, sprinklers, rockets on a pivoted rod**: each force at distance x from the centre gives Fx; two of them give $2Fx$ (not Fx).
- **Definitions and "which statement"**: a couple produces rotation only; its forces do **not** act through the same point and do **not** produce a resultant force.
- **Is it in equilibrium?**: a couple alone gives no resultant force but a resultant torque, so the object is not in equilibrium; it has an angular acceleration.

- **Four forces round a disc**: pair them up into couples, check whether the torques add or cancel, and check the resultant force separately.

5. Density (22 questions)

1. **Get mass and volume in SI units**: g → kg (÷1000), cm³ → m³ ($\times 10^{-6}$), or work in g cm⁻³ and multiply by 1000 at the end.
2. $\rho = \dfrac{m}{V}$, with V from the shape (cuboid xyz, cylinder $\pi r^2 l$, sphere $\frac{4}{3}\pi r^3$).

Variations you will meet:

- **Define density**: "mass per unit volume". Not "mass of one cubic metre".
- **Mixtures**: total mass ÷ total volume. To find how much of one liquid to add, write $\rho_{\text{mix}}(V_1 + V_2) = \rho_1 V_1 + \rho_2 V_2$ and solve for the unknown volume (Quick check 2).
- **Cubes of the same material**: mass $\propto$ side³, so doubling the mass multiplies the side by $\sqrt[3]{2} = 1.26$.
- **Atoms in a lattice**: each atom takes up a cube of side a, so $\rho = \frac{m_{\text{atom}}}{a^3}$ and $a = \sqrt[3]{m/\rho}$.
- **From a graph**: mass against length for rods of radius r has gradient $G = \rho \pi r^2$, so $\rho = \frac{G}{\pi r^2}$.
- **Mass flow**: the mass passing each second through a pipe of area A at speed v is $\rho A v$.
- **Density doesn't change with** g or with the size of the sample.

6. Hydrostatic pressure, $\Delta p = \rho g \Delta h$ (18 questions)

1. **Find the depth** below the liquid surface (or the difference in depth between two points), in metres.
2. $\Delta p = \rho g \Delta h$. Add atmospheric pressure only if asked for the **total** pressure.
3. **For two liquids** stacked in one container, add the two contributions: $\rho_1 g h_1 + \rho_2 g h_2$.

Variations you will meet:

- **Derive** $p = \rho g h$ (Example 4): volume Ah, mass $\rho A h$, weight $\rho A h g$, pressure = weight ÷ A. Name the definitions you use. Potential energy is not part of it.
- **Pressure is proportional to depth**: at $\frac{3}{4}$ of the depth it is $\frac{3}{4}$ of the pressure at the bottom, whatever the container's shape. Doubling the depth doubles the pressure due to the liquid.
- **U-tubes**: the pressure is the same at the same level in one continuous liquid. With two liquids, equate $\rho_1 g h_1 = \rho_2 g h_2$ for the columns above the boundary level (Example 4). With gas pressures on each side, $p_1 - p_2 = \rho g \times$ (difference in levels).
- **Depth from a force on a plate**: $p = \frac{F}{A}$, then $h = \frac{p}{\rho g}$.
- **Density from a pressure–depth graph**: gradient $= \rho g$. The intercept is the atmospheric pressure.
- **Pressure at the bottom of a submerged object** that just reaches the surface: its depth is its full height (for a sphere, $2r$).

7. Pressure from force and area (15 questions)

1. **Force**: usually a weight, mg. Share it if there are several feet or legs.

2. **Area** in m²: 1 cm² $= 10^{-4}$ m², 1 mm² $= 10^{-6}$ m². A circle from its circumference: $r = \frac{C}{2\pi}$, then πr^2.
3. $p = \dfrac{F}{A}$.

Variations you will meet:

- **Define pressure**: "force per unit area" (normal force). Not "per unit volume", not "mass per unit area".
- **Largest pressure from a block**: stand it on its **smallest** face. For a solid with vertical sides, $p = \rho g h$ with h its height as it stands.
- **Hemisphere or cone on a table**: weight $= \rho g V$, area = the base; divide.
- **Estimate** a person's pressure on the ground: about 600 N on about 0.05 m², so about 10^4 Pa.
- **Area from a pressure**: $A = \frac{F}{p}$ (convert kPa to Pa).
- **Work done by a pressure**: $W = F\Delta x = pA\Delta x = p\Delta V$.
- **A jet of water on a wall**: find the force from the change in momentum per second (Dynamics), then $p = \frac{F}{A}$. Not $\rho g h$.

8. Centre of gravity (8 questions)

- **Define it**: "the point where all the weight of an object may be taken to act". Say **weight**, not mass.
- **Uniform object**: centre of gravity at the geometric centre, so its weight has no moment about a pivot at its midpoint.
- **Hanging freely**: the centre of gravity is directly below the point of support, so it lies on the vertical line through it. For a square sheet hanging from a corner, that line is a diagonal.
- **Held away from that position by a torque**: the torque = weight × horizontal distance of the centre of gravity from the pivot, so that distance $= \frac{\tau}{W}$. Combine it with the line from hanging freely to pin down the point.
- **From a balance**: use the principle of moments with the centre of gravity's position as the unknown.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A uniform plank of length 4.0 m and weight 150 N rests horizontally on two supports: A at one end and B at 3.0 m from A. A painter of weight 700 N stands on the plank 1.0 m from A.

(a) By taking moments about A, calculate the force from support B on the plank. **[3]**

(b) Calculate the force from support A on the plank. **[1]**

(c) The painter walks along the plank, past B, towards the far end. Calculate how far beyond B he can stand before the plank starts to tip. **[2]**

Solution

(a) The plank's weight acts at its centre, 2.0 m from A. **B1** Clockwise moments = anticlockwise moments about A: $R_B \times 3.0 = 700 \times 1.0 + 150 \times 2.0$ **M1** $3.0R_B = 1000$, so $R_B = 333$ N. **A1**

(b) Up = down: $R_A = 700 + 150 - 333 = 517$ N. **A1**

(c) When it is about to tip about B, $R_A = 0$. Moments about B: the plank's weight is 1.0 m on the A side of B, so $700x = 150 \times 1.0$ **M1** $x = 0.21$ m beyond B. **A1**

Example 2

A uniform beam AB of length 1.2 m and weight 60 N is hinged to a vertical wall at A. It is held horizontal by a cable from B to a point on the wall above A; the cable makes 40° with the beam. A load of weight 90 N hangs from the beam 0.90 m from A.

(a) State the two conditions for the beam to be in equilibrium. **[2]**

(b) By taking moments about A, calculate the tension T in the cable. **[3]**

(c) Calculate the horizontal and vertical components of the force from the hinge on the beam. **[3]**

(d) The load is moved closer to B. State and explain what happens to the tension. **[1]**

Solution

(a) The resultant force on the beam is zero. **B1** The resultant torque (moment) about any point is zero. **B1**

(b) Only the vertical component $T \sin 40^\circ$ has a moment about A (the horizontal component acts along the beam, through A). **B1** $T \sin 40^\circ \times 1.2 = 60 \times 0.60 + 90 \times 0.90 = 36 + 81 = 117$ N m **M1** $T = \dfrac{117}{1.2 \times 0.643} = 152$ N. **A1**

(c) Horizontally: the cable pulls B towards the wall with $T \cos 40^\circ = 116$ N, so the hinge pushes the beam away from the wall with 116 N. **A1** Vertically: $T \sin 40^\circ = 97.5$ N up from the cable, and the weights total 150 N down **M1**, so the hinge pushes up with $150 - 97.5 = 52.5$ N. **A1**

(d) The load's moment about A increases, so the cable's moment must increase: the tension **increases. B1**

Example 3

A solid wooden cylinder has cross-sectional area 0.020 m², length 0.30 m and density 700 kg m^{-3}. It floats upright in water of density 1000 kg m^{-3}.

(a) Show that the weight of the cylinder is about 41 N. **[2]**

(b) Calculate the depth of the bottom of the cylinder below the water surface. **[2]**

(c) Use $\Delta p = \rho g \Delta h$ to show that the force of the water on the bottom face equals the weight. **[2]**

(d) Calculate the extra downward force needed to hold the cylinder with its top just level with the water surface. **[2]**

Solution

(a) Mass $= \rho V = 700 \times 0.020 \times 0.30 = 4.2$ kg **C1** Weight $= 4.2 \times 9.81 = 41.2$ N. **A1**

(b) Floating, so upthrust = weight: $1000 \times 9.81 \times 0.020 \times h = 41.2$ **C1** $h = 0.21$ m. **A1** (Check: $\frac{700}{1000} \times 0.30 = 0.21$ m.)

(c) Pressure on the bottom due to the water $= 1000 \times 9.81 \times 0.21 = 2060$ Pa **C1** Force $= pA = 2060 \times 0.020 = 41.2$ N, the same as the weight. **A1**

(d) Fully under, upthrust $= 1000 \times 9.81 \times 0.020 \times 0.30 = 58.9$ N **C1** Extra force $= 58.9 - 41.2 = 17.7$ N. **A1**

Example 4

(a) A liquid of density ρ fills a container to a depth h. Starting from the definitions of density and pressure, show that the pressure due to the liquid on the base is $\rho g h$. **[3]**

(b) A diver is 25 m below the surface of the sea. The sea water has density 1030 kg m^{-3} and the atmospheric pressure is 101 kPa. Calculate the total pressure on the diver. **[2]**

(c) A U-tube contains water of density 1000 kg m^{-3}. Oil of density 850 kg m^{-3} is poured into one arm until the oil column is 12.0 cm tall. The oil and water don't mix. Calculate how far the top of the oil is above the top of the water in the other arm. **[3]**

Solution

(a) Take the liquid above an area A of the base. Its volume is Ah, so its mass is $m = \rho A h$ (density = mass ÷ volume). **B1** Its weight is $mg = \rho A h g$. **B1** Pressure = force ÷ area $= \dfrac{\rho A h g}{A} = \rho g h$. **B1**

(b) Due to the water: $1030 \times 9.81 \times 25 = 2.53 \times 10^5$ Pa **C1** Total $= 101\,000 + 253\,000 = 354$ kPa. **A1**

(c) At the level of the oil–water boundary the pressures in the two arms are equal. **B1** $850 \times 9.81 \times 0.120 = 1000 \times 9.81 \times h_w$, so $h_w = 0.102$ m $= 10.2$ cm of water above that level. **C1** The oil surface is $12.0 - 10.2 = 1.8$ cm higher. **A1**

Common mistakes

- **Using the wrong distance for a moment.** The distance must be perpendicular to the line of action, and measured from the point you are taking moments about. Examiners often see the distance along a tilted beam used without $\sin$ or $\cos$, or the beam's weight placed at the wrong distance from a hinge.
- **Forgetting the beam's own weight.** A uniform beam's weight acts at its middle and has a moment about any point except the middle. In multiple choice, the most popular wrong answer often comes from leaving it out.
- **Stating the principle of moments without "about the same point"**, without "sum of", or without "in equilibrium". Some candidates write the conservation of momentum instead ("isolated system"): it is a different principle.
- **Half a couple.** For two 25 N forces each 1.5 m from the centre, the torque is $25 \times 3.0 = 75$ N m, not $25 \times 1.5 = 37.5$ N m. Use the distance **between** the two lines of action.
- **Vague equilibrium conditions.** Say "**resultant** force is zero" and "**resultant** torque is zero". "Turning effect" is not a physical quantity, and "the forces are balanced" doesn't mention rotation.
- **Centre of gravity where the weight "acts"** or where the "mass" acts. It is where the **weight may be taken to act**.
- **Putting the object's density or weight into $F = \rho g V$.** Upthrust uses the **fluid's** density. Examiners also see the upthrust confused with the weight, and the force from a support used in place of the upthrust.
- **Mixing up pressure and force.** $\rho g h$ is a pressure: multiply by the area to get a force. And $\rho g h$ is only for a fluid at a depth: the pressure of a solid on a table or of a jet on a wall is $\frac{F}{A}$. Dividing an object's weight by its base area does not give the pressure in the liquid under it.
- **Explaining upthrust with Newton's third law.** Upthrust comes from the pressure **difference**: more pressure on the bottom, because it is deeper, so a larger force up than down.
- **Unit slips**: g to kg, cm to m, cm^2 and cm^3 to m^2 and m^3 (power-of-ten errors are very common), and forgetting g when turning a mass into a weight. Recall $\frac{4}{3}\pi r^3$ for a sphere; it isn't on the data sheet.
- **In a "show that" question**, stopping at the rounded value. Show the full substitution and give one more significant figure than the value in the question.

How to get the marks

- **Definitions, word for word in meaning**:
  - Moment: force × perpendicular distance from the point to the line of action of the force.
  - Principle of moments: for an object in equilibrium, the sum of the clockwise moments about a point equals the sum of the anticlockwise moments about the same point.
  - Couple: two equal, opposite forces along different (parallel) lines, which produce rotation only.
  - Torque of a couple: one of the forces × the perpendicular distance between their lines of action.
  - Centre of gravity: the point where all the weight of the object may be taken to act.
  - Density: mass per unit volume. Pressure: (normal) force per unit area.
  - Equilibrium: resultant force zero and resultant torque zero.

- **"By taking moments about A"**: write the full equation with numbers (every moment on each side) before solving. That line earns the method mark even if the arithmetic slips.
- **"Show that"**: every step, the formula, the substitution and a final value to more significant figures than given. For the upthrust derivation or the $\rho g h$ derivation, write each step and name the definition it uses.
- **"Explain why there is an upthrust"**: two marks, two ideas: (1) the pressure is greater at the bottom (greater depth); (2) so the upward force on the bottom is greater than the downward force on the top, giving a resultant force upwards.
- **Vector triangles**: closed, tip to tail, every arrow labelled and pointing the same way round, sides drawn to the angles given (examiners allow a few degrees).
- **Arrows on a diagram**: start on the object, point the right way, and label them (weight, upthrust, drag, tension).
- **"State and explain"** (for example, what happens to a tension): the effect **and** the reason in terms of moments.
- **Directions**: for a resultant moment say clockwise or anticlockwise; for a hinge force give each component's direction.
- **Units**: moments and torques in N m (not N m^{-1}), pressure in Pa, density in kg m^{-3}.
- **Multiple choice**: work the number yourself first. The wrong options are usually a half-distance couple, a missing beam weight, a missing g, or the wrong density in $\rho g V$.

Quick check

1. The two hands on a steering wheel of diameter 0.38 m each push with 15 N, in opposite directions along parallel lines at the edge of the wheel. Calculate the torque.
2. 300 cm^3 of a liquid of density 1.20 g cm^{-3} is mixed with 200 cm^3 of water (1.00 g cm^{-3}). The volumes simply add. Calculate the density of the mixture in kg m^{-3}.
3. A tank holds water 3.5 m deep. Calculate the pressure on the bottom due to the water, and the total pressure there if the atmospheric pressure is 101 kPa.
4. A metal block of mass 2.0 kg and volume 2.5×10^{-4} m^3 hangs from a newton meter, fully under water. Calculate the upthrust, the reading on the newton meter and the density of the metal.
5. A uniform metre rule balances on a pivot at the 40.0 cm mark when a 50 g mass hangs at the 10.0 cm mark. Calculate the mass of the rule.

Answers

1. The forces form a couple; the distance between their lines of action is the diameter. Torque $= 15 \times 0.38 = 5.7$ N m.
2. Mass $= 300 \times 1.20 + 200 \times 1.00 = 360 + 200 = 560$ g. Volume $= 500$ cm^3. Density $= \frac{560}{500} = 1.12$ g cm^{-3} $= 1120$ kg m^{-3}.
3. $\Delta p = 1000 \times 9.81 \times 3.5 = 34\,300$ Pa $= 34.3$ kPa. Total $= 101 + 34.3 = 135$ kPa.
4. Upthrust $= 1000 \times 9.81 \times 2.5 \times 10^{-4} = 2.45$ N. Weight $= 2.0 \times 9.81 = 19.6$ N, so the reading is $19.6 - 2.45 = 17.2$ N. Density $= \frac{2.0}{2.5 \times 10^{-4}} = 8000$ kg m^{-3}.
5. The rule's weight acts at 50.0 cm, 10.0 cm from the pivot; the 50 g mass is 30.0 cm from it. $M \times 10.0 = 50 \times 30.0$, so $M = 150$ g.

Past questions to try

- **9702/22/F/M/25 Q2**: the principle of moments on a beam resting on a floating cylinder, then the upthrust and how deep the cylinder sits.
- **9702/21/M/J/23 Q4**: deriving $p = \rho g h$, density from a pressure–depth graph, and the change in a tension as a cylinder is lowered into a liquid.
- **9702/22/M/J/23 Q2**: centre of gravity, then moments with a string at an angle and the resultant moment when it is cut.
- **9702/23/O/N/25 Q2**: defining pressure, explaining upthrust, and the upthrust on a ball falling through oil.