Cambridge AS & A Level · Physics 9702 · Notes

Gravitational fields A2

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Read first: **Motion in a circle**, **Work, energy and power**

What you need to know

By the end of this topic you should be able to:

1. **Understand that a gravitational field is a field of force**, and **define gravitational field** (field strength g) as the force per unit mass on a mass placed at that point.
2. **Draw a gravitational field with field lines.**
3. **Treat a uniform sphere as a point mass at its centre** for any point outside it.
4. **Recall and use Newton's law of gravitation**, $F = \frac{Gm_1m_2}{r^2}$, for the force between two point masses.
5. **Analyse circular orbits** by setting the gravitational force equal to the force needed for the centripetal acceleration.
6. **Understand geostationary orbits**: the satellite stays above the same point on the Earth, with a period of 24 hours, moving from west to east, directly above the Equator.
7. **Derive** $g = \frac{GM}{r^2}$ for a point mass from Newton's law and the definition of g.
8. **Recall and use** $g = \frac{GM}{r^2}$**.**
9. **Explain why** g **is almost constant** for small changes in height near the Earth's surface.
10. **Define gravitational potential** at a point as the work done per unit mass in bringing a small test mass from infinity to that point.
11. **Use** $\phi = -\frac{GM}{r}$, and understand how it leads to the potential energy of two point masses, $E_P = -\frac{GMm}{r}$.

Centripetal acceleration and $\omega = \frac{2\pi}{T}$ come from Motion in a circle; kinetic energy and work done come from Work, energy and power.

Understand it

A field of force

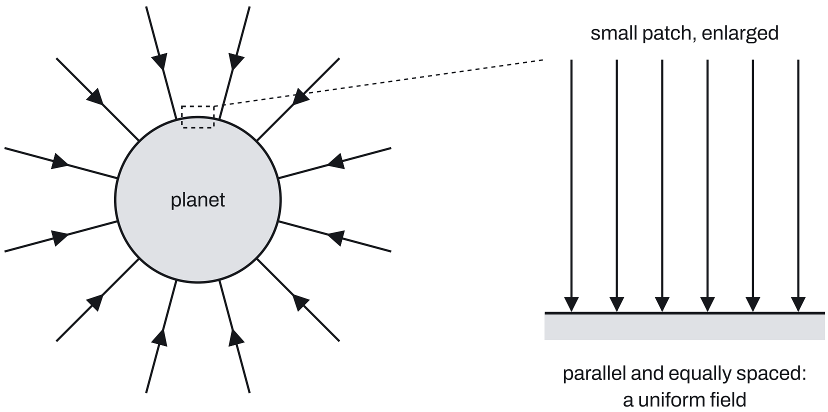
Any mass pulls on any other mass. We describe this by saying a mass sets up a **gravitational field** around itself: a region where another mass feels a force. Put a small "test" mass m at a point, measure the force F on it, and divide:

$$g = \frac{F}{m}$$

This is the **gravitational field strength** at that point, in N kg^{-1}. It is the force **per unit mass**, so it does not depend on the test mass you used. The field is a vector: it has the direction of the force on the mass. Near the Earth, $g = 9.81$ N kg^{-1}, which is the same number as the acceleration of free fall in m s^{-2} (a force of 9.81 N on each kilogram gives each kilogram an acceleration of 9.81 m s^{-2}).

Field lines

A **field line** shows the direction of the force on a small mass placed on it. The closer the lines, the stronger the field.



- Around a planet or any point mass the lines are **radial**: straight lines pointing **towards the centre**, with arrows inwards (gravity only attracts). They spread out with distance, which shows the field getting weaker.
- Draw at least four, equally spaced round the planet, each starting at the surface and meeting it at right angles.
- All round the surface the lines meet the ground at right angles and are equally spaced, so an observer anywhere on the surface of a uniform planet measures the same field, pointing straight down.
- Look at a patch of ground a few kilometres across. Compared with the planet's radius, that patch is tiny, so the lines over it are almost **parallel and equally spaced**: the field is **uniform** there.

A sphere acts as a point mass

For any point **outside** a uniform sphere, the sphere pulls exactly as if all its mass were at its centre. So a planet or star becomes a point mass, and every distance r in this topic is measured **from the centre**: radius of the planet plus height above the surface.

Newton's law of gravitation

The force between two point masses is proportional to the **product of their masses** and inversely proportional to the **square of their separation**:

$$F = \frac{Gm_1m_2}{r^2}$$

$G = 6.67 \times 10^{-11}$ N m² kg⁻² is the **gravitational constant** (on the data sheet). The force always **attracts**, and the two masses feel forces of the **same size** in opposite directions (Newton's third law), even if one mass is far bigger.

G is tiny, so the force between everyday objects is tiny. Two spheres of 150 kg and 12 kg with centres 0.40 m apart attract with 7.50×10^{-7} N; the Earth pulls the smaller one with 118 N. Gravity only matters when at least one mass is enormous.

Field strength of a point mass: $g = \dfrac{GM}{r^2}$

Put a test mass m at distance r from a point mass M. Newton's law gives the force on it, and the definition of g divides by m:

$$g = \frac{F}{m} = \frac{GMm/r^2}{m} = \frac{GM}{r^2}$$

State what G is when you derive this. The test mass cancels, which is why every object falls with the same acceleration.

- $g \propto \dfrac{1}{r^2}$: at twice the distance from the centre, g is a quarter; at three times, a ninth. On the other side of the mass at half the distance, the field points the opposite way and is four times as strong.
- The field points **towards** the mass. If you choose a direction as positive (say to the right), then to the right of the centre g is negative and to the left it is positive: g always has the opposite sign to the displacement from the centre, because the force is attractive.

Why g is almost constant near the surface

Climb 10 km on a planet of radius 5.0×10^6 m and r grows from 5.00×10^6 m to 5.01×10^6 m. Then g changes by the factor $\left(\frac{5.00}{5.01}\right)^2 = 0.996$, less than half a percent. The change in height is **tiny compared with the radius**, so r (and so g) hardly changes; the field lines over that small region are almost parallel. That is why $W = mg$ and $\Delta E_P = mg\Delta h$ work near the ground. Over large heights (a large fraction of the radius) they do not.

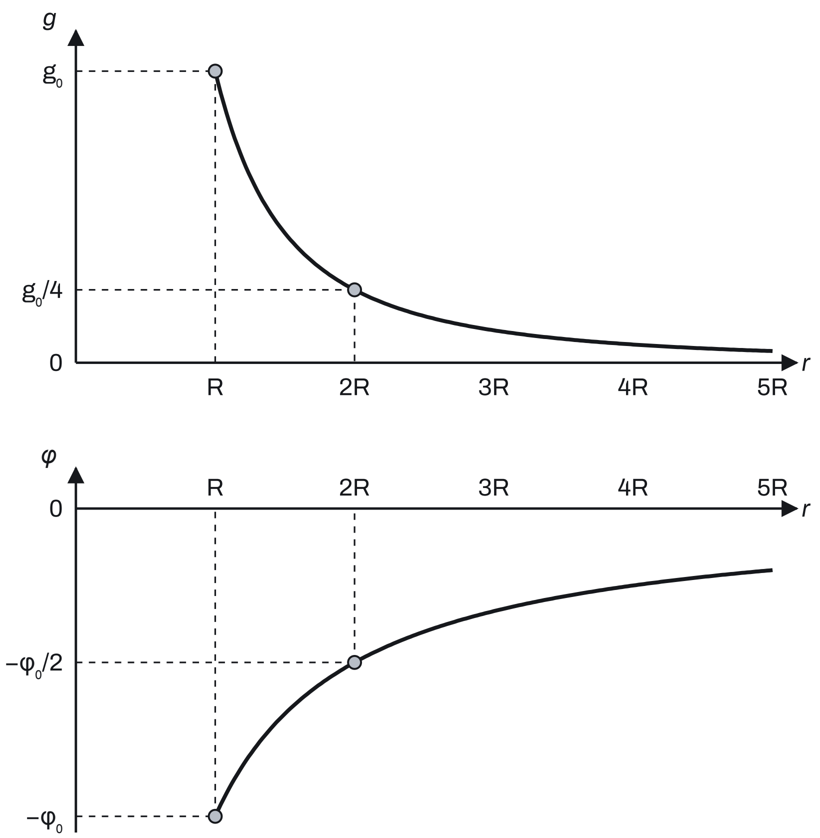
Gravitational potential

The **gravitational potential** ϕ at a point is the **work done per unit mass** in bringing a small test mass **from infinity** to that point. It is measured in J kg^{-1}.

Far away (at infinity) the potential is **zero**. Gravity attracts, so as a mass moves in from infinity the field pulls it along: the field does work on the mass, and the work done **by you** (or by any outside agent) is negative. So every potential near a mass is **negative**, and it is most negative at the surface of the planet. For a point mass:

$$\phi = -\frac{GM}{r}$$

- $\phi \propto -\frac{1}{r}$: at twice the distance the potential is half as negative. It rises (gets less negative) towards 0 as r grows.
- Potential is a **scalar**. With two or more masses, **add** the potentials, each one negative: $\phi = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2}$.
- Points at the same distance from the centre have the same potential, so a satellite in a circular orbit stays at constant potential.



Field and potential are linked. The field strength is minus the **potential gradient**: $g = -\frac{d\phi}{dr}$. The minus sign means the field points towards **lower** potential, which is towards the mass (the direction in which r decreases). The **size** of the field is the gradient: $g = \frac{d\phi}{dr}$ in magnitude. So the steeper the ϕ graph, the stronger the field, and where the ϕ graph is flat (at a maximum), $g = 0$. For a single point mass, $\phi = -\dfrac{GM}{r}$ gives $\dfrac{d\phi}{dr} = \dfrac{GM}{r^2}$, so the field strength is $g = \dfrac{GM}{r^2} = \dfrac{|\phi|}{r}$ (towards the mass), which lets you turn a potential into a field strength without finding M.

Gravitational potential energy

A mass m at a point where the potential is ϕ has **gravitational potential energy** $E_P = m\phi$. For two point masses:

$$E_P = m\phi = -\frac{GMm}{r}$$

It is the potential energy **of the pair**. It is zero when they are infinitely far apart and **always negative** at any finite separation, because the force is attractive: work has to be done **on** the masses to pull them apart to infinity (where $E_P = 0$). For example, a 2.0 kg mass at a point where $\phi = -3.0 \times 10^7$ J kg^{-1} has $E_P = -6.0 \times 10^7$ J, and the smallest energy needed to take it to infinity is 6.0×10^7 J.

Moving **away** from a planet, E_P becomes **less negative**, so it **increases**; moving closer, it decreases. The change between two distances is

$$\Delta E_P = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

which is positive (an increase) when $r_2 > r_1$. Don't use $mg\Delta h$ here unless the height change is tiny compared with r: g is not constant over a large height.

Circular orbits

A satellite (or moon, or planet round a star) in a circular orbit has **only one force** on it: gravity. That force points to the centre of the planet, which is always at right angles to the satellite's velocity, and at a fixed radius it has a fixed size. So it provides exactly the centripetal force (see Motion in a circle):

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2$$

The satellite's mass m cancels, so every satellite at the same radius has the same speed and period, whatever its mass. Useful results:

- **Speed:** $v^2 = \dfrac{GM}{r}$, so $v = \sqrt{\dfrac{GM}{r}}$. Higher orbits are **slower**.
- **Period:** with $\omega = \dfrac{2\pi}{T}$, $\dfrac{GM}{r^2} = r\dfrac{4\pi^2}{T^2}$, so

$$T^2 = \frac{4\pi^2}{GM} r^3 \qquad \text{or} \qquad 4\pi^2 r^3 = GMT^2$$

$T^2 \propto r^3$ for everything orbiting the same central mass. Four times the radius gives $4^{3/2} = 8$ times the period.

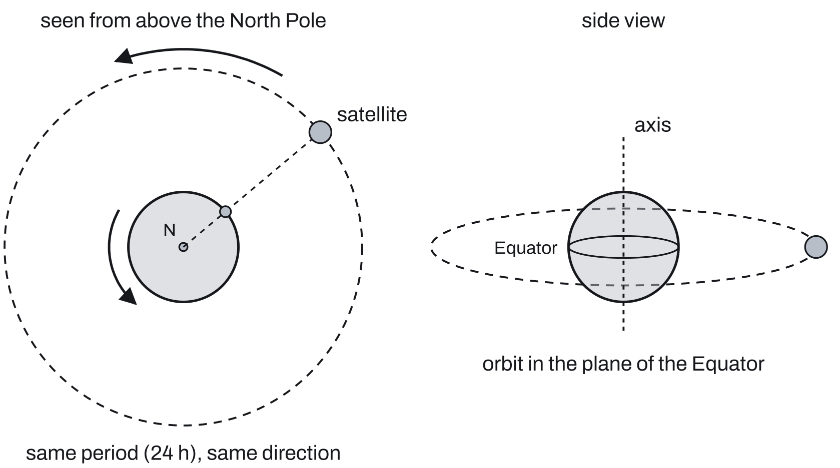
- **Kinetic energy:** $\frac{1}{2}mv^2 = \dfrac{GMm}{2r}$, exactly half the size of E_P.

- **Total energy:** $E_K + E_P = \dfrac{GMm}{2r} - \dfrac{GMm}{r} = -\dfrac{GMm}{2r}$. It is negative (the satellite is bound to the planet); moving to a higher orbit makes it less negative, so it needs extra energy from the rockets.

Geostationary orbits

A **geostationary** satellite stays above the **same point** on the Earth's surface, so a dish aimed at it never needs to move. For that, it must:

- have a period of **24 hours**, the same as the Earth's rotation;
- orbit **from west to east**, the same way the Earth spins;
- orbit **directly above the Equator** (its orbit is in the equatorial plane).



A period of 24 hours fixes the radius through $4\pi^2 r^3 = GMT^2$, so all geostationary satellites are at the same height. An orbit with the same 24-hour period that is tilted (for example over the poles), or goes from east to west, is **not** geostationary: the satellite would drift across the sky. The same idea works for any planet: an orbit that stays above one point has the planet's rotation period.

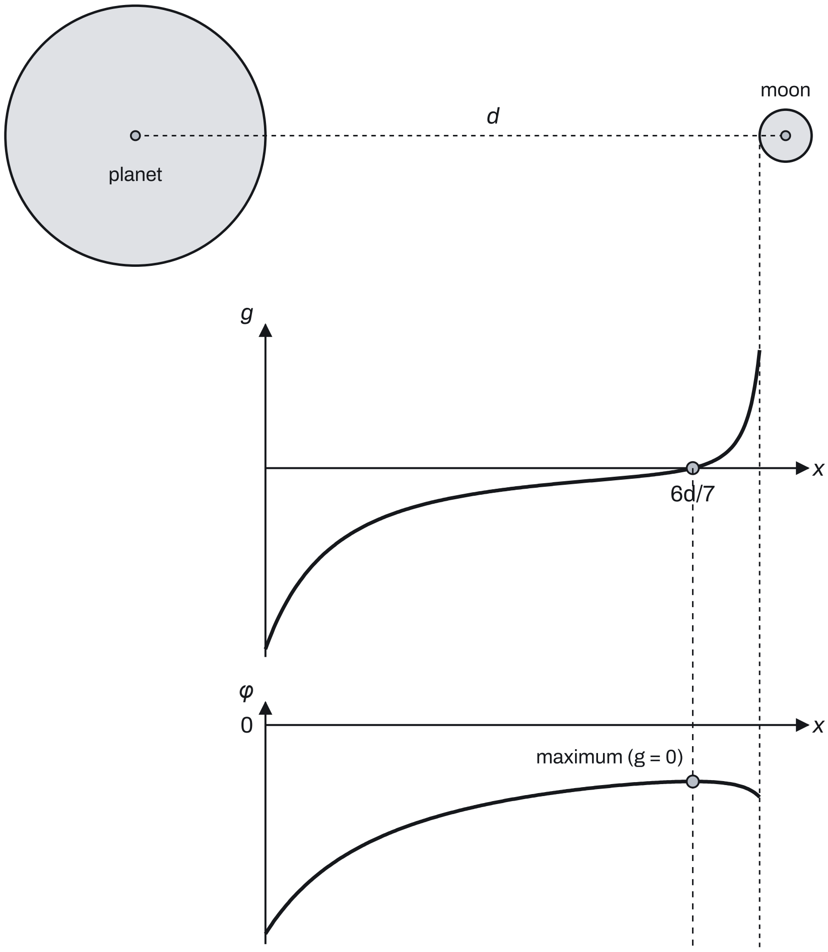
An object resting on a spinning planet

An object sitting on the Equator is **not** in orbit. It goes round once per rotation of the planet (24 hours for the Earth), with radius equal to the planet's radius, so its centripetal acceleration is $a = r\omega^2 = \dfrac{4\pi^2 r}{T^2}$, which is small. Two forces act: its weight (gravity, down) and the normal contact force (up). The contact force is **slightly less** than the weight; the difference is the resultant force ma towards the centre. The force per unit mass from the ground is $g - a$.

Two masses: where the field is zero

Between a planet and its moon, the planet pulls a test mass one way and the moon pulls it the other way. At one point the two fields are **equal in size and opposite in direction**, so the resultant field is zero. It is **nearer the smaller mass**. Set the two field strengths equal and take square roots:

$$\frac{GM_1}{x^2} = \frac{GM_2}{(d-x)^2} \quad \Rightarrow \quad \frac{x}{d-x} = \sqrt{\frac{M_1}{M_2}}$$



The potential does not go to zero there: it is the **sum of two negative potentials**, so it is still negative. On the potential graph this point is the **maximum** (where the gradient, and so g, is zero). The gradient of the ϕ graph is steeper at the surface of the body with the stronger surface field, and the field changes sign at the maximum.

Gravity and electric potential compared

Gravitational potential due to a point mass and electric potential due to a point charge are both proportional to $\frac{1}{r}$, both are zero at infinity, and both have equipotential surfaces that are concentric spheres. The difference: gravitational potential is always **negative** (gravity only attracts), while electric potential can be positive or negative.

Key facts and formulas

"Given" means it is on the Paper 4 data sheet. "Learn it" means you must remember it.

Formula	Status
Field strength $g = \dfrac{F}{m}$ (force per unit mass)	Learn it
Newton's law $F = \dfrac{Gm_1m_2}{r^2}$	Learn it
$g = \dfrac{GM}{r^2}$ for a point mass (and outside a uniform sphere)	Learn it
$\phi = -\dfrac{GM}{r}$	Given
$E_P = -\dfrac{GMm}{r}$	Given
$E_P = m\phi$; $\Delta E_P = GMm\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$	Learn it
$g = -\dfrac{d\phi}{dr}$ (field strength = minus potential gradient: it points towards lower potential); size $\dfrac{d\phi}{dr}$, so for a point mass $g = \dfrac{GM}{r^2} = \dfrac{\lvert\phi\rvert}{r}$	Learn it
Orbit: $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r} = mr\omega^2$, so $v = \sqrt{\dfrac{GM}{r}}$	Learn it
$T^2 = \dfrac{4\pi^2}{GM}r^3$ (so $T^2 \propto r^3$)	Learn it
Orbiting mass: $E_K = \dfrac{GMm}{2r}$, total energy $= -\dfrac{GMm}{2r}$	Learn it
Geostationary: $T = 24$ h, west to east, above the Equator	Learn it
$G = 6.67 \times 10^{-11}$ N m² kg⁻²; $g = 9.81$ m s⁻²	Given

How to do it

In the Paper 4 questions we have tagged for this topic (26 different questions, 2021–2025; 10 more appear on two papers), the types came up this often. 25 of the 26 questions mix two or more types, so the counts add up to more than 26.

Question type	Questions
Definitions, laws and explanations	25
Potential and potential energy	11
Field strength $g = \frac{GM}{r^2}$	10
Graphs: sketching and using them	10

Question type	Questions
Orbits: gravity provides the centripetal force	9
Geostationary orbits	6
Two masses: where the field is zero	4
The force between two masses	2

1. Definitions, laws and explanations (25 questions)

Learn these, in meaning:

- **Gravitational field (strength)**: force per unit mass.
- **Gravitational potential** at a point: work done per unit mass in bringing a small test mass from infinity to the point. Both "per unit mass" and "from infinity" are needed.
- **Newton's law of gravitation**: the force between two point masses is proportional to the product of their masses and inversely proportional to the square of their separation. (If asked for the equation, write $F = \frac{Gm_1m_2}{r^2}$ and say that G is the gravitational constant.)
- **A field line** shows the direction of the force on a (test) mass at that point.

Variations you will meet:

- **Draw the field** of a planet or point mass: at least four straight radial lines, evenly spaced, touching the surface at right angles, with arrows **towards** the centre.
- **Why g is constant near the surface**: the change in height is much smaller than the radius, so r barely changes; the field lines there are almost parallel (and equally spaced).
- **Why the field is the same all round a uniform planet's surface**: the lines are evenly spaced all round and meet the surface at right angles, pointing down, everywhere.
- **Why the potential (or potential energy) is negative**: it is zero at infinity; the force is attractive, so work is done **on** a mass to move it away to infinity (or: the field does work as a mass moves in, so energy is lost). For potential **energy**, talk about the energy of the masses, not just the potential.
- **Why an orbit is circular**: gravity is the only force; it points to the centre (along the radial field lines), is at right angles to the velocity and has constant size, so it provides the centripetal force. If told to refer to your field-line drawing, link the lines to the direction of the force, and the force to the velocity.
- **Derive $g = \frac{GM}{r^2}$** or **show $E_P = -\frac{GMm}{r}$**: start from $F = \frac{GMm}{r^2}$ and $g = \frac{F}{m}$, or from $\phi = -\frac{GM}{r}$ and $E_P = m\phi$; name G.
- **State the link between potential and field strength**: field strength equals minus the potential gradient, $g = -\frac{d\phi}{dr}$; the minus sign means the field points towards lower potential (towards the mass), and its size is the gradient $\frac{d\phi}{dr}$ ($= \frac{GM}{r^2}$ for a point mass). Give the relationship, not two separate definitions.
- **Similarity and difference with electric potential**: both $\propto \frac{1}{r}$, both zero at infinity, both have spherical equipotentials; gravitational potential is always negative, electric can be either sign. Talk about **potentials**, not forces.

- **A mass on the ground at the Equator**: the forces are the weight and the normal contact force; the contact force is slightly smaller, and the resultant gives the centripetal acceleration. Never add "centripetal force" as a third force.

2. Potential and potential energy (11 questions)

1. **Potential**: $\phi = -\dfrac{GM}{r}$ with r from the centre. Unit J kg^{-1}. Keep the minus sign.
2. **Potential energy**: $E_P = m\phi = -\dfrac{GMm}{r}$. Unit J.
3. **A change** between two distances: $\Delta E_P = GMm\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$. Positive (an increase) if the mass moves further away.
4. **Energy conservation** if the mass moves freely: loss in $E_P =$ gain in E_K.

Variations you will meet:

- **Heights, not distances**: add the planet's radius to each height before using $\frac{1}{r}$.
- **Find a mass from an energy change**: rearrange $\Delta E_P = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ for M.
- **Increase or decrease?** Moving away: E_P increases (it becomes less negative; work is done against the attractive force). Moving closer: it decreases (the force does work). "Because it is negative" is not a reason on its own.
- **Speed after falling in** (a comet or probe): $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + |\Delta E_P|$. Include the starting kinetic energy and convert km s^{-1} to m s^{-1} (and back). The mass cancels, so the path does not depend on the mass.
- **Escape from the surface**: the mass just reaches infinity with zero speed, so $\frac{1}{2}mv^2 + m\phi = 0$ and $v = \sqrt{-2\phi} = \sqrt{\dfrac{2GM}{R}}$.
- **Energy of an orbit**: $E_K = \dfrac{GMm}{2r}$ (from $\frac{GMm}{r^2} = \frac{mv^2}{r}$, multiply by $\frac{r}{2}$), $E_P = -\dfrac{GMm}{r}$, total $-\dfrac{GMm}{2r}$. The extra energy to move an orbiting satellite to infinity is $\dfrac{GMm}{r} - \frac{1}{2}mv^2$ (the size of the total energy).
- **Answers in terms of symbols**: if the potential at radius $4R$ is $-\Phi$, then at radius R it is -4Φ (as $\phi \propto \frac{1}{r}$). Then $E_P = m\phi$, and field strength $g = \frac{|\phi|}{r}$, for example $\frac{\Phi}{4R}$ at $4R$ and $\frac{4\Phi}{R}$ at R.
- **Two masses**: the potential at a point is the **sum** of the two negative potentials.

3. Field strength $g = \frac{GM}{r^2}$ (10 questions)

1. Use $g = \dfrac{GM}{r^2}$ with r from the centre, and **square** r.
2. Give the unit N kg^{-1} and the number of significant figures asked for.

Variations you will meet:

- **Given g at the surface, find the radius or mass**: $R = \sqrt{\dfrac{GM}{g}}$ or $M = \dfrac{gR^2}{G}$. Sometimes GM is given as one number.

- **Compare two points** without numbers: $g \propto \frac{1}{r^2}$; at half the distance, four times the field. On opposite sides of the mass, the fields point in opposite directions.
- **From a potential**: $g = \frac{|\phi|}{r}$ (or find M from ϕ first).
- **Rotating planet**: centripetal acceleration on the Equator $a = \dfrac{4\pi^2 R}{T^2}$ with T the rotation period in seconds; the force per unit mass from the ground is $g - a$.
- **Combined with another inverse-square law** (for example the intensity of light from a star): write x^2 from each and eliminate it.

4. Graphs: sketching and using them (10 questions)

Sketching (outside a sphere of radius R):

- g **against** r: starts at the surface value g_0, falls steeply then less steeply, never reaching zero; $\frac{g_0}{4}$ at $2R$, $\frac{g_0}{9}$ at $3R$.
- ϕ **against** r: always negative, starts at $-\phi_0$ at the surface and rises towards zero; $-\frac{\phi_0}{2}$ at $2R$, $-\frac{\phi_0}{3}$ at $3R$. The gradient gets smaller in size.
- **Signed** g **on both sides of a sphere**: to the right g is negative, to the left positive (opposite sign to the displacement), each curve getting smaller in size as $\frac{1}{x^2}$.
- **Between two bodies**: g goes from one sign at the first surface, through zero at the null point, to the other sign at the second; ϕ is negative throughout with its maximum at the null point. Two **identical** spheres give a symmetric graph with zero field halfway.
- ϕ **against time for a satellite in a circular orbit**: a horizontal line, because r does not change.

Using a graph: rearrange the equation into $y = mx + c$ with the graph's axes, then read off the gradient (with its powers of ten and units).

Plot	Gradient
T^2 against r^3	$\dfrac{4\pi^2}{GM}$
r^3 against T^2	$\dfrac{GM}{4\pi^2}$
r^3 against $\dfrac{1}{\omega^2}$	GM
ϕ against $\dfrac{1}{r}$	$-GM$
g against $\dfrac{1}{r^2}$	GM

- **Height instead of radius**: $r = h + R$. From $4\pi^2(h+R)^3 = GMT^2$, take cube roots: $T^{2/3} = \left(\dfrac{4\pi^2}{GM}\right)^{1/3}(h+R)$. A graph of $T^{2/3}$ against h is a straight line with gradient $\left(\dfrac{4\pi^2}{GM}\right)^{1/3}$; its intercept on the $T^{2/3}$ axis is gradient $\times R$, so $R = \dfrac{\text{intercept}}{\text{gradient}}$, and the gradient gives $M = \dfrac{4\pi^2}{G \times \text{gradient}^3}$.

- **Reading a value**: the potential at the surface is the value at $r = R$; convert km to m and read the axis's power of ten.
- **Describing a graph's features** (planet and moon): compare the potentials at the two surfaces, the gradients (field strengths) at the two surfaces, and say where the maximum is (nearer the smaller body, where $g = 0$). Say what each feature means for the two bodies; don't just repeat that ϕ is negative.

5. Orbits: gravity provides the centripetal force (9 questions)

1. **Say it**: "the gravitational force provides the centripetal force".
2. **Write** $\dfrac{GMm}{r^2} = mr\omega^2$ with $\omega = \dfrac{2\pi}{T}$ (or $= \dfrac{mv^2}{r}$ with $v = \dfrac{2\pi r}{T}$).
3. **Rearrange**: $4\pi^2 r^3 = GMT^2$. Show each line in a "show that".
4. **Substitute** with T in **seconds** and r from the centre; square π and T.

Variations you will meet:

- **"Show that $T^2 = kr^3$; explain your reasoning"**: say that gravity provides the centripetal force, then do the algebra and state $k = \frac{4\pi^2}{GM}$.
- **A new orbit from an old one**: $\dfrac{r^3}{T^2}$ is the same for both (same central mass): $r_2 = r_1 \left(\dfrac{T_2}{T_1}\right)^{2/3}$. In a "show that", write this step out.
- **Speed at a given radius**: $v = \sqrt{\dfrac{GM}{r}}$.
- **Mass of the central body** from one orbit: $M = \dfrac{4\pi^2 r^3}{GT^2}$, or from a graph's gradient (type 4).
- **Two stars orbiting each other** (a binary): they share one period, and each goes round the common centre on its own radius. Use the **separation** in Newton's law, but each star's **own radius** in $mr\omega^2$. Each star feels the same force, so the lighter star has the bigger acceleration ($a = \frac{F}{m}$) and the bigger radius; with the same ω it also has the bigger speed. Find the period from one star: $a = \dfrac{4\pi^2 r}{T^2}$, so $T = 2\pi\sqrt{\dfrac{r}{a}}$.
- **Angular speed** of an orbit: $\omega = \frac{2\pi}{T}$ in rad s^{-1}, not a speed in m s^{-1}.

6. Geostationary orbits (6 questions)

1. **Period**: $T = 24$ h $= 86\,400$ s (for another planet, its rotation period).
2. **Radius** from $4\pi^2 r^3 = GMT^2$; **height** $= r - R$.
3. **Conditions**: above the Equator, west to east, period 24 h.

Variations you will meet:

- **"State the two other conditions"** when the period is given: equatorial orbit, and west to east (the same direction as the Earth spins).
- **Same radius and period but not geostationary**: the orbit could be from east to west, or not above the Equator (for example over the poles). Don't list geostationary properties here.

- **An orbit that stays above one point of another planet**: its period **is** that planet's rotation period, so you can deduce the rotation period from the orbit, or the orbit radius from the rotation period.
- **Given the potential at the orbit**: $r = \dfrac{GM}{|\phi|}$, then $T = 2\pi\sqrt{\dfrac{r^3}{GM}}$.

7. Two masses: where the field is zero (4 questions)

1. **Explain**: the two fields are in **opposite directions** and at that point they are **equal in magnitude**. Talk about **fields**, not forces, if the question asks about field strength.
2. **Find the point**: $\dfrac{M_1}{x^2} = \dfrac{M_2}{(d-x)^2}$, then square-root **both sides**: $\dfrac{x}{d-x} = \sqrt{\dfrac{M_1}{M_2}}$. No quadratic needed.
3. **Potential there** (if asked): add the two negative potentials; it is **not** zero.

For example, masses in the ratio $49 : 1$ give $\frac{x}{d-x} = 7$, so $x = \frac{7}{8}d$ from the heavier body.

8. The force between two masses (2 questions)

1. Use $F = \dfrac{Gm_1m_2}{r^2}$ with the **centre-to-centre separation**, squared.
2. The force on each mass is the same size (Newton's third law).

Variations you will meet:

- **Compare with the weight** to judge whether the force matters: for two molecules the gravitational force is many orders of magnitude smaller than their weight, so it can be ignored.
- **Acceleration of one body**: $a = \frac{F}{m}$ for that body's own mass.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A planet is a uniform sphere of mass 4.8×10^{23} kg and radius 2.6×10^6 m.

(a) Calculate the gravitational field strength at the surface. **[2]**

(b) Calculate the gravitational potential at the surface. Give a unit with your answer. **[2]**

(c) A lander of mass 350 kg is lifted from the surface to a height of 1.1×10^6 m. Calculate the increase in its gravitational potential energy. **[3]**

(d) A student uses $mg\Delta h$ with the field strength from (a). Explain why this gives the wrong answer, and say whether it is too big or too small. **[2]**

Solution

(a) $g = \dfrac{GM}{R^2} = \dfrac{6.67 \times 10^{-11} \times 4.8 \times 10^{23}}{(2.6 \times 10^6)^2}$ **C1** $= 4.74$ N kg^{-1}. **A1**

(b) $\phi = -\dfrac{GM}{R} = -\dfrac{6.67 \times 10^{-11} \times 4.8 \times 10^{23}}{2.6 \times 10^6}$ **C1** $= -1.23 \times 10^7$ J kg^{-1}. **A1**

(c) New distance from the centre: $r_2 = 2.6 \times 10^6 + 1.1 \times 10^6 = 3.7 \times 10^6$ m. **C1** $\Delta E_P = GMm\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right) = 6.67 \times 10^{-11} \times 4.8 \times 10^{23} \times 350 \times \left(\dfrac{1}{2.6 \times 10^6} - \dfrac{1}{3.7 \times 10^6}\right)$ **C1** $= 1.28 \times 10^9$ J. **A1**

(d) The height is not small compared with the radius, so g is not constant: it falls to 2.34 N kg^{-1} at the top. **B1** Using the surface value all the way gives $350 \times 4.74 \times 1.1 \times 10^6 = 1.82 \times 10^9$ J, which is too big. **B1**

Example 2

A planet of mass 5.2×10^{24} kg and radius 5.9×10^6 m rotates once every 30.0 hours. A satellite moves in a circular orbit of radius r and period T.

(a) Show that $4\pi^2 r^3 = GMT^2$, where M is the mass of the planet. **[3]**

(b) The satellite stays above the same point on the planet's surface. Calculate its height above the surface. **[3]**

(c) Calculate the speed of the satellite. **[2]**

(d) State two other features that this orbit must have. **[2]**

(e) A second satellite orbits the planet at a quarter of the radius in (b). Without calculating its radius, find its period in hours. **[2]**

Solution

(a) The gravitational force provides the centripetal force: $\dfrac{GMm}{r^2} = mr\omega^2$ **M1**, with $\omega = \dfrac{2\pi}{T}$ **M1**. So $\dfrac{GM}{r^2} = \dfrac{4\pi^2 r}{T^2}$, and multiplying by r^2T^2 gives $4\pi^2 r^3 = GMT^2$. **A1**

(b) $T = 30.0 \times 3600 = 1.08 \times 10^5$ s. **C1** $r^3 = \dfrac{GMT^2}{4\pi^2} = \dfrac{6.67 \times 10^{-11} \times 5.2 \times 10^{24} \times (1.08 \times 10^5)^2}{4\pi^2}$, so $r = 4.68 \times 10^7$ m. **C1** Height $= 4.68 \times 10^7 - 5.9 \times 10^6 = 4.09 \times 10^7$ m. **A1**

(c) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi \times 4.68 \times 10^7}{1.08 \times 10^5}$ **C1** $= 2.72 \times 10^3$ m s^{-1}. **A1**

(d) It is directly above the planet's equator (equatorial orbit). **B1** It moves in the same direction as the planet rotates. **B1**

(e) $T^2 \propto r^3$, so $T \propto r^{3/2}$: $T_2 = 30.0 \times \left(\frac{1}{4}\right)^{3/2}$ **C1** $= \dfrac{30.0}{8} = 3.75$ hours. **A1**

Example 3

A moon is a uniform sphere of mass 9.0×10^{22} kg and radius 1.9×10^6 m. A probe of mass 45 kg is in a circular orbit of radius 2.5×10^6 m around the moon.

(a) Show that the kinetic energy of the probe is $\dfrac{GMm}{2r}$, where M is the mass of the moon and m is the mass of the probe. **[2]**

(b) Calculate the gravitational potential energy, the kinetic energy and the total energy of the probe. **[3]**

(c) State the minimum energy needed to move the probe from its orbit to infinity. **[1]**

(d) Calculate the minimum speed with which an object must leave the surface of the moon, moving upwards, to escape from its gravitational field. **[3]**

Solution

(a) Gravity provides the centripetal force: $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$, so $mv^2 = \dfrac{GMm}{r}$. **M1** Then $E_K = \frac{1}{2}mv^2 = \dfrac{GMm}{2r}$. **A1**

(b) $E_P = -\dfrac{GMm}{r} = -\dfrac{6.67 \times 10^{-11} \times 9.0 \times 10^{22} \times 45}{2.5 \times 10^6} = -1.08 \times 10^8$ J. **A1** $E_K = \dfrac{GMm}{2r} = 5.40 \times 10^7$ J. **A1** Total $= 5.40 \times 10^7 - 1.08 \times 10^8 = -5.40 \times 10^7$ J. **A1**

(c) At infinity the total energy is at least zero, so 5.40×10^7 J. **B1**

(d) It just reaches infinity, where $E_K = 0$ and $E_P = 0$: $\frac{1}{2}mv^2 - \dfrac{GMm}{R} = 0$. **M1** $v = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2 \times 6.67 \times 10^{-11} \times 9.0 \times 10^{22}}{1.9 \times 10^6}}$ **C1** $= 2.51 \times 10^3$ m s^{-1}. **A1**

Example 4

A planet of mass 5.4×10^{24} kg and its moon, of mass 1.5×10^{23} kg, may be treated as point masses. Their centres are 2.8×10^8 m apart. Point X, on the line joining their centres, is where the resultant gravitational field strength is zero.

(a) Explain how the field strength can be zero at X. **[2]**

(b) Show that X is 2.4×10^8 m from the centre of the planet. **[3]**

(c) Calculate the gravitational potential at X. **[3]**

Solution

(a) At X the fields due to the planet and the moon are in opposite directions **B1** and equal in magnitude, so they cancel. **B1**

(b) $\frac{GM_P}{x^2} = \frac{GM_M}{(2.8\times10^8 - x)^2}$ **C1**, so $\frac{x}{2.8\times10^8 - x} = \sqrt{\frac{5.4\times10^{24}}{1.5\times10^{23}}} = \sqrt{36} = 6$. **M1** $x = 6(2.8\times10^8 - x)$, so $7x = 1.68\times10^9$ and $x = 2.4\times10^8$ m. **A1**

(c) X is $2.8\times10^8 - 2.4\times10^8 = 4.0\times10^7$ m from the moon's centre. Potentials are scalars, so add them: **C1** $\phi = -6.67\times10^{-11}\left(\frac{5.4\times10^{24}}{2.4\times10^8} + \frac{1.5\times10^{23}}{4.0\times10^7}\right)$ **C1** $= -1.75\times10^6$ J kg^{-1}. **A1**

Common mistakes

- **Loose definitions.** Reports in 2021, 2022 and 2023 found potential defined as an energy ("work done to move a mass from infinity") without **per unit mass**, and gravitational field given as a force, or as a description of what a field is, instead of **force per unit mass**. Learn the exact wording.
- **Leaving words out of Newton's law.** The March 2021 report found answers without "product" of the masses, with "radius" or plain "distance" instead of separation, or with no mention of force. The June 2022 report found the separation not squared.
- **Mixing up potential and potential energy.** In March 2021 many explained why the **potential** is negative when asked about potential energy, and added a potential to a kinetic energy. The June 2021 report found the change in potential worked out when the change in potential energy was asked for.
- **Thinking E_P falls as r rises.** It is negative, so a bigger r makes it **less negative**: it increases (June 2021). "Because it is negative" does not explain a decrease either (June 2022): give the physics (the attractive force does work).
- **Using $\Delta E_P = \frac{GMm}{r_2 - r_1}$.** Subtract $\frac{1}{r}$ values, not distances (June 2021).
- **Forgetting the planet's radius.** r is from the centre: add the radius to a height, and take it away to get a height. The June 2023 report found the planet's radius put in for the orbit radius.
- **Units and powers.** Reports in 2021 and 2022 found hours or minutes not converted to seconds, T or π not squared, and powers of ten dropped when reading graphs.
- **Subtracting potentials.** With two masses, add the two negative potentials; potential is a scalar (November 2021).
- **Forces instead of fields.** When asked why the field is zero at a point, say the two **fields** are equal and opposite. "They cancel out" alone was not enough (November 2021, June 2022).
- **Solving a quadratic for the null point.** Take square roots of both sides first (November 2021, June 2022).
- **Treating an object on the ground as a satellite.** At the Equator it goes round once a day, and the contact force is slightly less than the weight (November 2022). The "centripetal force" is not an extra force.
- **Answering from an old mark scheme.** The November 2022 and June 2023 reports found answers recalled from similar past questions that did not fit the question asked; read what is actually asked (for example, a satellite that is **not** geostationary).

How to get the marks

- **Definitions**: one mark each for the key phrases: "force per unit mass"; "work done per unit mass" + "moving a (test) mass from infinity"; "proportional to the product of the masses" + "inversely proportional to the square of the separation".
- **Show that / derive**: start from a stated law (Newton's law, $g = \frac{F}{m}$, $\phi = -\frac{GM}{r}$), write "gravitational force provides the centripetal force" for orbits, show each algebra step, and name G.
- **Calculations**: the mark scheme gives **C1** for the equation with numbers in, then **A1** for the answer. Give an answer to 2 or 3 significant figures to match the data, or to the number asked for ("to three significant figures").
- **"Give a unit"**: J kg^{-1} for potential, N kg^{-1} (or m s^{-2}) for field strength, rad s^{-1} for angular speed.
- **Signs**: potentials and potential energies are negative; a "change" or "energy required" is usually asked as a magnitude, but say whether it is an increase or decrease when asked.
- **Sketches**: mark the start and end points exactly (such as $(R, -\phi_0)$), make the curve pass through the key values ($\frac{1}{4}$ at $2R$ for g, $\frac{1}{2}$ at $2R$ for ϕ), and show the right curvature. Keep ϕ below the axis.
- **"Explain" with forces and directions**: name the force, give its direction (towards the centre) and relate it to the velocity (perpendicular).
- **"State, with a reason"**: the decision earns nothing without the reason.

Quick check

1. Two spheres of mass 150 kg and 12 kg have centres 0.40 m apart. Calculate the gravitational force between them.
2. At what height above the surface of a planet of radius R is the gravitational field strength one ninth of its value at the surface?
3. The gravitational potential at a point is -4.5×10^6 J kg^{-1}. Calculate the minimum energy needed to move a 20 kg mass from this point to infinity.
4. Satellite A orbits a planet with radius r and period 2.0 hours. Satellite B, of the same mass, orbits the same planet with radius $4r$. Find the period of B, and say which satellite has more gravitational potential energy.
5. A climber rises 900 m on a planet of radius 5.5×10^6 m. By what percentage does the gravitational field strength change? What does this show?

Answers

1. $F = \dfrac{6.67 \times 10^{-11} \times 150 \times 12}{0.40^2} = 7.50 \times 10^{-7}$ N.
2. $g \propto \dfrac{1}{r^2}$, so g is a ninth when $r = 3R$. The height is $3R - R = 2R$.
3. $E_P = m\phi = 20 \times (-4.5 \times 10^6) = -9.0 \times 10^7$ J, so 9.0×10^7 J is needed to bring it up to 0.
4. $T^2 \propto r^3$: $T_B = 2.0 \times 4^{3/2} = 2.0 \times 8 = 16$ hours. B has more E_P: $-\frac{GMm}{4r}$ is less negative than $-\frac{GMm}{r}$.
5. $\left(\dfrac{5.5 \times 10^6}{5.5009 \times 10^6}\right)^2 = 0.99967$, a fall of about 0.033%. The height is tiny compared with the radius, so g is effectively constant.

Past questions to try

- **9702/43/M/J/21 Q1**: the radius of an orbit from its period, a second orbit, and the change in potential energy.
- **9702/42/M/J/22 Q1**: potential energy between two distances from a star, and the speed of a comet by energy conservation.
- **9702/43/O/N/21 Q2**: the point between two bodies where the field is zero, and the potential there.
- **9702/42/O/N/21 Q2**: the potential graph between a planet and its moon, and sketching the field from it.