Cambridge AS & A Level · Physics 9702 · Notes

# Ideal gases A2

Read online at pastopic.com/cambridge/physics-9702/notes/ideal-gases

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Read first: **Temperature**



## What you need to know

By the end of this topic you should be able to:

1. **Know that amount of substance is an SI base quantity**, with base unit the **mole** (mol).
2. **Use molar quantities**: one mole of any substance contains $N_\mathrm{A}$ particles, where $N_\mathrm{A}$ is the Avogadro constant ($6.02 \times 10^{23}$ mol$^{-1}$).
3. **Know what an ideal gas is**: a gas that obeys $pV \propto T$, where $T$ is the thermodynamic temperature.
4. **Recall and use the equation of state** in both forms: $pV = nRT$ ($n$ = amount of substance in moles) and $pV = NkT$ ($N$ = number of molecules).
5. **Recall that the Boltzmann constant is** $k = \dfrac{R}{N_\mathrm{A}}$.
6. **State the basic assumptions of the kinetic theory of gases.**
7. **Explain how moving molecules cause the pressure** of a gas, and **derive and use** $pV = \frac{1}{3}Nm\langle c^2\rangle$, where $\langle c^2\rangle$ is the mean-square speed (one molecule colliding in one dimension, then extended to three dimensions with $\langle c_x^2\rangle = \frac{1}{3}\langle c^2\rangle$).
8. **Understand that the root-mean-square speed** is $c_\mathrm{r.m.s.} = \sqrt{\langle c^2\rangle}$.
9. **Compare** $pV = \frac{1}{3}Nm\langle c^2\rangle$ with $pV = NkT$ to show that the **average translational kinetic energy of a molecule is** $\frac{3}{2}kT$, and recall and use this.

Paper 4 questions on this topic often go on to the first law of thermodynamics (in the Thermodynamics topic) or to the kelvin scale (in the Temperature topic).

## Understand it

### Moles and the Avogadro constant

Gases are made of huge numbers of molecules, so we count them in **moles**. Amount of substance is one of the SI **base quantities** (like mass, length and time) and its base unit is the **mol**.

- **One mole** of any substance contains $N_\mathrm{A} = 6.02 \times 10^{23}$ particles (atoms or molecules). $N_\mathrm{A}$ is the **Avogadro constant**: the number of particles per mole.

- Number of molecules: $N = nN_{\mathrm{A}}$. Amount of substance: $n = \dfrac{N}{N_{\mathrm{A}}}$.
- **Mass of one molecule** from its nucleon number: a molecule of mass $28$ u has mass $28 \times 1.66 \times 10^{-27} = 4.65 \times 10^{-26}$ kg ($1$ u is on the data sheet). Or divide the mass of a sample by its number of molecules.
- Keep $n$ (moles) and $N$ (molecules) apart. A number of molecules is a huge whole number, never $0.33$.

## The ideal gas and its equation of state

For a fixed amount of a real gas at low pressure, experiments show that $pV$ is proportional to the **thermodynamic temperature** $T$ (in kelvin). A gas that obeys $pV \propto T$ at **all** pressures, volumes and temperatures is an **ideal gas**. Real gases behave almost ideally when the pressure is low and the temperature is well above the point where they would liquefy.

For an ideal gas the constant of proportionality depends only on how much gas there is, which gives the **equation of state**:

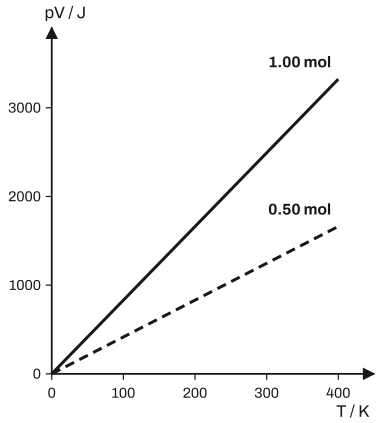
$$pV = nRT \qquad \text{or} \qquad pV = NkT$$

- $p$ = pressure (Pa), $V$ = volume (m³), $T$ = thermodynamic temperature (K), with $T/\mathrm{K} = \theta/^{\circ}\mathrm{C} + 273.15$; absolute zero is $0$ K $= -273.15\,^{\circ}$C.
- $n$ = amount of substance (mol) with $R = 8.31$ J K$^{-1}$ mol$^{-1}$, the **molar gas constant**.
- $N$ = number of molecules with $k = 1.38 \times 10^{-23}$ J K$^{-1}$, the **Boltzmann constant**.
- The two forms agree because $nR = Nk$. With $N = nN_{\mathrm{A}}$, this gives $k = \dfrac{R}{N_{\mathrm{A}}}$: the Boltzmann constant is the gas constant **per molecule**.

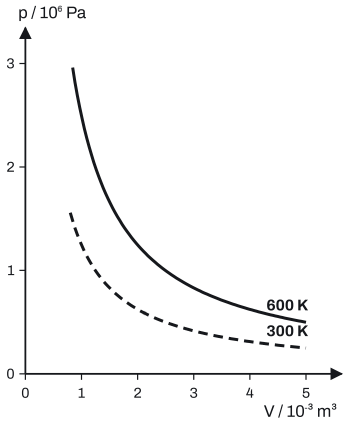
**Using it for a change of state.** For a fixed amount of gas, $nR$ is constant, so

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

- Constant volume: $p \propto T$. Constant pressure: $V \propto T$. Constant temperature: $pV$ is constant ($p \propto \frac{1}{V}$).
- Every $T$ must be in **kelvin**. A gas at $27\,^{\circ}$C heated to $327\,^{\circ}$C has its thermodynamic temperature **doubled** ($300$ K to $600$ K), not multiplied by $12$.
- **Showing a gas is ideal from data**: work out $\frac{pV}{T}$ (with $T$ in K) for each state; equal values (the same $nR$) show it behaves as an ideal gas.



straight lines through 0 K: pV ∝ T



0.50 mol: pV constant on each curve

- A graph of $pV$ against $T$ is a **straight line through the origin** with gradient $nR$ (or $Nk$). A graph of $pV$ against $kT$ has gradient $N$, the number of molecules.
- At constant temperature, $p$ against $V$ is a curve with $pV$ constant: a hotter gas has a curve further out.
- **Average separation of molecules**: each molecule has, on average, a volume $\frac{V}{N}$ to itself, so the average separation is about $\sqrt[3]{V/N}$.

## The kinetic theory of gases

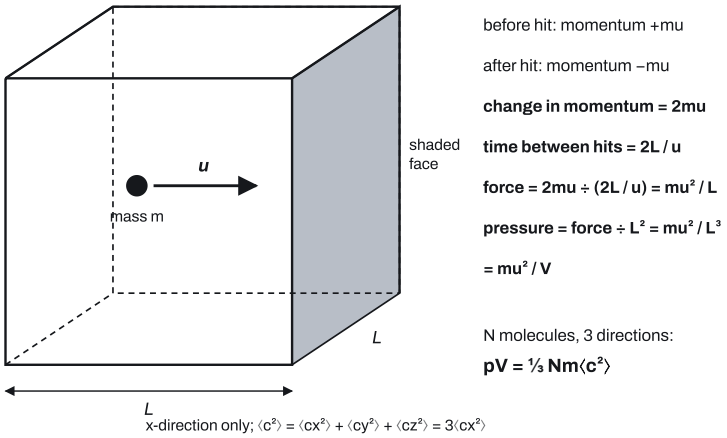
The kinetic theory explains the behaviour of a gas from the motion of its molecules. It models an ideal gas with these **basic assumptions** (all about the **molecules**, not the gas as a whole):

- the molecules are in **continuous random motion**;
- there are **no forces between the molecules** except during collisions;
- the **volume of the molecules is negligible** compared with the volume occupied by the gas;
- **all collisions** (with each other and with the walls) are **perfectly elastic**: no kinetic energy is lost;
- the **time of a collision is negligible** compared with the time between collisions.

A gas at very high pressure (for example at the surface of a star) is not ideal: its molecules are very close together, so the forces between them, and their own volume, are no longer negligible.

## How moving molecules cause pressure

A molecule hits a wall and bounces back. Its **momentum changes**, so the wall exerts a **force** on it. By Newton's third law the molecule exerts an equal and opposite force on the wall. Enormous numbers of molecules hit every part of the wall each second, so there is a steady **average force per unit area**: the pressure.




**Deriving $pV = \frac{1}{3}Nm\langle c^2\rangle$.**

1. **One molecule, one dimension.** A molecule of mass $m$ in a cube of side $L$ moves with velocity $u$ towards a face. It rebounds elastically with velocity $-u$. Change in momentum $= mu - (-mu) = 2mu$.
2. **Time between hits.** It crosses the box and back, a distance $2L$, before hitting the same face again: $t = \dfrac{2L}{u}$.
3. **Average force.** Force = rate of change of momentum: $F = \dfrac{2mu}{2L/u} = \dfrac{mu^2}{L}$.
4. **Pressure from one molecule.** Divide by the area of the face, $L^2$: $p = \dfrac{mu^2}{L^3} = \dfrac{mu^2}{V}$.
5. **Many molecules.** $N$ molecules have different $x$-velocities, so replace $u^2$ by the mean of their squares: $p = \dfrac{Nm\langle c_x^2\rangle}{V}$.
6. **Three dimensions.** Each speed satisfies $c^2 = c_x^2 + c_y^2 + c_z^2$. The motion is random, so no direction is special: $\langle c_x^2\rangle = \langle c_y^2\rangle = \langle c_z^2\rangle = \frac{1}{3}\langle c^2\rangle$. So

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle \qquad \text{or} \qquad p = \tfrac{1}{3}\rho\langle c^2\rangle$$

where $\rho = \frac{Nm}{V}$ is the density of the gas ($Nm$ is the total mass).

- $\langle c^2\rangle$ is the **mean-square speed**: square every molecule's speed, then take the mean (unit $m^2\ s^{-2}$).
- The **root-mean-square (r.m.s.) speed** is its square root: $c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}$ (unit $m\ s^{-1}$). It is not the same as the mean speed.
- The derivation uses the assumptions: elastic collisions (rebound at the same speed), negligible molecular volume (the full length $L$ is travelled), no forces between molecules and negligible collision time (steady motion between hits), random motion (the $\frac{1}{3}$).

**Why a gas cools when it expands against a moving piston.** Molecules that hit a piston moving **away** from them rebound with a **smaller speed**. Their mean kinetic energy falls, so the temperature falls. (A piston moving in makes them rebound faster, and the gas warms.)

## Temperature and molecular kinetic energy

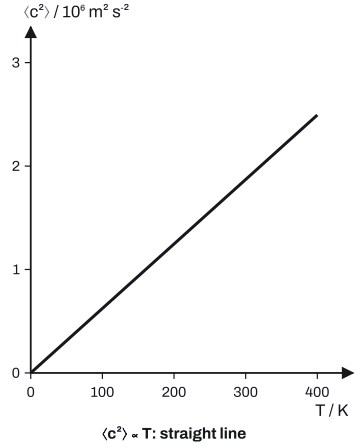
Set the two expressions for $pV$ equal:

$$\tfrac{1}{3}Nm\langle c^2\rangle = NkT \quad \Rightarrow \quad m\langle c^2\rangle = 3kT \quad \Rightarrow \quad \tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

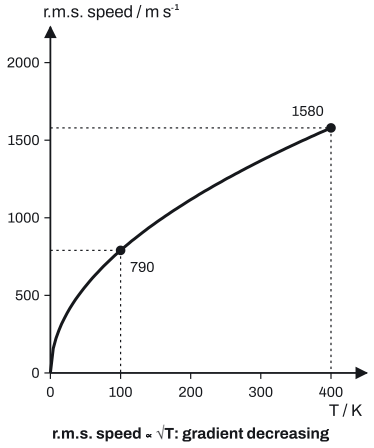
The **average translational kinetic energy of a molecule** is

$$E_\mathrm{K} = \tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

- It depends **only on the thermodynamic temperature**, not on the type of gas, the pressure or the volume. At $300$ K every ideal-gas molecule has mean kinetic energy $\frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21}$ J.
- **Temperature is a measure of the mean kinetic energy of the molecules.** At $0$ K it would be zero.
- **R.m.s. speed**: $c_\mathrm{r.m.s.} = \sqrt{\dfrac{3kT}{m}}$, so $c_\mathrm{r.m.s.} \propto \sqrt{T}$ for a given gas, and $c_\mathrm{r.m.s.} \propto \dfrac{1}{\sqrt{m}}$ at a given temperature.
- **Two gases at the same temperature** have the same mean kinetic energy, so lighter molecules move faster: $\dfrac{c_1}{c_2} = \sqrt{\dfrac{m_2}{m_1}}$. A molecule $100$ times heavier moves $10$ times slower.
- **Compressing at constant temperature** changes $p$ and $V$ but **not** $c_\mathrm{r.m.s.}$: it depends only on $T$.
- Starting from $pV = nRT$ instead works too, but then you must use $n = \frac{N}{N_\mathrm{A}}$ and $k = \frac{R}{N_\mathrm{A}}$ to reach $\frac{3}{2}kT$.



⟨c²⟩ ∝ T: straight line



r.m.s. speed ∝ √T: gradient decreasing

## The total kinetic energy of the gas

In an ideal gas there are **no forces between molecules**, so the molecules have **no potential energy**. The internal energy (the sum of the random kinetic and potential energies of the molecules) is therefore just their **total kinetic energy**:

$$U = N \times \tfrac{3}{2}kT = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT = \tfrac{3}{2}pV$$

- $U \propto T$: the internal energy of an ideal gas depends only on its temperature. If $T$ returns to its starting value (a full cycle), the change in internal energy is **zero**.
- $U = \frac{3}{2}pV$, so on a $p$–$V$ graph the internal energy rises wherever $pV$ rises.
- At constant pressure, $U = \frac{3}{2}pV$ is proportional to $V$: a straight line through the origin.
- **Change in internal energy**: $\Delta U = \frac{3}{2}Nk\,\Delta T$, using the temperature **change** (no $+273$).
- Multiply by $N$: the mean kinetic energy of **one** molecule is not the internal energy of the gas.

## Key facts and formulas

"Given" means it is on the data sheet in the exam. "Learn it" means you must remember it. The constants $N_A$, $R$, $k$ and 1 u are on the data sheet.

| Formula | Status |
|---|---|
| $N = nN_A$, with $N_A = 6.02 \times 10^{23}$ mol$^{-1}$ | Learn it (value given) |
| Ideal gas: $pV \propto T$, with $T$ in kelvin | Learn it |
| Equation of state $pV = nRT$ | Learn it |
| Equation of state $pV = NkT$ | Learn it |
| Boltzmann constant $k = \dfrac{R}{N_A}$ | Learn it |
| Fixed amount of gas: $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$ | Learn it |
| Pressure of an ideal gas $p = \dfrac{1}{3}\dfrac{Nm}{V}\langle c^2\rangle$, so $pV = \frac{1}{3}Nm\langle c^2\rangle$ and $p = \frac{1}{3}\rho\langle c^2\rangle$ | Given |
| R.m.s. speed $c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}$ | Learn it |
| Mean translational kinetic energy $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ | Learn it |
| $c_{\text{r.m.s.}} = \sqrt{\dfrac{3kT}{m}}$ | Learn it |
| Total kinetic energy (internal energy) of an ideal gas $U = \frac{3}{2}NkT = \frac{3}{2}nRT$ | Learn it |

## How to do it

In the Paper 4 questions we have tagged for this topic (25 different questions, 2021–2025; 9 more appear on two papers), the types came up this often. 22 of the 25 questions mix two or more types, so the counts add up to more than 25. Many also have parts on the first law of thermodynamics.

| Question type | Questions |
|---|---|
| Equation of state: find $N$, $n$, $p$, $V$ or $T$ | 16 |
| Total kinetic energy (internal energy) of the gas | 14 |
| Mean kinetic energy and r.m.s. speed | 12 |
| Ideal gas definition and kinetic theory assumptions | 10 |
| Graphs and sketches | 8 |
| Symbols, constants and deriving $E = \frac{3}{2}kT$ | 7 |
| Pressure from molecular movement | 4 |

## 1. Equation of state: find $N$, $n$, $p$, $V$ or $T$ (16 questions)

1. **Convert**: $T$ to kelvin ($+273$), volumes to m³ ($1\ \text{cm}^3 = 10^{-6}\ \text{m}^3$), pressures to Pa.
2. **Choose the form**: $pV = NkT$ if you want (or have) the number of molecules; $pV = nRT$ for moles.
3. **Substitute every value** into the equation, then rearrange and give the answer with its unit.

**Variations you will meet:**

- **"Show that the number of molecules is …"**: start from $pV = NkT$, substitute all the values including $k$, and give the result to more figures than printed. If you use $pV = nRT$, you must then show $N = n \times 6.02 \times 10^{23}$ as a separate step.
- **Number of molecules asked, not moles**: $n$ alone earns nothing. Then **mass of one molecule** = mass of gas $\div N$ (it should come out around $10^{-26}$ kg, not $10^{26}$).
- **Two states of a fixed mass**: $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$ to find the missing value; or find $T$ in one state and use $pV \propto T$.
- **Constant pressure**: $V \propto T$, so $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ (use the **new total volume**, not the change in volume).
- **Show a gas is ideal**: calculate $\frac{pV}{T}$ for both states (both temperatures in kelvin) and show they are equal.
- **Gases in thermal equilibrium**: same temperature, so find $T$ from one container and use it for the other.
- **An expression in symbols**: for example the gas at $3p$ and $\frac{1}{2}V$ has $T = \dfrac{3pV}{2nR}$, with $R$ identified as the molar gas constant.
- **Read a graph**: the gradient of $pV$ against $kT$ is $N$; of $pV$ against $T$ is $nR$; then $n = \frac{N}{N_\text{A}}$.
- **Average separation**: $\sqrt[3]{V/N}$.

## 2. Total kinetic energy (internal energy) of the gas (14 questions)

1. **Explain why $U$ is only kinetic energy**: no intermolecular forces, so **zero potential energy**.
2. **Multiply the mean kinetic energy by the number of molecules**: $U = \frac{3}{2}NkT = \frac{3}{2}nRT = \frac{3}{2}pV$.
3. **For a change**, use $\Delta U = \frac{3}{2}Nk\,\Delta T$ and give the sign (a fall in $T$ means a **negative** increase in internal energy).

**Variations you will meet:**

- **"Explain why the internal energy is proportional to $T$"**: potential energy is zero, so $U$ is the total kinetic energy, and the mean kinetic energy of a molecule is proportional to $T$.
- **Internal energy from $p$ and $V$**: $U = \frac{3}{2}pV$ (no temperature needed); a full cycle back to the start has $\Delta U = 0$.
- **Tables of energy changes with the first law** ($\Delta U = q + W$): the ideal-gas part is $\Delta U$. At constant temperature $\Delta U = 0$; if $T$ doubles, $U$ doubles (increase $= U$); if $T$ falls from $4T$ to $2T$, $\Delta U = -2U$. The first law itself is in the Thermodynamics topic.
- **Direction of change from a $p$–$V$ graph**: $U \propto pV$, so $U$ decreases where $pV$ decreases.
- **Sketch $U$ against $V$ at constant pressure**: straight line through the origin.
- **Specific heat capacity of a gas**: $\Delta U = \frac{3}{2}Nk\,\Delta T$ with the total mass $Nm$ gives $c = \dfrac{3k}{2m}$ at constant volume.

## 3. Mean kinetic energy and r.m.s. speed (12 questions)

1. **Write** $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ with $T$ in kelvin and $m$ the mass of **one molecule** in kg.
2. **Solve** for the unknown ($\langle c^2\rangle$, $T$ or $m$).
3. **For the r.m.s. speed, take the square root** of $\langle c^2\rangle$.

**Variations you will meet:**

- **R.m.s. speed from temperature**: $c_{\text{r.m.s.}} = \sqrt{3kT/m}$. Don't divide by $\sqrt{2}$ afterwards: $\sqrt{\langle c^2\rangle}$ is already the r.m.s. speed.
- **Mass given in u**: multiply by $1.66 \times 10^{-27}$ kg. **Mass asked in u**: find $m$ in kg, then divide by $1.66 \times 10^{-27}$.
- **From pressure and density**: $p = \frac{1}{3}\rho\langle c^2\rangle$ gives $c_{\text{r.m.s.}} = \sqrt{3p/\rho}$; or use $pV = \frac{1}{3}(\text{mass of gas})\langle c^2\rangle$.
- **Temperature from r.m.s. speed** (for example a star's surface): $T = \dfrac{m\,c_{\text{r.m.s.}}^2}{3k}$; give the number of significant figures asked.
- **Ratio of r.m.s. speeds, same gas, two temperatures**: $\sqrt{T_2/T_1}$ with both in kelvin. Take the square root: the ratio of mean-square speeds is not the answer.
- **Ratio, two gases at the same temperature**: $\sqrt{m_2/m_1}$; for molecules of nucleon number $4$ and $100$ it is $\sqrt{100/4} = 5.0$.
- **Molecules nine times heavier at the same temperature**: same mean kinetic energy, so one third of the r.m.s. speed. State the link (kinetic energy $\propto T$, so the same; $\frac{1}{2}m\langle c^2\rangle$ the same, so $c_{\text{r.m.s.}} \propto \frac{1}{\sqrt{m}}$) before the conclusion.
- **Mean kinetic energy of one molecule**: $\frac{3}{2}kT$, for example $5.59 \times 10^{-21}$ J at $270$ K.

## 4. Ideal gas definition and kinetic theory assumptions (10 questions)

1. **Define an ideal gas**: a gas that obeys $pV \propto T$ (for all $p$, $V$ and $T$), **where $T$ is the thermodynamic temperature**.
2. **State the assumptions** asked for, each about the **molecules**.

**Variations you will meet:**

- **"State what is meant by an ideal gas"**: $pV \propto T$ earns one mark, "$T$ is thermodynamic temperature" the other. $pV = nRT$ only counts if you say $n$ and $R$ are constants. Don't list kinetic theory assumptions here, and don't add "only at constant temperature".
- **State two or three assumptions**: pick from continuous random motion; no intermolecular forces (except in collisions); negligible volume **of the molecules** compared with the volume **of the gas**; perfectly elastic collisions; collision time negligible.
- **What is an elastic collision?** One in which total kinetic energy is conserved.
- **Potential energy of the molecules**: no intermolecular forces, so potential energy is zero.
- **Why a gas at very high pressure (or near liquefying) is not ideal**: the molecules are very close together, so the intermolecular forces are not negligible and the volume of the molecules is not negligible compared with the gas volume.

- **Test an assumption with numbers**: for example, two molecules of mass $6.6 \times 10^{-26}$ kg that are $3.0 \times 10^{-9}$ m apart attract with a gravitational force of $3.2 \times 10^{-44}$ N, far smaller than a molecule's weight of $6.5 \times 10^{-25}$ N: it is negligible, which supports "no forces between molecules".

## 5. Graphs and sketches (8 questions)

1. **Decide what is proportional to what** at the stated condition: $pV \propto T$; $\langle c^2 \rangle \propto T$; $c_{\text{r.m.s.}} \propto \sqrt{T}$; $U \propto T$, and $U \propto V$ at constant $p$.
2. **Sketch** from the correct starting point, with the right shape, and stop at the limit of the data.

**Variations you will meet:**

- **R.m.s. speed against $T$**: a curve starting at the **origin**, rising with a **decreasing positive gradient**, never turning back towards an axis. If the data stop at $350$ K, draw it from $(0, 0)$ to $(350, c)$ only.
- **R.m.s. speed against pressure at constant temperature**: a **horizontal straight line** through the starting point, because $c_{\text{r.m.s.}}$ depends only on $T$.
- **Internal energy against $V$ at constant pressure**: straight line through the origin.
- **$p$ against $V$ for a cycle**: constant volume is a vertical line, constant pressure a horizontal line, constant temperature a curve with $pV$ constant (steeper at small $V$).
- **Draw conclusions from graphs of $pV$ and $\langle c^2 \rangle$ against $T$**: straight lines through the origin show the gases are ideal; the gradient of $\langle c^2 \rangle$ against $T$ is $\frac{3k}{m}$, giving the mass of a molecule; the gradient of $pV$ against $T$ is $nR$, giving the amount (then $N$ and total mass). Give numbers where you can.

## 6. Symbols, constants and deriving $E = \frac{3}{2}kT$ (7 questions)

1. **Name every symbol** precisely.
2. **For the derivation**: equate $\frac{1}{3}Nm\langle c^2 \rangle$ and $NkT$ (**M1**), then use $E = \frac{1}{2}m\langle c^2 \rangle$ to reach $\frac{3}{2}kT$ (**A1**).

**Variations you will meet:**

- **Symbols in $pV = NkT$**: $p$ pressure of the gas; $V$ volume of the gas; $N$ **number of molecules** (not moles); $k$ Boltzmann constant; $T$ **thermodynamic** temperature.
- **Symbols in $pV = \frac{1}{3}Nm\langle c^2 \rangle$**: $m$ mass of **one** molecule; $\langle c^2 \rangle$ **mean-square speed** of the molecules.
- **Avogadro constant**: the number of particles per mole (per unit amount of substance). **Relationship**: $N_{\text{A}} = \dfrac{R}{k}$.
- **Unfamiliar letters**: rearrange the given equation into the shape of $pV = nRT = \frac{N}{N_{\text{A}}}RT$ or $pV = NkT$ and match it term by term to name each constant. Then redo the $\frac{3}{2}kT$ derivation with the new letters in place of $k$ (or of $\frac{R}{N_{\text{A}}}$). For example, if $pV = nLZT$ with $L$ the Avogadro constant, then $LZ = R$, so $Z = \frac{R}{L}$ is the Boltzmann constant, and the mean kinetic energy of a molecule is $\frac{3}{2}ZT$.
- **Tables for two samples**: write each quantity from the formulas: $p = \frac{NkT}{V}$, $n = \frac{N}{N_{\text{A}}}$, $\langle c^2 \rangle = \frac{3kT}{m}$, $U = \frac{3}{2}NkT$, then put in the other sample's $N$, $m$, $V$.

### 7. Pressure from molecular movement (4 questions)

1. **Explain in steps**: molecules hit the walls; their momentum changes; so the wall exerts a force on them; so they exert a force on the wall (Newton's third law); many molecules hitting the walls give a force per unit area, the pressure.
2. **For the one-dimensional model**: change in momentum $2mu$; time between hits $\frac{2L}{u}$; force $\frac{mu^2}{L}$; pressure $\frac{mu^2}{L^3}$.

**Variations you will meet:**

- **Expressions in $m$, $u$, $L$**: keep the factor $2$ in both the momentum change ($2mu$) and the time ($2L/u$); use only the letters given.
- **R.m.s. speed from $p$ and $\rho$**: $p = \frac{1}{3}\rho\langle c^2\rangle$.
- **Why a gas cools when it expands** (use kinetic theory, not the first law): molecules rebound from the retreating piston with less speed, so their mean kinetic energy falls, so $T$ falls.

## Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

### Example 1

A cylinder of volume $2.50 \times 10^{-3}$ m³ contains an ideal gas at a pressure of $1.80 \times 10^{5}$ Pa and a temperature of $17\,^{\circ}$C.

(a) Calculate the amount of gas in the cylinder. **[2]**

(b) Calculate the number of molecules of gas. **[1]**

(c) The cylinder is sealed and heated to $87\,^{\circ}$C. Its volume does not change. Calculate the new pressure. **[2]**

**Solution**

(a) $T = 17 + 273 = 290$ K and $pV = nRT$ **C1** $n = \dfrac{1.80 \times 10^{5} \times 2.50 \times 10^{-3}}{8.31 \times 290} = 0.187$ mol **A1**

(b) $N = nN_{\mathrm{A}} = 0.1867 \times 6.02 \times 10^{23} = 1.12 \times 10^{23}$ **A1**

(c) At constant volume $p \propto T$, with $T_2 = 87 + 273 = 360$ K **C1** $p_2 = 1.80 \times 10^{5} \times \dfrac{360}{290} = 2.23 \times 10^{5}$ Pa **A1**

## Example 2

Nitrogen may be treated as an ideal gas. A nitrogen molecule has a mass of $28$ u.

(a) Show that the average translational kinetic energy of a molecule of an ideal gas is $\frac{3}{2}kT$. **[2]**

(b) Calculate the r.m.s. speed of nitrogen molecules at $300$ K. **[3]**

(c) Determine the temperature, in °C, at which the r.m.s. speed is twice the value in (b). **[2]**

**Solution**

(a) $pV = NkT$ and $pV = \frac{1}{3}Nm\langle c^2\rangle$, so $\frac{1}{3}Nm\langle c^2\rangle = NkT$ **M1** $m\langle c^2\rangle = 3kT$, and $E = \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ **A1**

(b) $m = 28 \times 1.66 \times 10^{-27} = 4.648 \times 10^{-26}$ kg **C1** $\frac{1}{2} \times 4.648 \times 10^{-26} \times c_{\text{r.m.s.}}^2 = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300$ **C1** $c_{\text{r.m.s.}} = 517$ m s$^{-1}$ **A1**

(c) $c_{\text{r.m.s.}} \propto \sqrt{T}$, so doubling the speed needs $4 \times 300 = 1200$ K **C1** $1200 - 273 = 927\,^\circ$C **A1**

## Example 3

A container of volume $2.00 \times 10^{-3}$ m³ holds helium, an ideal gas, at a temperature of $320$ K and a pressure of $1.20 \times 10^5$ Pa. The mass of a helium atom is $6.64 \times 10^{-27}$ kg.

(a) Calculate the r.m.s. speed of the helium atoms. **[2]**

(b) Use your answer to (a) to determine the density of the gas. **[2]**

(c) Determine the total kinetic energy of the helium atoms. Explain why this is the internal energy of the gas. **[3]**

**Solution**

(a) $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, so $\langle c^2\rangle = \dfrac{3 \times 1.38 \times 10^{-23} \times 320}{6.64 \times 10^{-27}} = 1.995 \times 10^6$ m² s$^{-2}$ **C1** $c_{\text{r.m.s.}} = \sqrt{1.995 \times 10^6} = 1410$ m s$^{-1}$ **A1**

(b) $p = \frac{1}{3}\rho\langle c^2\rangle$, so $\rho = \dfrac{3p}{\langle c^2\rangle} = \dfrac{3 \times 1.20 \times 10^5}{1.995 \times 10^6}$ **C1** $\rho = 0.180$ kg m$^{-3}$ **A1**

(c) Total kinetic energy $= N \times \frac{3}{2}kT = \frac{3}{2}pV = 1.5 \times 1.20 \times 10^5 \times 2.00 \times 10^{-3}$ **C1** $= 360$ J **A1** There are no forces between the atoms of an ideal gas, so their potential energy is zero and the internal energy is just this total kinetic energy. **B1**

## Example 4

Two containers X and Y are at the same thermodynamic temperature $T$. X has volume $V$ and holds $N$ molecules of an ideal gas, each of mass $m$. Y has volume $\frac{1}{2}V$ and holds $2N$ molecules of another ideal gas, each of mass $10m$.

(a) Determine the ratio $\frac{\text{pressure in Y}}{\text{pressure in X}}$. **[2]**

(b) Determine the ratio $\frac{\text{r.m.s. speed in Y}}{\text{r.m.s. speed in X}}$. Explain your reasoning. **[3]**

(c) State the ratio $\frac{\text{internal energy of Y}}{\text{internal energy of X}}$. **[1]**

**Solution**

(a) $p = \frac{NkT}{V}$, so $p_{\text{X}} = \frac{NkT}{V}$ and $p_{\text{Y}} = \frac{2NkT}{V/2} = \frac{4NkT}{V}$ **C1** ratio $= 4$ **A1**

(b) Same temperature, so the same mean kinetic energy $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$ for a molecule in each **B1** so $c_{\text{r.m.s.}} \propto \frac{1}{\sqrt{m}}$ **C1** ratio $= \frac{1}{\sqrt{10}} = 0.316$ **A1**

(c) $U = \frac{3}{2}NkT$, and Y has twice as many molecules at the same $T$: ratio $= 2$ **B1**

## Common mistakes

- **Moles instead of molecules.** The March 2021 and June 2023 reports describe candidates calculating the amount in moles when the number of molecules was asked; a number of moles with no conversion earned nothing. The November 2022 report adds that many confused $N$ (number of molecules) with amount of substance.
- **Mean-square speed given as the r.m.s. speed.** The March 2021, November 2021 and March 2025 reports all describe candidates stopping at $\langle c^2\rangle$, or finding the ratio of mean-square speeds and forgetting the square root. The November 2022 report describes the opposite error: dividing the correct r.m.s. speed by $\sqrt{2}$.
- **Celsius in a gas equation.** The November 2021, November 2022, June 2022 and March 2023 reports describe answers lost by leaving $T$ in °C. The March 2023 report also saw $273$ added to a temperature **change**.
- **One molecule instead of the whole gas.** The June 2021 and March 2023 reports say most candidates found the change in kinetic energy of a **single** molecule when the question asked for the total kinetic energy (internal energy) of the gas: multiply by $N$. The June 2021 report adds that few noticed the internal energy was **decreasing**, so its increase was negative.
- **Mixing up the ideal gas definition and the kinetic theory assumptions.** The March 2023 and June 2023 reports say many answered "what is an ideal gas" with assumptions. The June 2023 report says weaker candidates added conditions such as "only at constant temperature", and many forgot "$T$ is thermodynamic temperature". The June 2022 report says $pV = nRT$ alone needs $n$ and $R$ stated to be constants.
- **Assumptions about the gas, not the molecules.** The November 2021 report says a common error was not recognising that the assumptions are about the particles; the June 2023 report describes "negligible volume of the gas" or "of a single particle" instead of the total volume of the molecules compared with the gas.

- **Losing the factors of 2 in the derivation.** The November 2021 report says many omitted the $2$ in $2mu$ or in $\frac{2L}{u}$, and invented letters that were not defined.
- **"Show that" without full substitution.** The June 2021 report says the marks go to substituting every value (including $k$) into the starting equation; lists of numbers joined by arrows earn nothing. Starting from $pV = NkT$ rather than $pV = nRT$ saves the extra step to $N$.
- **Mass of gas versus mass of a molecule.** The March 2021 and June 2023 reports describe confusion between the two, and answers for the mass of a molecule of about $10^{26}$ kg that a quick sense check would have caught.
- **Wrong volume in $pV = NkT$.** The March 2025 report describes candidates using a **change** in volume as $V$.
- **Sketches of r.m.s. speed.** The June 2023 report says lines often ran past the last temperature given and only the strongest got the curvature right (decreasing gradient). The November 2022 report says few saw that at constant temperature the r.m.s. speed is constant, a horizontal line.
- **Kinetic theory explanations answered with the first law.** The June 2021 report says many explained cooling on expansion with the first law instead of molecular motion, and some thought collisions produce thermal energy.
- **Internal energy and $pV$.** The June 2023 report says many did not see that the internal energy of an ideal gas is proportional to $pV$, so its change can be read straight from a $p$–$V$ graph.

## How to get the marks

- **Definitions** (key words):
  - Ideal gas: obeys $pV \propto T$ **and** $T$ is thermodynamic temperature (two marks).
  - Avogadro constant: number of particles **per mole**.
  - Mean-square speed: the mean of the squares of the speeds; r.m.s. speed is its square root.
  - Internal energy of an ideal gas: total kinetic energy of the random motion of the molecules, **because** the potential energy is zero (no intermolecular forces).
- **Derivation of $\frac{3}{2}kT$**: two marks, **M1** for equating $\frac{1}{3}Nm\langle c^2\rangle$ with $NkT$ and **A1** for using $\frac{1}{2}m\langle c^2\rangle$ to reach the result. Write each step; if you start from $nRT$, show $n = N/N_\text{A}$ and $k = R/N_\text{A}$.
- **Calculations (C1 then A1)**: write the starting equation ($pV = NkT$, $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$), then the full substitution with $T$ in K, then the answer with a unit.
- **"Show that"**: substitute every value, including constants, and give the answer to one more significant figure than the value printed.
- **Significant figures**: data to three significant figures means an answer to three (for example $1.12 \times 10^{23}$ molecules, $517$ m s$^{-1}$ in Worked examples 1 and 2). When the question says "to three significant figures", give exactly three.
- **Explain your reasoning** (internal energy): the mark for "potential energy is zero, so $U$ is the total kinetic energy" comes before the number.
- **"Explain how … compares"** (r.m.s. speeds): give the link (same temperature so same mean kinetic energy; $c_\text{r.m.s.} \propto 1/\sqrt{m}$), then the numerical conclusion ("one third").

- **Sketches**: one mark for the start point and extent (through the origin, or through the given point), one for the shape (straight, or decreasing gradient, or horizontal).
- **Graph conclusions**: each correct statement earns a mark; say the gases are ideal, then add numerical conclusions from gradients (mass of a molecule, amount, number of molecules).
- **Assumptions**: each one about the **molecules**, with the comparison where needed ("volume of molecules negligible **compared with the volume of the gas**").

## Quick check

1. How many molecules are there in $1.00$ m³ of an ideal gas at $1.01 \times 10^{5}$ Pa and $273$ K?
2. Explain why the internal energy of an ideal gas is proportional to its thermodynamic temperature.
3. A gas is at $250$ K. To what temperature, in °C, must it be heated to triple the r.m.s. speed of its molecules?
4. The mean-square speed of the molecules of an ideal gas at $300$ K is $2.50 \times 10^{5}$ m² s⁻². Find the r.m.s. speed and the mass of a molecule, in kg and in u.
5. A fixed mass of ideal gas at $2.0 \times 10^{5}$ Pa occupies $4.0 \times 10^{-3}$ m³ at $300$ K. It is slowly compressed to $1.6 \times 10^{-3}$ m³, staying at $300$ K. Find its new pressure, and say what happens to the r.m.s. speed of its molecules.

## Answers

1. $N = \dfrac{pV}{kT} = \dfrac{1.01 \times 10^5 \times 1.00}{1.38 \times 10^{-23} \times 273} = 2.68 \times 10^{25}$.
2. There are no forces between the molecules of an ideal gas, so their potential energy is zero and the internal energy is the total kinetic energy of the molecules. The mean kinetic energy of a molecule is $\frac{3}{2}kT$, so $U = \frac{3}{2}NkT$, proportional to $T$.
3. $c_{\text{r.m.s.}} \propto \sqrt{T}$, so $T$ must be multiplied by $3^2 = 9$: $9 \times 250 = 2250$ K, which is $1977\,^\circ$C.
4. $c_{\text{r.m.s.}} = \sqrt{2.50 \times 10^5} = 500$ m s$^{-1}$. $m = \dfrac{3kT}{\langle c^2 \rangle} = \dfrac{3 \times 1.38 \times 10^{-23} \times 300}{2.50 \times 10^5} = 4.97 \times 10^{-26}$ kg, and $\dfrac{4.97 \times 10^{-26}}{1.66 \times 10^{-27}} = 29.9$ u (about 30 u).
5. At constant temperature $pV$ is constant: $p = \dfrac{2.0 \times 10^5 \times 4.0 \times 10^{-3}}{1.6 \times 10^{-3}} = 5.0 \times 10^5$ Pa. The r.m.s. speed does not change, because it depends only on the temperature, which stays at $300$ K.

## Past questions to try

- **9702/42/O/N/24 Q4**: kinetic theory assumptions, how molecules cause pressure, and conclusions from graphs of $\langle c^2 \rangle$ and $pV$ against $T$.
- **9702/42/O/N/21 Q3**: the one-dimensional derivation of pressure step by step, then $\frac{3}{2}kT$ and an r.m.s. speed.
- **9702/43/M/J/24 Q3**: defining an ideal gas, number of molecules, mean kinetic energy, internal energy and a sketch.
- **9702/42/O/N/25 Q2**: pressure from $pV = nRT$, the average separation of molecules, and testing the no-forces assumption.