Cambridge AS & A Level · Physics (9702) · Notes

Kinematics AS

Read online at pastopic.com/cambridge/physics-9702/notes/kinematics

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Read first: Physical quantities and units

What you need to know

By the end of this topic you should be able to:

1. **Define and use distance, displacement, speed, velocity and acceleration**, and say which are scalars and which are vectors.
2. **Show motion on graphs**: distance–time, displacement–time, speed–time, velocity–time and acceleration–time, and read what each one tells you.
3. **Find displacement from the area under a velocity–time graph.**
4. **Find velocity from the gradient of a displacement–time graph.**
5. **Find acceleration from the gradient of a velocity–time graph.**
6. **Derive the equations of uniformly accelerated motion** from the definitions of velocity and acceleration (using a velocity–time graph).
7. **Solve problems with these equations** for motion in a straight line with constant acceleration, including objects falling freely near the Earth with no air resistance.
8. **Describe an experiment to measure the acceleration of free fall** g using a falling object.
9. **Describe and explain motion with a constant velocity in one direction and a constant acceleration at right angles to it**, such as a ball thrown through the air (a projectile).

Every calculation here assumes no air resistance and a constant acceleration. How forces cause the acceleration (and how air resistance changes it) is in Dynamics.

Understand it

Distance and displacement, speed and velocity

Distance is how far something has travelled along its path. It is a scalar: it has size only. **Displacement** is the distance in a straight line from the starting point **in a stated direction**. It is a vector.

Walk 30 m east and then 40 m north. You have covered a distance of 70 m, but your displacement is only $\sqrt{30^2 + 40^2} = 50$ m, at $\tan^{-1} \frac{40}{30} = 53.1^\circ$ north of east. Walk once round a circular track and your distance is the whole circumference, but your displacement is zero, because you are back where you started.

The same split works for how fast:

- **Speed** is the rate of change of distance: $\text{speed} = \dfrac{\text{distance}}{\text{time}}$. A scalar.
- **Velocity** is the rate of change of displacement: $v = \dfrac{\Delta s}{\Delta t}$. A vector, so it has a direction. Two cars at 20 m s^{-1} going opposite ways have the same speed but different velocities, often written $+20$ and -20 m s^{-1}.

If that walk took 50 s, the average speed was $\frac{70}{50} = 1.4$ m s^{-1} and the average velocity $\frac{50}{50} = 1.0$ m s^{-1} at 53.1° north of east. **Average speed** is always total distance ÷ total time, never the average of the speeds.

Acceleration

Acceleration is the rate of change of velocity:

$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{t}$$

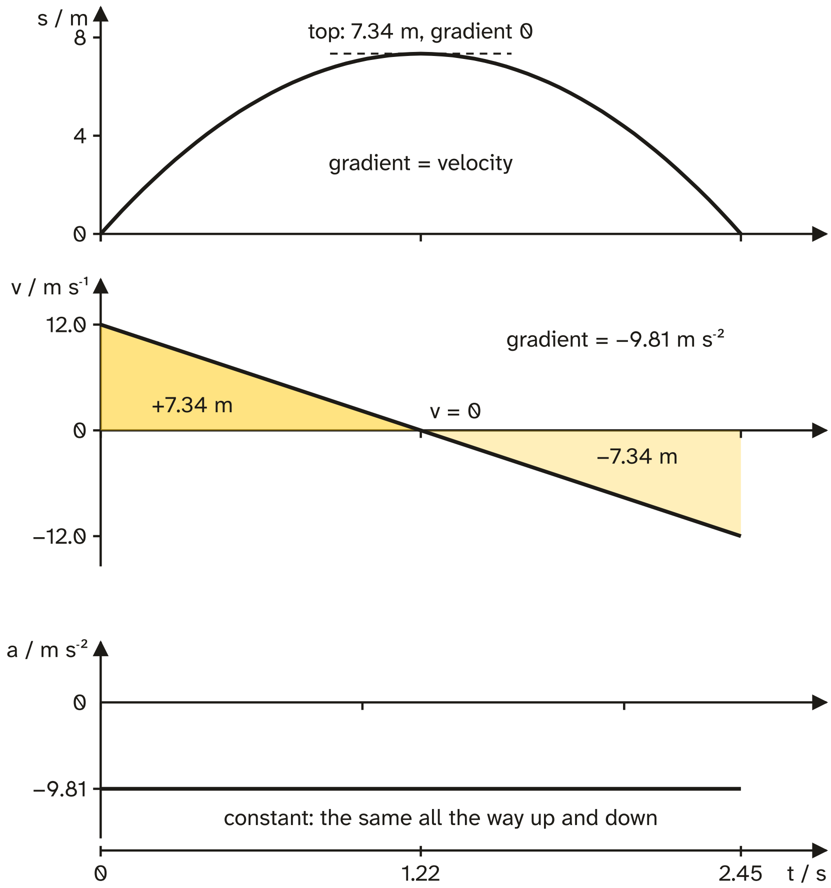
where u is the starting velocity and v the final one. Units: (m s^{-1}) per s, so m s^{-2}. It is a vector. Because velocity includes direction, an object speeding up, slowing down **or changing direction** is accelerating. A negative acceleration in a direction you chose as positive is sometimes called a deceleration.

Reading motion graphs

The two rules that do almost everything:

- **Gradient of a displacement–time graph = velocity.** A straight line means constant velocity; a curve means the velocity is changing; a flat line means at rest. To find the gradient of a curve at one instant, draw the **tangent** there and find its gradient with a large triangle.
- **Gradient of a velocity–time graph = acceleration**, and **area under a velocity–time graph = displacement.** Area below the time axis counts as negative displacement (moving the other way).

The same rules give two more: the area under an acceleration–time graph is the **change in velocity**, and a distance–time graph (which never goes down) has gradient equal to the speed.



Look at the ball thrown straight up at 12.0 m s⁻¹ (upwards positive):

- The **velocity–time** graph is one straight line of gradient -9.81 m s⁻². The acceleration is the same on the way up, at the top and on the way down.
- At the top the velocity is zero for an instant, but the acceleration is **not** zero. It takes $\frac{12.0}{9.81} = 1.22$ s to get there.
- The area of the triangle above the axis, $\frac{1}{2} \times 1.223 \times 12.0 = 7.34$ m, is the height reached. The triangle below the axis has the same area, so the total displacement after 2.45 s is zero: the ball is back in your hand.
- The **displacement–time** graph is a curve. Its gradient (the velocity) starts steep, is zero at the top and is steeply negative at the end.

Deriving the equations of motion

For a **constant acceleration** a in a straight line, draw the velocity–time graph: a straight line from u at time 0 to v at time t.

1. **From the gradient**: $a = \dfrac{v-u}{t}$, so $v = u + at$.
2. **From the area**: the area under the line is a trapezium, so the displacement is $s = \frac{(u+v)}{2}\,t$. The average velocity is $\frac{u+v}{2}$ only because the graph is a straight line.
3. **Put $v = u + at$ into the area equation**: $s = \dfrac{u+u+at}{2}\,t$, so $s = ut + \frac{1}{2}at^2$. (Or split the area into a rectangle ut and a triangle $\frac{1}{2} \times t \times at$.)

4. **Get rid of** t: $t = \dfrac{v-u}{a}$ in $s = \dfrac{(u+v)}{2}t$ gives $s = \dfrac{v^2-u^2}{2a}$, so $v^2 = u^2 + 2as$.

Each equation leaves out one of the five quantities s, u, v, a, t. That is how you pick one: list the three you know and the one you want, and use the equation that doesn't need the fifth. They only work when the acceleration is constant, and $s = \frac{1}{2}at^2$ also needs the object to start from rest ($u = 0$).

Falling freely

With no air resistance, everything near the Earth's surface falls with the same acceleration, $g = 9.81$ m s^{-2} downwards, whatever its mass. A heavy ball and a light ball dropped together land together. On another planet or the Moon, g is different, so use the value you are given.

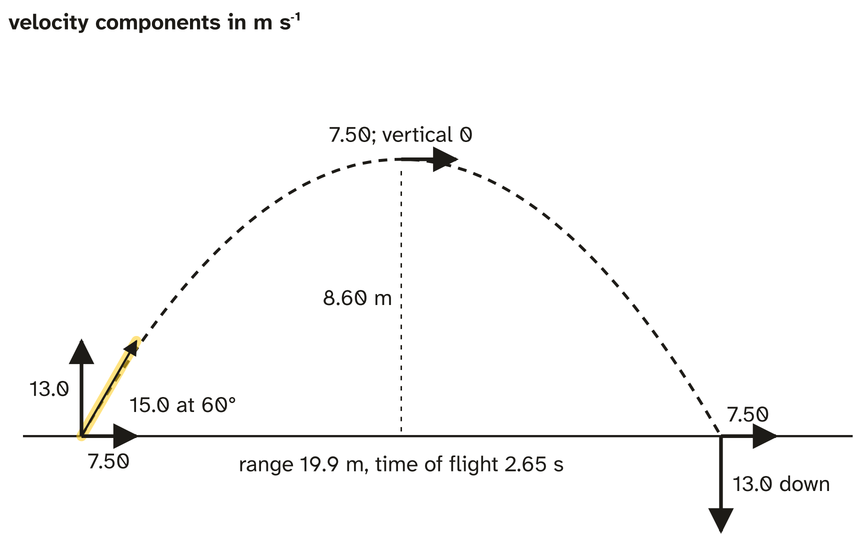
Choose a positive direction and stick to it. With **upwards positive**, a ball thrown up at 12.0 m s^{-1} has $u = +12.0$ and $a = -9.81$; a displacement of -25 m means 25 m **below** the start.

Projectiles: two motions at once

A projectile moves sideways and up or down at the same time. With no air resistance, the only force is its weight, which acts straight down. So:

- **Horizontally**: no force, no acceleration. The horizontal velocity stays the same all the way.
- **Vertically**: a constant acceleration g downwards, exactly as if the object had been thrown straight up or dropped.

The two motions happen **independently**, and the only thing they share is the **time**. Split the launch velocity into components, work out each direction on its own, and link them through t.



For a ball launched from the ground at 15.0 m s^{-1} at 60° to the horizontal:

- horizontal component $u_x = 15.0\cos 60^\circ = 7.50$ m s^{-1}, the same all the way;
- vertical component $u_y = 15.0\sin 60^\circ = 13.0$ m s^{-1} upwards, falling by 9.81 m s^{-1} every second;

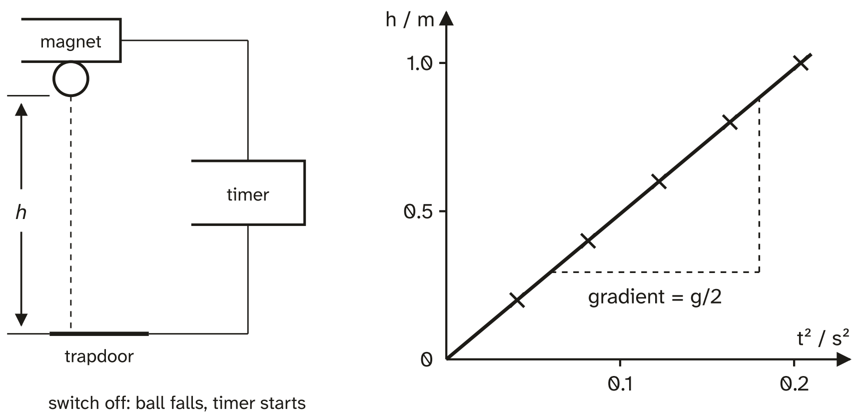
- time to the top (where $v_y = 0$): $t = \frac{12.99}{9.81} = 1.324$ s; time of flight $2 \times 1.324 = 2.65$ s;
- maximum height: $0 = 12.99^2 - 2(9.81)h$, so $h = 8.60$ m;
- range: $7.50 \times 2.65 = 19.9$ m.

At the top the ball is still moving at 7.50 m s^{-1} sideways, so its speed is **not** zero there. The path is a curve (a parabola) because the horizontal displacement grows steadily with t while the vertical displacement changes with t^2.

A ball thrown horizontally has $u_y = 0$, so the time to fall depends only on the height: $h = \frac{1}{2}gt^2$. Throwing it faster sideways makes it land further away but not sooner.

Final speed and energy. A projectile's final speed comes from its components: $v = \sqrt{v_x^2 + v_y^2}$. Since $v_y^2 = u_y^2 + 2gh$ for a drop h, and $v_x = u_x$, this gives $v^2 = u^2 + 2gh$. So the speed on landing depends only on the launch speed and the height dropped, not on the launch angle. It is the same result you get from energy: the gain in kinetic energy equals the loss in gravitational potential energy.

Measuring g



switch off: ball falls, timer starts
ball hits trapdoor: timer stops

1. An electromagnet holds a small steel ball. Switching the magnet off releases the ball and starts an electronic timer. The ball hits a trapdoor (or passes a light gate) which stops the timer.
2. Measure the height h fallen with a metre rule, from the **bottom of the ball** to the trapdoor, and record the time t.
3. Repeat each height and take the mean time, then do it for five or more heights.
4. The ball starts from rest, so $h = \frac{1}{2}gt^2$. Plot h against t^2: a straight line through the origin with **gradient** $= \frac{g}{2}$, so $g = 2\times$ gradient.

A small, dense ball keeps air resistance negligible. Timing by hand with a stopwatch adds your reaction time, which is why electronic timing or light gates are used. The magnet takes a moment to let go of the ball, so every time reading is a little too large and g comes out too small. This is a systematic error: repeating and averaging does not remove it. Measuring h to the top of the ball instead of the bottom adds the same length to every height instead. On the h–t^2 graph that shifts the line so it misses the origin, but the gradient, and so g, is unchanged. For one reading, $g = \dfrac{2h}{t^2}$, so the percentage uncertainty in g is the percentage uncertainty in h **plus twice** that in t.

Key facts and formulas

"Given" means it is on the data sheet in the exam. "Learn it" means you must remember it.

Formula	Status
Speed $= \dfrac{\text{distance}}{\text{time}}$; velocity $v = \dfrac{\Delta s}{\Delta t}$ (rate of change of displacement)	Learn it
Acceleration $a = \dfrac{\Delta v}{\Delta t}$ (rate of change of velocity)	Learn it
Average speed $= \dfrac{\text{total distance}}{\text{total time}}$; average velocity $= \dfrac{\text{total displacement}}{\text{total time}}$	Learn it
Gradient of s–t graph = velocity; gradient of v–t graph = acceleration	Learn it
Area under v–t graph = displacement; area under a–t graph = change in velocity	Learn it
$v = u + at$	Learn it
$s = \frac{1}{2}(u+v)t$	Learn it
$s = ut + \frac{1}{2}at^2$	Given
$v^2 = u^2 + 2as$	Given
$g = 9.81$ m s^{-2}	Given
Launch at θ to the horizontal: $u_x = u\cos\theta$, $u_y = u\sin\theta$	Learn it
Speed from components: $v = \sqrt{v_x^2 + v_y^2}$, angle to horizontal $= \tan^{-1}\dfrac{v_y}{v_x}$	Learn it
Free-fall experiment: $h = \frac{1}{2}gt^2$, so gradient of h against $t^2 = \frac{g}{2}$	Learn it
Uncertainty in $g = \dfrac{2h}{t^2}$: $\%g = \%h + 2 \times \%t$	Learn it

The equations of motion are for **constant acceleration in a straight line** only. For a projectile, use them in each direction separately.

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (125 different questions, 2021–2025; 5 more appear on two papers), the types came up this often. 17 of the 125 questions mix two or three types, so the counts add up to more than 125.

Question type	Questions
Motion graphs: gradients, areas and sketches	37

Question type	Questions
Projectiles	37
Equations of motion in a straight line	30
Distance, displacement, speed, velocity and the definitions	23
Deriving the equations and when they apply	6
Falling with air resistance: reading the curves	6
Measuring g by free fall	5

1. Motion graphs: gradients, areas and sketches (37 questions)

Reading values from a velocity–time graph:

1. **Acceleration** = gradient $= \dfrac{\Delta v}{\Delta t}$ for a straight section. Read both values carefully from the grid and check the units on the axes (km h^{-1}, cm s^{-1} or ms need converting).
2. **Displacement** = area under the line. Split it into rectangles and triangles, or use the trapezium $\frac{1}{2}(u+v)t$. Area **below** the axis is displacement the other way.
3. **Distance** travelled = the sum of all the areas, all counted as positive. **Displacement** = areas above minus areas below.

For example, a runner goes from 3.0 m s^{-1} to 7.0 m s^{-1} steadily in 5.0 s: acceleration $= \frac{7.0-3.0}{5.0} = 0.80$ m s^{-2}, and displacement $= \frac{1}{2}(3.0+7.0) \times 5.0 = 25$ m (a trapezium, not a triangle, because she didn't start from rest).

Variations you will meet:

- **Which graph matches the motion?** Work section by section: at rest is flat on an s–t graph and on the axis of a v–t graph; constant velocity is a straight sloping s–t line and a flat v–t line; constant acceleration is a curve on s–t and a straight sloping line on v–t. On a **distance**–time graph the line never goes down; a return journey still climbs, and a faster leg is steeper.
- **Changing one graph into another.** From s–t to v–t: sketch the gradient at each moment. From v–t to a–t: sketch the gradient again. From a–t to v–t: a constant positive a from rest gives a straight line up; zero a gives a flat line.
- **Sketching v–t for an object thrown up** (upwards positive): one straight line with negative gradient, crossing the time axis at the top and continuing below it to the end of the motion. Use a ruler.
- **"At which point is the object at rest?"** Where $v = 0$ (the line crosses the time axis). **"Zero acceleration?"** Where the v–t graph is flat (a peak, a trough or a flat section).
- **Bouncing ball** (upwards positive): straight lines of the same gradient $-g$, with sudden jumps from negative to positive velocity at each bounce. Each triangle's area is a height. If the rebound speed is less than the impact speed, the bounce is inelastic.
- **Non-uniform acceleration**: draw a tangent at the time asked, and find its gradient from a large triangle.

- **Momentum–time graphs** for a constant mass: velocity $= \frac{p}{m}$, so divide the graph by the mass to get velocities, then use the rules above.

2. Projectiles (37 questions)

1. **Resolve the launch velocity**: $u_x = u\cos\theta$, $u_y = u\sin\theta$ (for θ measured from the horizontal). Thrown horizontally: $u_x = u$, $u_y = 0$.
2. **Horizontal**: constant velocity, so $x = u_x t$.
3. **Vertical**: constant acceleration g downwards. Use the equations of motion with u_y, choosing up or down as positive.
4. **Link the two through the time.** Often you find t from one direction (for example $t = \frac{x}{u_x}$ to reach a wall) and use it in the other.
5. **Speed at any moment**: find v_x (still u_x) and v_y (from $v_y = u_y - gt$), then $v = \sqrt{v_x^2 + v_y^2}$ and angle $\tan^{-1}\frac{v_y}{v_x}$.

Variations you will meet:

- **Thrown horizontally from a height h**: time from $h = \frac{1}{2}gt^2$, then range $= u_x t$. If the height is 4 times as big, the time is $\sqrt{4} = 2$ times as long and so is the range.
- **Maximum height**: at the top $v_y = 0$, so $0 = u_y^2 - 2gH$ and $H = \dfrac{u_y^2}{2g}$; time to the top $= \dfrac{u_y}{g}$.
- **Clearing a wall or hitting a target**: time to reach it from $t = \frac{x}{u_x}$, then its height from $y = u_y t - \frac{1}{2}gt^2$ (plus the launch height). If v_y is negative there, the ball is already on its way down.
- **Displacement** from launch to landing: $\sqrt{x^2 + y^2}$, which is less than the distance along the curved path.
- **Descriptions and graphs**: horizontal velocity constant (a flat line), vertical velocity changing steadily (a straight sloping line), acceleration always g downwards. Horizontal displacement–time is a straight line; vertical displacement–time is a curve. Speed decreases on the way up to a minimum of u_x (not zero) at the top, then increases.
- **Same speed, different angle**: the time in the air depends only on the vertical motion, so a smaller upward component means less time. The landing speed depends only on the launch speed and the height dropped, $v^2 = u^2 + 2gh$, so it doesn't change with the angle.
- **Landing on a slope**: find x and y separately; the slope angle is $\tan^{-1}\frac{y}{x}$.
- **Formula options** in multiple choice, such as the range: time of flight $\frac{2u\sin\theta}{g}$ times $u\cos\theta$ gives $\dfrac{2u^2\sin\theta\cos\theta}{g}$. Build it from the steps rather than guessing.

3. Equations of motion in a straight line (30 questions)

1. **Pick a positive direction** (the direction of motion, or upwards for things thrown up).
2. **List s, u, v, a, t**: three known values and the one you want. "From rest" means $u = 0$; "comes to rest" means $v = 0$; "at its highest point" means $v = 0$; free fall means $a = \pm 9.81$ m s^{-2}.
3. **Choose the equation** that leaves out the fifth quantity, substitute with signs, and solve.

4. **Check** that the answer makes sense, and give the unit.

For example, a stone thrown down a well at 4.0 m s⁻¹ hits the water 2.0 s later (downwards positive): $s = 4.0 \times 2.0 + \frac{1}{2} \times 9.81 \times 2.0^2 = 27.6$ m.

Variations you will meet:

- **Thrown up from a height, lands below**: use one equation for the whole flight with signs, for example $-25 = 8.0t - 4.905t^2$, and solve the quadratic (Example 3).
- **Two stages**: the final velocity of one stage is the initial velocity of the next. Braking after a reaction time: distance at constant speed during the reaction time, plus the braking distance.
- **"How much further to stop?"** For the same deceleration, $v^2 = u^2 + 2as$ gives the deceleration from the first stage. For example, slowing from 10 to 8 m s^{-1} in 9.0 m gives $a = \frac{8^2-10^2}{2\times9.0} = -2.0$ m s^{-2}, and stopping from 8 m s^{-1} then needs $\frac{8^2}{2\times2.0} = 16$ m more.
- **Catching up**: write each object's displacement in terms of t and set them equal. A car at a steady 15 m s^{-1} passes a second car at rest, which then accelerates at 2.5 m s^{-2}: $15t = \frac{1}{2} \times 2.5 \times t^2$, so $t = 12$ s. For two constant velocities, divide the gap by the difference in speeds.
- **Ratios**: from rest, $s = \frac{1}{2}at^2$ so $t \propto \sqrt{\frac{s}{a}}$, and $v^2 = 2as$ so $v \propto \sqrt{as}$. A fall on a planet where g is one quarter as big takes twice as long. In time $\frac{T}{2}$ a dropped object falls only $\frac{1}{4}$ of the height it falls in T.
- **Echo or splash timing**: total time = falling time + time for sound to come back ($\frac{\text{depth}}{\text{speed of sound}}$). A stone dropped into a 45 m well takes $\sqrt{\frac{2\times45}{9.81}} = 3.03$ s to fall and $\frac{45}{330} = 0.14$ s for the sound, 3.17 s in all.
- **Unit conversions**: 1 km h^{-1} $= \frac{1000}{3600}$ m s^{-1}. Convert before using the equations.
- **Up a slope or along a road**: the equations work in any straight line as long as the acceleration along it is constant.

4. Distance, displacement, speed, velocity and the definitions (23 questions)

1. **Definitions** in words: velocity is the **rate of change of displacement**; acceleration is the **rate of change of velocity**; displacement is the **distance in a straight line from a point, in a stated direction**.
2. **Average speed** $= \dfrac{\text{total distance}}{\text{total time}}$. Work out the time for each part first. For example, 60 km at 40 km h^{-1} (1.5 h) then 60 km at 60 km h^{-1} (1.0 h) gives $\frac{120}{2.5} = 48$ km h^{-1}, not 50.
3. **Average velocity** $= \dfrac{\text{displacement}}{\text{time}}$. Half way round a circle of radius 5.0 m, the distance is $\pi \times 5.0 = 15.7$ m but the displacement is the diameter, 10.0 m. After a full lap the average velocity is zero.

Variations you will meet:

- **Change in velocity** between two directions: a vector subtraction, $\Delta\mathbf{v} = \mathbf{v} - \mathbf{u}$. Draw $\mathbf{v}$ and $-\mathbf{u}$ tip to tail; at right angles the size is $\sqrt{u^2 + v^2}$.
- **Pulses and echoes**: distance $=$ speed $\times$ time, halved when the signal goes there and back. Light and radio travel at 3.00×10^8 m s^{-1}.
- **Which statement is wrong?** Watch for "velocity is the rate of change of distance" or "acceleration is rate of change of speed per unit time": both are wrong.

- **Scalar or vector?** Distance and speed are scalars; displacement, velocity and acceleration are vectors.

5. Deriving the equations and when they apply (6 questions)

1. **Which equation comes from the definition of acceleration** (or the gradient of the v–t graph)? $v = u + at$.
2. **Which comes from the area** under the graph? $s = \frac{1}{2}(u+v)t$ (and, with $v = u + at$, $s = ut + \frac{1}{2}at^2$).
3. **"State two conditions"** for a simplified equation: say what each missing term needs. $s = \frac{1}{2}at^2$ needs the object to **start from rest** and to have **constant acceleration**. $s = \frac{v^2}{2a}$ needs the same two conditions. Don't give two that contradict each other (a constant acceleration and zero resultant force can't both be true).
4. **Using an equation inside a proof**, for example showing that work done $Fs = mas$ equals $\frac{1}{2}mv^2$: from rest, $v^2 = 2as$, so $as = \frac{1}{2}v^2$ and $mas = \frac{1}{2}mv^2$.

6. Falling with air resistance: reading the curves (6 questions)

At AS you don't calculate with air resistance, but you do read the graphs it makes.

- **Falling from rest with air resistance**: the v–t graph starts with gradient g, then gets less steep and levels off at the **terminal velocity**, where the acceleration is zero. The a–t graph starts at g and falls to zero.
- **Opening a parachute**: the velocity drops quickly to a new, lower terminal velocity. The gradient is large and negative just after opening.
- **Acceleration at one instant**: draw the tangent to the curve and find its gradient.
- **Distance fallen**: area under the curve. Count the grid squares (a square more than half inside counts as one) and multiply by the displacement one square stands for, or split it into straight-sided shapes.
- **Thrown up with air resistance**: the object slows faster on the way up (weight and drag both act down) than it speeds up on the way down, so the v–t line is steeper above the axis than below. Why the forces change is covered in Dynamics.

7. Measuring g by free fall (5 questions)

1. **Describe** the method: the electromagnet, ball and timer set-up in "Understand it", with h measured and t timed for several heights.
2. **Analyse**: $h = \frac{1}{2}gt^2$, so $g = \frac{2h}{t^2}$ for one reading, or $g = 2\times$ the gradient of h against t^2. If the graph is h against t^2 with gradient G, then $g = 2G$.
3. **Uncertainty**: $\%g = \%h + 2 \times \%t$ (the time is squared). For example, h to 0.5% and t to 1% gives $0.5 + 2 = 2.5\%$.
4. **Errors**: both of these are **systematic** (the same every time, so averaging doesn't remove them). A delay in the magnet letting go makes every t too large, so g comes out too small. Measuring h to the top of the ball instead of the bottom makes every h wrong by the same length: the h–t^2 line misses the origin, but

its gradient still gives g. Reaction time with a stopwatch varies from go to go (**random**) and is usually bigger than the fall time's own uncertainty. Results can be **precise** (close together) but not **accurate** (far from the true value) when a systematic error is present.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A car is travelling at 25 m s^{-1} along a straight, level road when the driver sees an obstacle. After a reaction time of 0.70 s the driver brakes, and the car decelerates uniformly, stopping 50 m after the brakes are applied.

(a) Calculate the deceleration of the car. **[2]**

(b) Calculate the time for which the brakes are applied. **[1]**

(c) Calculate the total distance the car travels from the moment the driver sees the obstacle. **[2]**

Solution

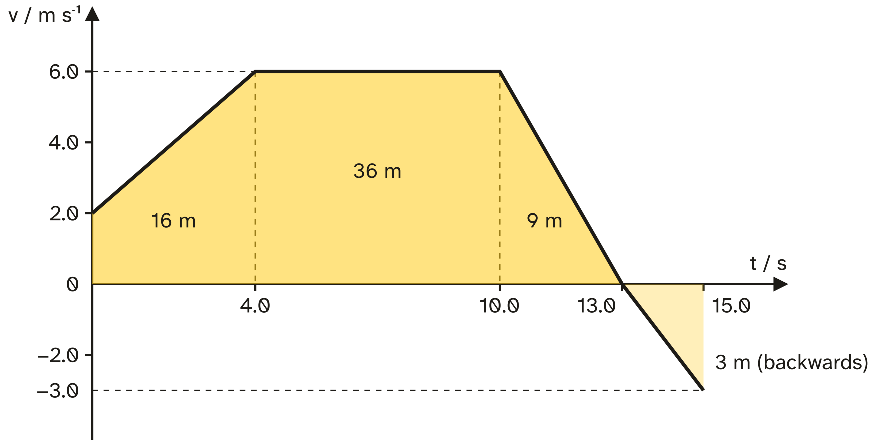
(a) While braking: $u = 25$, $v = 0$, $s = 50$. Use $v^2 = u^2 + 2as$: $0 = 25^2 + 2a(50)$. **M1** $a = -\frac{625}{100} = -6.25$ m s^{-2}, a deceleration of 6.25 m s^{-2}. **A1**

(b) $v = u + at$: $0 = 25 - 6.25t$, so $t = 4.0$ s. **A1**

(c) During the reaction time the speed is constant: $25 \times 0.70 = 17.5$ m. **M1** Total $= 17.5 + 50 = 67.5$ m, or 68 m to 2 significant figures. **A1**

Example 2

A toy car moves along a straight track. Its velocity–time graph is shown.



(a) Calculate the acceleration of the car from $t = 0$ to $t = 4.0$ s. **[1]**

(b) State the time at which the car is momentarily at rest. **[1]**

(c) Determine the distance travelled and the displacement from the start at $t = 15.0$ s. **[4]**

(d) Calculate the acceleration of the car at $t = 12.0$ s. **[1]**

Solution

(a) Gradient $= \dfrac{6.0 - 2.0}{4.0} = 1.0$ m s^{-2}. **A1**

(b) At rest means $v = 0$: $t = 13.0$ s, where the line crosses the time axis. **B1**

(c) Areas above the axis: 0–4 s trapezium $\frac{1}{2}(2.0 + 6.0) \times 4.0 = 16$ m; 4–10 s rectangle $6.0 \times 6.0 = 36$ m; 10–13 s triangle $\frac{1}{2} \times 3.0 \times 6.0 = 9$ m. Total forwards $= 61$ m. **M1 A1** Below the axis, 13–15 s: $\frac{1}{2} \times 2.0 \times 3.0 = 3$ m backwards. **M1** Distance $= 61 + 3 = 64$ m; displacement $= 61 - 3 = 58$ m in the original direction. **A1**

(d) From 10 s to 15 s is not one straight line, but 10–13 s is: gradient $= \dfrac{0 - 6.0}{13.0 - 10.0} = -2.0$ m s^{-2}. **A1**

Example 3

A ball is thrown vertically upwards at 8.0 m s^{-1} from the edge of a cliff 25 m above the sea. Air resistance is negligible.

(a) Calculate the maximum height of the ball above the sea. **[2]**

(b) Calculate the time from the throw until the ball hits the sea. **[3]**

(c) Calculate the speed at which the ball hits the sea. **[2]**

Solution

Take upwards as positive, so $a = -9.81$ m s^{-2}.

(a) At the top $v = 0$: $0 = 8.0^2 - 2(9.81)s$, so $s = \dfrac{64}{19.62} = 3.26$ m above the cliff. **M1** Height above the sea $= 25 + 3.26 = 28.3$ m. **A1**

(b) The sea is 25 m **below** the start, so $s = -25$ m. **B1** $s = ut + \frac{1}{2}at^2$: $-25 = 8.0t - 4.905t^2$, so $4.905t^2 - 8.0t - 25 = 0$. **M1**

$$t = \frac{8.0 + \sqrt{8.0^2 + 4(4.905)(25)}}{2(4.905)} = \frac{8.0 + 23.55}{9.81} = 3.22 \text{ s}$$

(the other root is negative). **A1**

(c) $v^2 = u^2 + 2as = 8.0^2 + 2(-9.81)(-25) = 554.5$. **M1** $v = 23.5$ m s-1 (downwards). **A1**

Check: $v = u + at = 8.0 - 9.81 \times 3.216 = -23.5$ m s-1. ✓

Example 4

A ball is kicked from horizontal ground at 18 m s-1 at 35° above the horizontal, towards a vertical wall 20 m away. The wall is 4.5 m high. Air resistance is negligible.

(a) Calculate the horizontal and vertical components of the initial velocity. **[2]**

(b) Calculate the time taken for the ball to reach the wall. **[1]**

(c) Show that the ball goes over the wall, and state whether it is rising or falling as it does so. **[4]**

(d) Calculate the speed of the ball as it passes over the wall. **[2]**

Solution

(a) $u_x = 18\cos 35^\circ = 14.7$ m s-1 **B1**; $u_y = 18\sin 35^\circ = 10.3$ m s-1 **B1**.

(b) Horizontally the velocity is constant: $t = \dfrac{20}{14.74} = 1.36$ s. **A1**

(c) Vertically (upwards positive): $y = u_y t - \frac{1}{2}gt^2 = 10.32 \times 1.356 - \frac{1}{2} \times 9.81 \times 1.356^2$ **M1** $= 14.00 - 9.02 = 4.98$ m. **A1** 4.98 m is more than 4.5 m, so the ball clears the wall (by about 0.5 m). **A1** $v_y = u_y - gt = 10.32 - 9.81 \times 1.356 = -2.98$ m s-1. It is negative, so the ball is already **falling**. **B1** (Or: it reaches the top at $\frac{10.32}{9.81} = 1.05$ s, before it gets to the wall.)

(d) $v = \sqrt{14.74^2 + 2.98^2}$ **M1** $= 15.0$ m s-1. **A1**

Keep the unrounded components (14.74, 10.32) in the working; rounding to 14.7 and 10.3 first moves the height in (c) to 4.93 m.

Common mistakes

- **Loose definitions.** "Velocity is speed in a direction" or "rate of change of distance" earns nothing; examiners want "rate of change of displacement". Acceleration is the "rate of change of velocity" ("change in velocity per unit time" is also correct). The trap is mixing the two: "rate of change of velocity per unit time" divides by time twice, and "rate of change of speed" leaves out direction.

- **Using a triangle when the graph starts above zero.** Examiner reports mention candidates finding the area under a v–t graph as a triangle instead of a trapezium, or putting $u = 0$ into an equation when the object was already moving.
- **Misreading the graph.** Read values at the exact times asked, check the axis units and powers of ten, and use a large triangle for a gradient.
- **Forgetting the sign convention.** A ball thrown up and landing below its start has a negative displacement. Mixing up signs in $s = ut + \frac{1}{2}at^2$ is the commonest way to lose the final mark.
- **Using the horizontal velocity as the initial vertical velocity** in a projectile. Examiners see this often; a ball thrown horizontally has $u_y = 0$.
- **Adding components instead of using Pythagoras.** Components of 12 m s^{-1} across and 16 m s^{-1} down give a speed of 20 m s^{-1}, not 28.
- **Thinking a projectile stops at the top.** Only the vertical velocity is zero; the speed at the top is u_x. "Its speed decreases to zero" is a popular wrong option.
- **Thinking heavier objects fall faster.** With no air resistance the acceleration is g for every mass, so the time and landing speed don't depend on mass.
- **Drawing a curve for constant acceleration.** On a v–t graph constant acceleration is a straight line, drawn with a ruler, and for a ball thrown up it must carry on through zero to the end of the motion.
- **Mistaking the area under an a–t graph** for the average or final velocity. It is the change in velocity.
- **Using 9.81 m s^{-2} on another planet.** Use the g given for that planet, or find it from the graph.
- **Averaging speeds.** Average speed is total distance over total time, not the mean of two speeds.
- **Rounding too early** in multi-step projectile questions. Keep at least one more figure than the data until the last line.

How to get the marks

- **"Define"**: a precise word equation with the key word: "rate of change of **displacement**", "rate of change of **velocity**". Symbols alone need each one explained.
- **Show the equation first.** Physics mark schemes give a **C1** mark for the right equation with the numbers in (shown as **M1** in the examples above), then **A1** for the answer. If the final answer slips, the C1 mark is still earned from visible working.
- **"Show that"**: work it through and give your answer to at least one more significant figure than the value printed in the question, and don't use the given value in your own working.
- **Units and figures**: give the unit (m s^{-1}, m s^{-2}), and 2 or 3 significant figures to match the data. Carry unrounded values through the working.
- **Vectors**: a velocity or displacement answer may need a direction ("58 m in the original direction", "23.5 m s^{-1} downwards"). "Magnitude" means the size only.
- **Sketches**: straight lines with a ruler where the gradient is constant; start and end at the right values and times; cross the axis at the right time; label lines if asked. Mark schemes give separate marks for the start value, the end value and the shape.

- **"State and explain"** (projectiles): give the conclusion ("less time") **and** the reason in physics terms ("the initial vertical component of velocity is smaller"). A reason without the conclusion, or the other way round, earns only one mark.
- **"By considering energy"**: compare the total initial energy and the change in gravitational potential energy, then conclude about the speed.
- **Multiple choice**: work the numbers yourself before looking at the options; the wrong options are often the results of common slips (a missing $\frac{1}{2}$, a forgotten square root, a sign).

Quick check

1. A runner completes one lap of a 400 m circular track in 80 s. State her average speed and her average velocity.
2. A train accelerates uniformly from 12 m s^{-1} to 30 m s^{-1} while travelling 630 m. Calculate its acceleration and the time taken.
3. A stone is dropped from rest from a bridge and hits the water 2.2 s later. Air resistance is negligible. Calculate the height of the bridge and the speed of the stone as it hits the water.
4. A dart is thrown horizontally at 12 m s^{-1} at the centre of a dartboard 2.4 m away. How far below the centre does it land?
5. In a free-fall experiment a ball falls 0.800 m from rest in 0.404 s. The percentage uncertainty in the height is 0.5% and in the time is 1%. Calculate g and its absolute uncertainty.

Answers

1. Average speed $= \frac{400}{80} = 5.0$ m s^{-1}. She finishes where she started, so the displacement is zero and the average velocity is 0.
2. $v^2 = u^2 + 2as$: $30^2 = 12^2 + 2a(630)$, so $a = \frac{900-144}{1260} = 0.60$ m s^{-2}. Then $t = \frac{v-u}{a} = \frac{18}{0.60} = 30$ s. (Check: $\frac{1}{2}(12+30) \times 30 = 630$ m.)
3. $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.81 \times 2.2^2 = 23.7$ m. $v = gt = 9.81 \times 2.2 = 21.6$ m s^{-1}.
4. Time to reach the board $= \frac{2.4}{12} = 0.20$ s. Vertical drop $= \frac{1}{2} \times 9.81 \times 0.20^2 = 0.196$ m, about 0.20 m below the centre.
5. $g = \frac{2h}{t^2} = \frac{2\times0.800}{0.404^2} = 9.80$ m s^{-2}. Percentage uncertainty $= 0.5 + 2 \times 1 = 2.5\%$, so the absolute uncertainty is $0.025 \times 9.80 = 0.245$, and $g = (9.8 \pm 0.2)$ m s^{-2}.

Past questions to try

- **9702/22/F/M/21 Q1**: acceleration and distance from a velocity–time graph.
- **9702/22/M/J/22 Q3**: a ball thrown up from a wall, then a velocity–time sketch and what its gradient means.
- **9702/21/M/J/24 Q2**: a projectile that just clears a wall.
- **9702/23/M/J/23 Q1**: timing a stone falling down a well, and why the measured time differs.