Cambridge AS & A Level · Physics 9702 · Notes

Medical physics A2

Read online at **pastopic.com/cambridge/physics-9702/notes/medical-physics**

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Read first: **Quantum physics**, **Nuclear physics**

What you need to know

By the end of this topic you should be able to:

Ultrasound

1. **Understand the piezoelectric effect**: a p.d. across a piezoelectric crystal changes its shape, and changing its shape makes it generate an e.m.f.
2. **Understand how a piezoelectric transducer generates and detects ultrasound.**
3. **Understand how ultrasound pulses reflected at boundaries between tissues** give diagnostic information about structures inside the body.
4. **Define specific acoustic impedance** $Z = \rho c$, where c is the speed of sound in the medium.
5. **Use the intensity reflection coefficient** $\dfrac{I_R}{I_0} = \dfrac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$ for a boundary between two media.
6. **Recall and use** $I = I_0 e^{-\mu x}$ for the attenuation of ultrasound in matter.

X-rays

7. **Explain that X-rays are produced when electrons bombard a metal target**, and calculate the minimum X-ray wavelength from the accelerating p.d.
8. **Understand how X-rays are used to image structures inside the body**, including what **contrast** means in an X-ray image.
9. **Recall and use** $I = I_0 e^{-\mu x}$ for the attenuation of X-rays in matter.
10. **Understand how computed tomography (CT) scanning builds a 3D image**: X-ray images of one section taken from many angles are combined into a 2D image of that section, and this is repeated along an axis so the 2D images of many sections combine into a 3D image.

PET scanning

11. **Understand that a tracer** is a substance containing radioactive nuclei that is put into the body and absorbed by the tissue being studied.
12. **Recall that PET (positron emission tomography) uses a tracer that decays by β^+ decay.**
13. **Understand annihilation**: a particle meets its antiparticle, and mass–energy and momentum are conserved.

14. **Explain that in PET the positrons from the tracer annihilate with electrons in the tissue**, making two gamma-ray photons that travel in opposite directions.
15. **Calculate the energy of the gamma-ray photons** from an electron–positron annihilation.
16. **Understand that the photons leave the body and are detected**, and that processing their arrival times builds an image of the tracer concentration in the tissue.

Photon energy $E = hf = \dfrac{hc}{\lambda}$ and the electronvolt come from Quantum physics; $E = mc^2$, β^+ decay and activity from Nuclear physics.

Understand it

Doctors want to see inside the body without cutting it open. This topic covers three ways to do it: **ultrasound** (sound echoes), **X-rays** (shadows) and **PET** (radiation sent out from inside).

Piezoelectric crystals

A **piezoelectric** crystal, such as quartz, links shape and voltage both ways:

- put a **p.d. across it** and it **changes shape** (it gets slightly thicker or thinner, depending on the p.d.'s direction);
- **squash or stretch it** and it **generates an e.m.f.** across itself.

Generating ultrasound. Ultrasound is sound above the range of human hearing (above about 20 kHz); medical scans use a few MHz. An **alternating p.d.** across the crystal makes it change shape back and forth: it **vibrates**, at the frequency of the p.d. When that frequency equals the crystal's **natural frequency**, the crystal **resonates** and vibrates with a large amplitude. A crystal is cut so its natural frequency is in the ultrasound range, so it sends out an ultrasound wave.

Detecting ultrasound. An ultrasound wave arriving at the crystal pushes and pulls on it, so its shape changes back and forth (it vibrates). The changing shape generates an **alternating e.m.f.**, which is amplified and processed.

In a scan the **same crystal does both jobs**: it sends a short **pulse**, then stops and "listens" for the echoes before the next pulse. The whole unit is a **transducer**, because it turns electrical energy into sound energy and back.

Specific acoustic impedance and reflection

How much ultrasound is reflected at a boundary depends on the **specific acoustic impedance** Z of the two media:

$$Z = \rho c$$

the **product of the density** ρ of the medium **and the speed** c of ultrasound in that medium. Its unit is kg m^{-3} × m s^{-1} $=$ **kg m^{-2} s^{-1}**. Water, with $\rho = 1000$ kg m^{-3} and $c = 1500$ m s^{-1}, has $Z = 1.50 \times 10^6$ kg m^{-2} s^{-1}.

At a boundary between media 1 and 2, the fraction of the incident intensity that is **reflected** is the **intensity reflection coefficient**

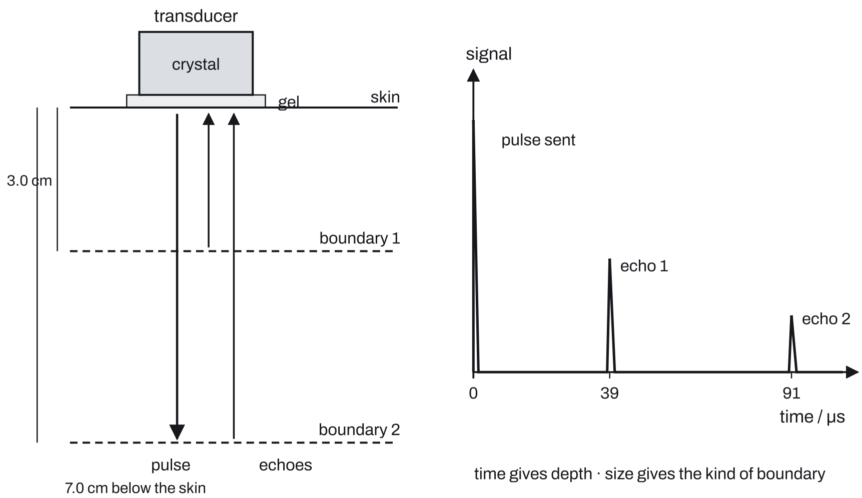
$$\alpha = \frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

and the fraction transmitted across the boundary is $1 - \alpha$.

- Z_1 and Z_2 **nearly equal**: α close to 0; almost all the ultrasound goes **through**.
- Z_1 and Z_2 **very different**: α close to 1; almost all of it is **reflected**.
- Only the **difference** matters, not which side the wave comes from: swapping Z_1 and Z_2 gives the same α.

Why gel is used. Air has a tiny Z (about 4.3×10^2 kg m^{-2} s^{-1}) and skin a large one (about 1.6×10^6). With air between the transducer and the skin, $\alpha = 0.999$: almost all the ultrasound would be reflected at the skin and none would get in. A **gel** with Z close to skin's fills the gap and pushes out the air, so almost all the ultrasound is transmitted into the body. For the same reason ultrasound can't image inside air-filled organs such as the lungs, and it is reflected strongly by bone.

Ultrasound scanning: pulse and echo



time gives depth · size gives the kind of boundary

1. The transducer sends a short pulse of ultrasound into the body.
2. At **each boundary between tissues** with different Z, part of the pulse is **reflected** back and the rest carries on.
3. The echoes return to the transducer, which detects them.

Two things are measured for each echo:

- the **time delay** between sending the pulse and receiving the echo gives the **depth** of the boundary. The pulse goes there and back, so depth $= \frac{ct}{2}$. With $c = 1540$ m s^{-1}, boundaries 3.0 cm and 7.0 cm deep give echoes after 39 μs and 91 μs;
- the **intensity** of the echo gives information about the **nature of the boundary** (which tissues meet there, from how different their Z values are).

Many pulses sent in different directions build a picture of the structures.

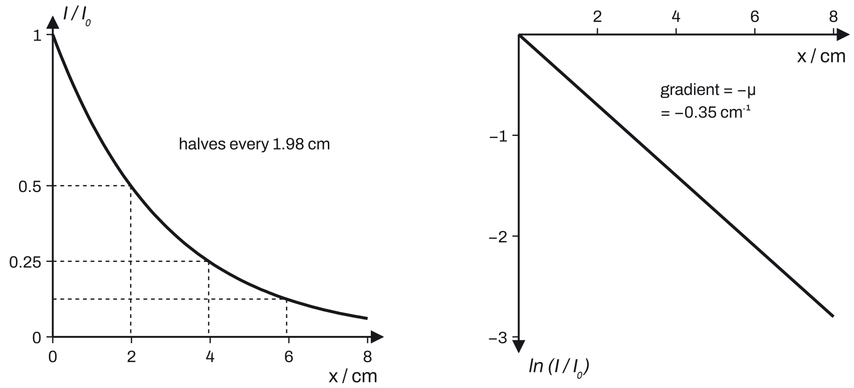
Attenuation

As ultrasound or X-rays travel through matter, energy is absorbed and scattered, so the intensity of a parallel beam falls **exponentially** with the distance x travelled:

$$I = I_0 e^{-\mu x}$$

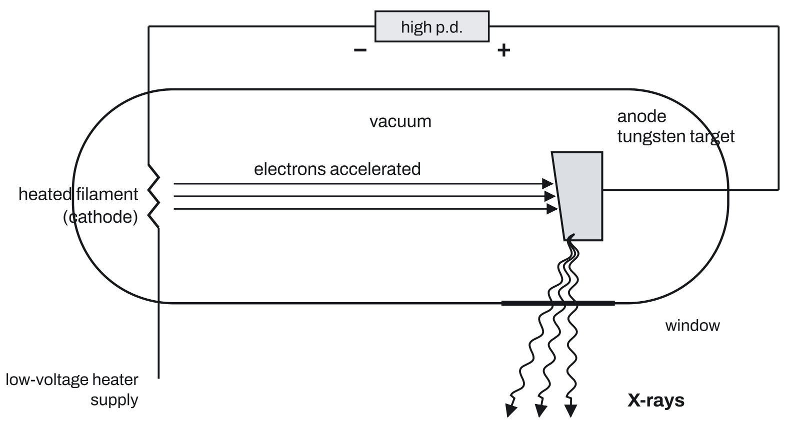
μ is the **linear attenuation coefficient** of that material (for that beam). μx must have no unit, so if x is in cm, μ is in **cm^{-1}**; there is no need to change to metres. Each equal extra thickness cuts the intensity by the **same fraction**.

Taking logs gives $\ln\left(\frac{I}{I_0}\right) = -\mu x$, so a graph of $\ln\left(\frac{I}{I_0}\right)$ against x is a straight line through the origin with **gradient** $-\mu$.



- The fraction **transmitted** is $\frac{I}{I_0} = e^{-\mu x}$; the fraction **absorbed** (attenuated) is $1 - e^{-\mu x}$.
- Through **two layers**, the fractions **multiply**: $I = I_0 e^{-\mu_1 x_1} e^{-\mu_2 x_2} = I_0 e^{-(\mu_1 x_1 + \mu_2 x_2)}$. Add the exponents; never add the exponentials.

Producing X-rays



1. A **heated filament** gives off electrons (thermionic emission).
2. A large p.d. V (tens of kV) **accelerates the electrons** across a vacuum towards a metal **target** (the anode). Each electron arrives with kinetic energy eV.

3. The electrons **bombard the target** and are **decelerated** very rapidly. Energy they lose is emitted as **X-ray photons**.

Why a spread of photon energies? Electrons are slowed by different amounts in different interactions, and many lose their energy in several steps, so the photons have a continuous range of energies. The **most** an electron can give one photon is all its kinetic energy, eV. So the **highest photon energy** and the **shortest wavelength** are:

$$eV = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} \quad \Rightarrow \quad \lambda_{\text{min}} = \frac{hc}{eV}$$

For 30 kV, $\lambda_{\text{min}} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{1.60 \times 10^{-19} \times 30 \times 10^{3}} = 4.14 \times 10^{-11}$ m. A bigger accelerating p.d. gives a shorter λ_{min}.

Only about 1% of the electrons' energy becomes X-rays; the rest heats the target, so it is made of a metal with a very **high melting point**, such as tungsten.

X-ray images and contrast

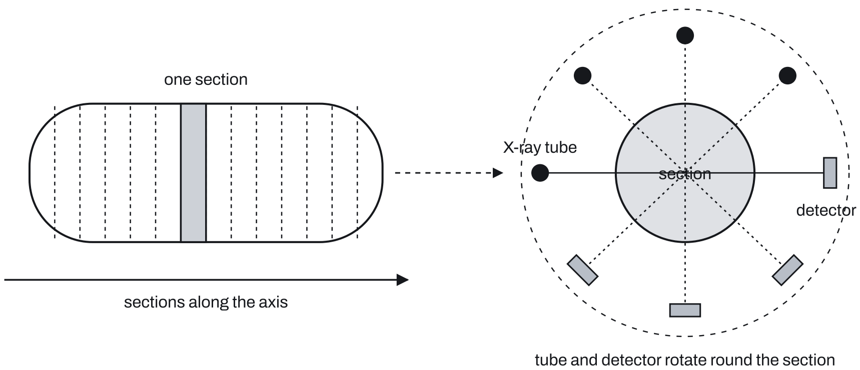
X-rays pass through the body and the **transmitted** X-rays are detected (on a detector or film) on the far side. Where the body absorbs strongly, few X-rays arrive; where it absorbs weakly, many arrive. The image is a shadow of the structures the beam passed through.

Contrast is the **difference in degree of blackening** between different areas of the image. Good contrast needs neighbouring structures whose transmitted intensities are **very different**, which happens when their attenuation coefficients (and thicknesses) are very different:

- **bone** has a much larger μ than soft tissue, so bone next to soft tissue gives **good contrast**;
- two soft tissues (blood and muscle, say) have **similar** μ, so they give **poor contrast**.

To judge contrast from numbers, compare the **transmitted intensities** along the two paths: a large ratio means good contrast; similar values mean poor contrast. (Older papers also asked about **sharpness**: how clearly the **edges** of structures can be made out.)

CT scanning



An ordinary X-ray image is flat: everything along the beam's path is overlaid. **Computed tomography** builds a **3D image**:

1. An X-ray tube and detectors **rotate around** the patient, taking X-ray images of **one thin section** from **many different angles**.
2. A computer **combines** these into a **2D image of that section**.
3. The process is **repeated for many sections** along an axis through the body.
4. The 2D images of all the sections are **combined into a 3D image**, which can be viewed from any angle.

The 3D image shows where things are in depth, and soft tissues can be told apart; the cost is a **much larger radiation dose** than one X-ray image.

PET scanning

The tracer. A **tracer** is a substance containing **radioactive nuclei** that is **introduced into the body** (usually injected) and is **absorbed by the tissue being studied**. PET uses a tracer that decays by β^+ **decay**, for example fluorine-18 built into a glucose-like molecule, which collects where cells use a lot of energy. Its decay gives out a positron and a neutrino:

$$^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + {}^{0}_{+1}\mathrm{e} + \nu$$

The half-life has to suit the scan: long enough for the tracer to be made, delivered and absorbed before too much has decayed, and short enough that the patient isn't left radioactive for long.

Annihilation. The positron is the **antiparticle** of the electron. Almost at once (within a millimetre or so) it meets an electron in the tissue and they **annihilate**: a particle and its antiparticle meet and **all of their mass** becomes energy, as **two gamma-ray photons**. (Many reactions turn some mass into energy; in annihilation the whole mass goes.)

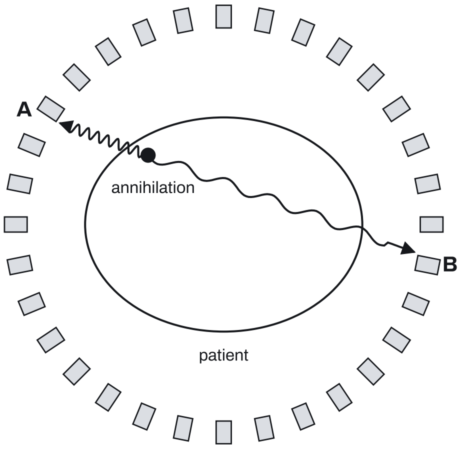
Two conservation laws fix the photons:

- **Momentum**: the electron and positron have almost no momentum, so the photons' total momentum is zero. A single photon can't do that, so there are **two**, with **equal energies**, travelling in **opposite directions**.
- **Mass–energy**: the energy of the two photons is the energy of the two masses, $2m_ec^2$ (the particles' kinetic energy is tiny by comparison). Each photon has

$$E = m_ec^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.20 \times 10^{-14}\ \mathrm{J} = 0.51\ \mathrm{MeV}$$

and wavelength $\lambda = \dfrac{hc}{E} = 2.43 \times 10^{-12}$ m.

Making the image.



1. Gamma photons pass out of the body and are detected by a **ring of detectors** around the patient.
2. Two photons arriving at almost the same time come from one annihilation, which lies on the **line joining the two detectors**.
3. The **difference in arrival times** shows where on the line it happened: the photon to the nearer detector arrives first. If one arrives Δt sooner, the event is $\frac{c\,\Delta t}{2}$ from the midpoint of the line, towards that detector.
4. A computer processes millions of such events to build an **image of the tracer concentration** in the tissue.

Each decay gives one positron and so **two** photons: the rate of photon production is **twice the activity** of the tracer.

Key facts and formulas

"Given" means it is on the data sheet printed in Paper 4. "Learn it" means you must remember it.

Formula	Status
Specific acoustic impedance $Z = \rho c$; unit kg m^{-2} s^{-1}	Learn it
Intensity reflection coefficient $\frac{I_R}{I_0} = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$	Given
Fraction transmitted at a boundary $= 1 - \frac{I_R}{I_0}$	Learn it
Echo depth $= \frac{ct}{2}$ (t = time from pulse to echo)	Learn it
Attenuation $I = I_0 e^{-\mu x}$; $\ln\left(\frac{I}{I_0}\right) = -\mu x$	Learn it
Two layers: $I = I_0 e^{-(\mu_1 x_1 + \mu_2 x_2)}$	Learn it

Formula	Status
Minimum X-ray wavelength $\lambda_{\min} = \dfrac{hc}{eV}$ (from $eV = \dfrac{hc}{\lambda}$)	Learn it
Photon energy $E = hf = \dfrac{hc}{\lambda}$; 1 MeV $= 1.60 \times 10^{-13}$ J	Learn it
Energy of each annihilation photon $E = m_e c^2$ (two photons, total $2m_e c^2$)	Learn it
β^+ decay: nucleus → daughter $+ {}^{0}_{+1}\mathrm{e} + \nu$	Learn it
Radioactive decay $x = x_0 e^{-\lambda t}$, decay constant $\lambda = \dfrac{0.693}{t_{1/2}}$	Given
$h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m s⁻¹, $e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg, $N_A = 6.02 \times 10^{23}$ mol⁻¹	Given

How to do it

In the Paper 4 questions we have tagged for this topic (22 different questions, 2021–2025; 8 more appear on two papers), the types came up this often. 17 of the 22 questions mix two or more types, so the counts add up to more than 22.

Question type	Questions
Attenuation calculations ($I = I_0 e^{-\mu x}$)	10
Producing X-rays and the minimum wavelength	8
Specific acoustic impedance and the reflection coefficient	7
Contrast in X-ray images	6
The piezoelectric transducer	6
Using ultrasound echoes; gel	5
PET scanning	4
CT scanning	4

1. Attenuation calculations (10 questions)

1. Write $I = I_0 e^{-\mu x}$, or $\dfrac{I}{I_0} = e^{-\mu x}$.
2. **Find** μ: $\mu = -\dfrac{\ln(I/I_0)}{x}$. Keep x in cm and give μ in cm⁻¹ (with the unit if asked).
3. **Find the fraction left**: $e^{-\mu x}$. **Fraction absorbed**: $1 - e^{-\mu x}$. Read which one is asked for.
4. **Find a thickness**: $x = -\dfrac{\ln(I/I_0)}{\mu}$. To **absorb 80%**, the fraction left is 0.20, not 0.80.

Variations you will meet:

- **Two materials in the beam** (bone inside soft tissue): add μx for each layer along that path. If bone of thickness b sits inside tissue of total width w, the tissue on that path is $w - b$, not w.
- **Equal thicknesses** x of two materials: $e^{-(\mu_1+\mu_2)x}$, so $x = -\dfrac{\ln(I/I_0)}{\mu_1 + \mu_2}$.
- **An unknown** μ for a second layer: divide out the known layer's factor first, then solve for the rest.
- **A graph of** $\ln(I/I_0)$ **against** x: $\mu = -$gradient. A line through $(0, 0)$ and $(8.0 \text{ cm}, -2.4)$ gives $\mu = 0.30$ cm^{-1}.
- **Ultrasound** is attenuated by the same law: an echo from depth d has travelled $2d$ through the tissue.

2. Producing X-rays and the minimum wavelength (8 questions)

1. **The particles** are **electrons**, from a heated filament, accelerated by a large p.d. The filament (cathode) is **negative** and the target (anode) **positive**.
2. **How X-rays are made**: the electrons hit the metal target and **decelerate**; the **kinetic energy** they lose is emitted as **X-ray photons**.
3. **Minimum wavelength**: $eV = \dfrac{hc}{\lambda_{\text{min}}}$. The electron's energy is eV joules, not V: multiply the p.d. by 1.60×10^{-19}. In MeV, the maximum photon energy is just the p.d. in MV (32 kV gives 0.032 MeV).

Variations you will meet:

- **Why a continuous spectrum**: electrons have a **range of decelerations**, and each photon's energy depends on the deceleration that made it.
- **Why tungsten**: most of the electrons' kinetic energy becomes **thermal energy** in the target, and tungsten's **high melting point** stops it melting.
- **Temperature rise of the target**: energy per second $= IV$, so energy $= IVt$; take the fraction that becomes heat, then $\Delta T = \dfrac{E}{mc}$. For 0.20 A at 50 kV for 0.050 s, 99% to heat and a 20 g target with $c = 140$ J kg^{-1} K^{-1}: $E = 495$ J and $\Delta T = 177$ K. Use the 99% (or 95%), not the X-ray fraction; convert g and ms.
- **Maximum photon momentum**: $p = \dfrac{E}{c}$ (from Quantum physics).
- **Intensity and hardness** (older papers): the **filament current** (heater p.d.) controls the number of electrons and so the **intensity**; the **accelerating p.d.** controls the photon energies, so the **hardness** (penetrating power).

3. Specific acoustic impedance and the reflection coefficient (7 questions)

1. **Definition**: the **product of the density** of the medium **and the speed of ultrasound in the medium**.
2. **Calculate** Z for each medium: $Z = \rho c$, unit kg m^{-2} s^{-1}. To find a missing ρ or c, rearrange: $\rho = \dfrac{Z}{c}$.
3. **Reflection coefficient**: $\alpha = \dfrac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$. Check it is between 0 and 1.
4. **Transmitted**: $1 - \alpha$ (as a percentage if asked).

Variations you will meet:

- **"Describe how α depends on Z_1 and Z_2"**: it depends on the **difference**: nearly equal gives $\alpha \approx 0$; very different gives $\alpha \approx 1$. Say what happens to the **value** of α, not "more reflects".
- **"Explain, without calculation"**: compare the two Z values and state the consequence: similar Z, so **almost all** transmitted and almost none reflected.
- **Why a real sample transmits less than $1 - \alpha$**: there is **more than one boundary** (more reflections), and ultrasound is **attenuated** in the tissue.
- **The unit of Z** from a table: kg m^{-3} × m s^{-1} = kg m^{-2} s^{-1}.

4. Contrast in X-ray images (6 questions)

1. **Definition**: contrast is the **difference in degree of blackening** between areas of the image.
2. **Calculate** the transmitted intensity along each path (type 1).
3. **Compare and conclude**: quote your two values, say whether they are very different (good contrast) or similar (poor contrast), consistent with your numbers.

Variations you will meet:

- **Without numbers**: compare **attenuation coefficients**: bone and muscle are very different, so very different amounts are absorbed and transmitted: good contrast; blood and muscle are similar: poor contrast. Compare the two values; don't just say one is large.
- **Why X-rays give good images of bone**: bone and soft tissue have very different μ, so very different intensities are transmitted.
- **Sharpness** (older papers): how easily the **edges** of structures can be seen.

5. The piezoelectric transducer (6 questions)

- **Generating**: an **alternating p.d.** makes the crystal **vibrate**; at the crystal's **natural frequency** it **resonates**; that frequency is in the **ultrasound** range.
- **Detecting**: the returning ultrasound makes the crystal **vibrate** (change shape), which **generates an e.m.f.** that is detected.
- **Its function**: it **both** generates the pulse and detects the reflected ultrasound. Answer the part asked (generation or detection).

Variations you will meet:

- **"Method of production"** in a comparison: ultrasound from a **vibrating piezoelectric (quartz) crystal**; X-rays from **electrons hitting a metal target**. For imaging, the ultrasound detected is **reflected**; the X-rays detected are **transmitted**.
- **The alternating p.d. across the probe**: peak $V_0 = \sqrt{2}\, V_{\mathrm{r.m.s.}}$ and $\omega = 2\pi f$ (from Alternating currents). For 50 V r.m.s. at 3.5 MHz: $V_0 = 70.7$ V and $\omega = 2.20 \times 10^7$ rad s^{-1}.

6. Using ultrasound echoes; gel (5 questions)

For "outline" or "describe how ultrasound gives diagnostic information", give these points:

1. A **pulse** of ultrasound is sent into the body by a piezoelectric transducer.
2. It is **reflected at boundaries between tissues**.
3. The reflected pulses are **detected** by the same transducer.
4. The **time delay** gives the **depth** of the boundary.
5. The **intensity** of the echo gives the **nature of the boundary** (the tissues on each side).
6. **Gel** on the skin removes air, so reflection at the skin is minimised.

Variations you will meet:

- **Why gel**: calculate or compare α for air–skin (almost 1: almost all reflected) and gel–skin (almost 0: almost all transmitted).
- **Why not the lungs**: air and soft tissue have very different Z, so $\alpha \approx 1$ and **almost no** ultrasound enters the air-filled cavity. Say "almost all" or "almost none", not "most".
- **Depth from a time**: depth $= \dfrac{ct}{2}$.

7. PET scanning (4 questions)

1. **Tracer**: a substance containing **radioactive nuclei**, **introduced into the body** and **absorbed by the tissue** being studied. For PET it is a β^+ **emitter**.
2. **Where the positrons come from**: the tracer nuclei decay by β^+ decay, emitting positrons.
3. **Where the gamma photons come from**: a positron meets an **electron** and they **annihilate**; the **whole mass** of both particles becomes the energy of **two gamma photons**, travelling in **opposite directions**.
4. **Photon energy**: each photon has $E = m_e c^2 = 8.20 \times 10^{-14}$ J; total $2m_e c^2 = 1.64 \times 10^{-13}$ J. **Wavelength**: $\lambda = \dfrac{hc}{E} = \dfrac{h}{m_e c} = 2.43 \times 10^{-12}$ m.
5. **The image**: the photons are **detected outside the body**; their **arrival times** are processed to find where each annihilation happened, building an **image of the tracer concentration**.

Variations you will meet:

- **"Show that $\lambda \approx 2.4$ pm; explain your reasoning"**: say that the masses of the electron and positron become **two** photons, so each has $m_e c^2$; then $E = \dfrac{hc}{\lambda}$. Using the total $2m_e c^2$ for one photon is the classic error.
- **"Explain annihilation"**: a **particle** meets its **antiparticle** (an electron and a positron; not "two antiparticles"); all their mass becomes energy.
- **The decay equation**: fluorine-18 → oxygen-18: ${}^{18}_{9}\mathrm{F} \to {}^{18}_{8}\mathrm{O} + {}^{0}_{+1}\mathrm{e} + \nu$; the other particle is an (electron) **neutrino**.
- **Rate of photon production**: two photons per decay, so rate $= 2A = 2\lambda N$. For amount n (in mol) and half-life T: $R_0 = \dfrac{2 \ln 2 \, n N_A}{T}$, and R then falls exponentially with the same half-life.

- **Detected particles**: the particles that leave the body and are detected are **gamma photons**, not positrons.
- **Choosing the half-life**: long enough to make, deliver and absorb the tracer; short enough to keep the patient's dose low. Give both halves.

8. CT scanning (4 questions)

1. **Purpose**: to produce a **3D image** of an internal structure (an image, not a model).
2. **Principles**: X-rays; scan in thin **sections**; each section from **many angles**; images combined into a **2D image of the section**; **repeated for many sections**; 2D images **combined into a 3D image**.

Variations you will meet:

- **"Why images of multiple sections?"**: so they can be **combined** into a 3D image.
- **Advantage and disadvantage** against a single X-ray image: gives a **3D image** (depth, soft tissue); but a **much greater radiation dose** (and it takes longer).

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

An ultrasound transducer is placed on the skin above a layer of fat, beneath which is muscle.

Tissue	Density / kg m^{-3}	Speed of ultrasound / m s^{-1}
fat	950	1450
muscle	1080	1590

(a) Define specific acoustic impedance. **[2]**

(b) Calculate the intensity reflection coefficient at the fat–muscle boundary. **[3]**

(c) The echo from the fat–muscle boundary is received 65 μs after the pulse is sent. Calculate the depth of the boundary below the skin. **[2]**

(d) The linear attenuation coefficient of ultrasound in fat is 0.12 cm^{-1}. Ignoring reflection at the skin, calculate the fraction of the original intensity that returns to the skin as this echo. **[3]**

Solution

(a) The product of the density of a medium **M1** and the speed of ultrasound in that medium. **A1**

(b) $Z_{\text{fat}} = 950 \times 1450 = 1.38 \times 10^6$ kg m^{-2} s^{-1} and $Z_{\text{muscle}} = 1080 \times 1590 = 1.72 \times 10^6$ kg m^{-2} s^{-1}. **C1** $\alpha = \dfrac{(1.72 - 1.38)^2}{(1.72 + 1.38)^2}$ **C1** $= 0.0120$. **A1**

(c) The pulse travels to the boundary and back: depth $= \dfrac{ct}{2} = \dfrac{1450 \times 65 \times 10^{-6}}{2}$ **C1** $= 0.047$ m (4.7 cm). **A1**

(d) The path through fat is $2 \times 4.7 = 9.4$ cm, so attenuation leaves $e^{-0.12 \times 9.4} = 0.323$. **C1** Only 0.0120 of the intensity is reflected at the boundary. **C1** Fraction returning $= 0.0120 \times 0.323 = 3.9 \times 10^{-3}$. **A1**

Example 2

An X-ray tube has an accelerating potential difference of 60 kV.

(a) Explain how X-rays are produced in the tube, and why they have a continuous range of wavelengths. **[3]**

(b) Calculate the minimum wavelength of the X-rays. **[2]**

(c) A parallel beam of the X-rays, of intensity I_0, passes through a model limb 8.0 cm thick. Along one path it passes through soft tissue only; along another it passes through 2.0 cm of bone and the rest soft tissue. The linear attenuation coefficients are 0.25 cm^{-1} for soft tissue and 2.5 cm^{-1} for bone. Calculate, in terms of I_0, the transmitted intensity along each path. **[4]**

(d) Use your answers to (c) to suggest whether the image would have good contrast. **[1]**

Solution

(a) Electrons are accelerated by the p.d. and hit the metal target, where they decelerate. **B1** The kinetic energy they lose is emitted as X-ray photons. **B1** Electrons have a range of decelerations, so the photons have a range of energies and wavelengths. **B1**

(b) $eV = \dfrac{hc}{\lambda_{\min}}$, so $\lambda_{\min} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{1.60 \times 10^{-19} \times 60 \times 10^3}$ **C1** $= 2.07 \times 10^{-11}$ m. **A1**

(c) Soft tissue only: $I = I_0 e^{-0.25 \times 8.0}$ **C1** $= 0.135 I_0$ (about $0.14 I_0$). **A1** Through bone: the soft tissue on this path is $8.0 - 2.0 = 6.0$ cm, so $I = I_0 e^{-(0.25 \times 6.0 + 2.5 \times 2.0)} = I_0 e^{-6.5}$ **C1** $= 1.5 \times 10^{-3} I_0$. **A1**

(d) The intensities differ by a factor of about 90, so the image has good contrast. **B1**

Example 3

A patient is given a tracer containing fluorine-18, which decays by β^+ decay, for a PET scan.

(a) State what is meant by a tracer. **[2]**

(b) Explain how the decay of fluorine-18 leads to gamma-ray photons leaving the body. **[3]**

(c) Calculate the energy of each gamma-ray photon, in MeV, and its wavelength. **[3]**

(d) Two detectors A and B, on opposite sides of the patient, are 80 cm apart. They detect the photons from one annihilation; the photon reaches A 0.60 ns before the photon reaches B. Determine how far from A, along the line AB, the annihilation happened. **[2]**

(e) At one moment the tracer's activity is 3.0×10^8 Bq. State the rate at which gamma-ray photons are produced. **[1]**

Solution

(a) A substance containing radioactive nuclei, introduced into the body, **B1** that is absorbed by the tissue being studied. **B1**

(b) Each decay emits a positron, which meets an electron in the tissue. **B1** They annihilate: the whole mass of the electron and positron becomes energy **B1** as two gamma-ray photons travelling in opposite directions, which pass out of the body. **B1**

(c) Each photon has $E = m_e c^2 = 9.11 \times 10^{-31} \times (3.00 \times 10^8)^2 = 8.20 \times 10^{-14}$ J **C1** $= \dfrac{8.20 \times 10^{-14}}{1.60 \times 10^{-13}} =$ 0.512 MeV. **A1** $\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{8.20 \times 10^{-14}} = 2.43 \times 10^{-12}$ m. **A1**

(d) The extra distance to B is $c\,\Delta t = 3.00 \times 10^8 \times 0.60 \times 10^{-9} = 0.18$ m, so the event is 0.090 m from the midpoint, towards A. **C1** Distance from A $= 0.40 - 0.090 = 0.31$ m. **A1**

(e) Two photons per decay: 6.0×10^8 photons per second. **B1**

Common mistakes

- **Half an answer on the transducer.** June 2021 and November 2021: candidates described only how the crystal makes ultrasound, not that it also **detects** the echoes. March 2023, asked about detection, many wrote about production instead.
- **Wrong speed in Z.** November 2021: c is the speed of **ultrasound in the medium**, not the speed of light.
- **"Most" instead of "almost all".** March 2023 and June 2023: with $\alpha \approx 1$ **almost all** the ultrasound is reflected (or, with gel, almost all transmitted). "Most" could mean just over half and loses the mark.
- **Vague about α.** November 2021: say how the **difference** between the Z values affects the **value** of the coefficient, not "amounts" reflected.
- **Two layers of absorber.** June 2021 and June 2022: the exponentials must be **multiplied** (exponents added), and the soft tissue on the bone path is the total width **minus** the bone.
- **Transmitted or absorbed?** November 2021 and March 2022: to absorb 80%, $I/I_0 = 0.20$. 12% emerging means $I/I_0 = 0.12$, not 0.88.
- **Units of μ.** November 2021: keep x in cm and give μ in cm^{-1}; converting to metres just invites mistakes. June 2022: evaluate $e^{\cdots}$ to a number.

- **Contrast without comparison.** March 2022, June 2021 and June 2022: compare the two **values** (intensities or μ) and draw a conclusion consistent with your own numbers.
- **The p.d. is not the energy.** June 2023: $\lambda_{\min} = \dfrac{hc}{eV}$, multiplying V by e. Some also said the charged particle was a positron: in an X-ray tube it is an **electron**.
- **Target heating.** March 2022: use the fraction that becomes heat (95% there, not 5%), and convert g and ms.
- **Annihilation described loosely.** March 2022, June 2022 and November 2022: a **particle and its antiparticle**, and the **whole** mass becomes the energy of the gamma photons. "Mass is converted to energy" alone happens in every nuclear reaction.
- **The factor of two.** March 2022 and November 2022: there are **two** photons, each with $m_e c^2$. Don't use de Broglie's $\lambda = \frac{h}{p}$ for an electron as the starting point.
- **Mixing up the scans.** March 2022: PET answers drifted into CT. CT answers (June 2021) mixed up "image" and "section".

How to get the marks

- **Definitions**: tracer (radioactive, introduced into the body, absorbed by the tissue studied); specific acoustic impedance (density × speed of ultrasound in the medium); contrast (difference in degree of blackening); annihilation (particle meets its antiparticle, all mass becomes energy).
- **"Explain" and "outline" answers**: each bold point in types 5–8 is usually one mark. Give them separately, in order.
- **Calculations**: the **C1** is for the right equation with numbers in; the **A1** is the answer. Keep 2–3 significant figures (3 if the data are to 3).
- **"Show that"**: state the physics (two photons, each $m_e c^2$), show every substitution, and give one more significant figure than printed (2.43 pm, not 2.4).
- **"Give a unit"**: μ in cm^{-1} (or m^{-1}); Z in kg m^{-2} s^{-1}; α and fractions have none.
- **"Use your answers"** (contrast): quote your numbers, compare, conclude.
- **"Suggest two reasons"**: give two different physics reasons (more boundaries; attenuation), not one idea twice.
- **"Without calculation"**: compare the given values in words and state what follows.

Quick check

1. Bone has density 1900 kg m^{-3} and ultrasound travels through it at 4080 m s^{-1}. Calculate its specific acoustic impedance.
2. Two tissues have $Z = 1.50 \times 10^6$ and 1.70×10^6 kg m^{-2} s^{-1}. Calculate the intensity reflection coefficient, and the percentage of the intensity transmitted.
3. Calculate the minimum wavelength of X-rays from a tube with an accelerating p.d. of 25 kV.

4. X-rays pass through 2.5 cm of a material with $\mu = 0.60$ cm^{-1}. What percentage of the intensity is absorbed?
5. Explain why an electron–positron annihilation produces two photons travelling in opposite directions, rather than one.

Answers

1. $Z = \rho c = 1900 \times 4080 = 7.75 \times 10^6$ kg m^{-2} s^{-1}.
2. $\alpha = \dfrac{(1.70 - 1.50)^2}{(1.70 + 1.50)^2} = \dfrac{0.04}{10.24} = 3.9 \times 10^{-3}$; transmitted $= 1 - 0.0039$, so 99.6%.
3. $\lambda_{\min} = \dfrac{hc}{eV} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{1.60 \times 10^{-19} \times 25 \times 10^3} = 4.97 \times 10^{-11}$ m.
4. Fraction transmitted $= e^{-0.60 \times 2.5} = e^{-1.5} = 0.22$, so 22% gets through and 78% is absorbed.
5. The electron and positron have (almost) zero total momentum, and momentum is conserved, so the photons' total momentum must be zero. One photon always carries momentum, so there must be two, with equal energy, travelling in opposite directions.

Past questions to try

- **9702/43/M/J/23 Q8**: acoustic impedances from a table, reflection at air–tissue and gel–tissue boundaries, and why gel is used.
- **9702/43/M/J/22 Q9**: X-ray production, CT sections, and intensity through bone inside soft tissue.
- **9702/42/F/M/22 Q11**: tracer, annihilation, photon energy and wavelength, and forming a PET image.
- **9702/42/M/J/21 Q11**: the purpose and principles of CT scanning.