Cambridge AS & A Level · Physics 9702 · Notes

Motion in a circle A2

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Read first: **Dynamics**, **Work, energy and power**

What you need to know

By the end of this topic you should be able to:

1. **Define the radian** and measure angles turned (angular displacement) in radians.
2. **Understand angular speed**: the angle turned per second, in rad s^{-1}.
3. **Recall and use** $\omega = \frac{2\pi}{T}$ **and** $v = r\omega$, linking angular speed, period and speed.
4. **Understand that a force of constant size that is always at right angles to the velocity** makes an object accelerate towards the centre of a circle (centripetal acceleration).
5. **Understand that this centripetal acceleration produces circular motion** at a constant angular speed.
6. **Recall and use** $a = r\omega^2$ **and** $a = \frac{v^2}{r}$**.**
7. **Recall and use** $F = mr\omega^2$ **and** $F = \frac{mv^2}{r}$ for the resultant force needed.

The force itself always comes from something you already know: a tension, friction, a contact force, gravity, or (later in the course) an electric or magnetic force. Newton's law of gravitation, Coulomb's law and the magnetic force $F = BQv$ belong to their own topics; here you only need the step "this force **is** the centripetal force".

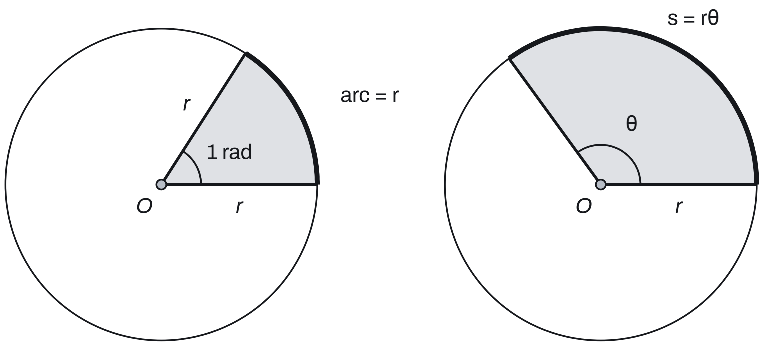
Understand it

Radians and angular displacement

An angle measured in **radians** compares the arc it cuts off with the radius:

$$\theta = \frac{s}{r}$$

where s is the arc length and r the radius. **One radian** is the angle at the centre of a circle when the arc length equals the radius.



A full turn has arc length $2\pi r$, so it is $\dfrac{2\pi r}{r} = 2\pi$ rad. That gives the conversions:

- $360^\circ = 2\pi$ rad, $180^\circ = \pi$ rad, and 1 rad $= \dfrac{180^\circ}{\pi} = 57.3^\circ$.
- An arc of 0.30 m on a circle of radius 0.20 m means the radius has turned through $\dfrac{0.30}{0.20} = 1.5$ rad (85.9°).

How far an object has turned round the circle is its **angular displacement** θ, and it is always given in radians in this topic. The radian is a ratio of two lengths, so it has no real unit; "rad" is written to remind you what the number means.

Angular speed

The **angular speed** ω (omega) is the angle turned per unit time:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

in rad s^{-1}. In one period T the object turns 2π rad, so

$$\omega = \frac{2\pi}{T} = 2\pi f$$

where f is the number of turns per second. A ceiling fan making 2.6 turns per second has $\omega = 2\pi \times 2.6 = 16.3$ rad s^{-1} and period $T = \frac{1}{2.6} = 0.385$ s. From revolutions per minute: multiply by 2π, then divide by 60.

Every point on a rigid spinning object (a wheel, a disc, a clock hand) turns through the same angle in the same time, so **every point has the same** ω, wherever it is.

Speed and angular speed: $v = r\omega$

In time Δt a point at radius r turns through $\Delta\theta$ and moves along an arc $\Delta s = r\Delta\theta$. Divide by Δt:

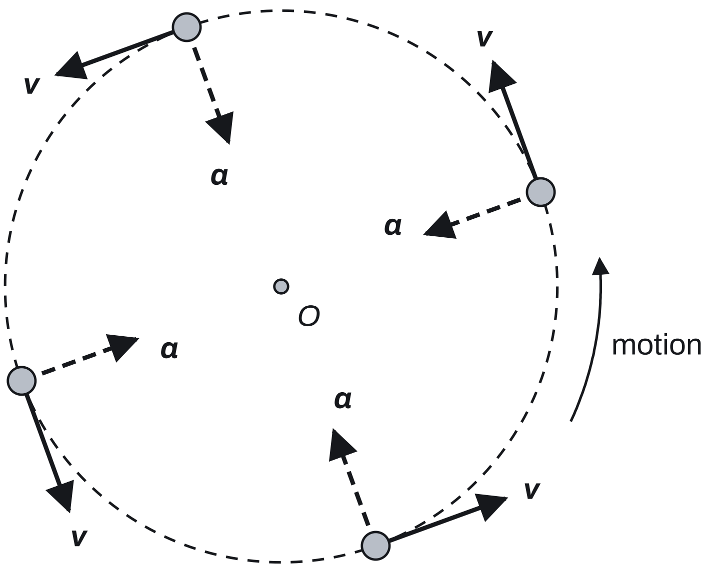
$$v = r\omega$$

Here v is the (linear) speed along the circle, in m s^{-1}. On the fan above, a blade tip 0.55 m from the axis moves at $v = 0.55 \times 16.34 = 8.98$ m s^{-1}, while a point halfway along the blade has the same ω but half the speed. Keep the two ideas apart: **angular speed** is in rad s^{-1} and is the same across the whole fan; **speed** is in m s^{-1} and grows with r.

The same rule, arc $= r\times$ angle, works for belts and chains. A chain that runs over two cogs without slipping moves the same distance at the rim of each cog, so $r_1\theta_1 = r_2\theta_2$ and $r_1\omega_1 = r_2\omega_2$: the smaller cog turns through the bigger angle.

Why moving in a circle is accelerating

An object going round a circle at constant speed is **accelerating**, because its velocity keeps changing direction. The velocity always points along the tangent, at right angles to the radius. Over a short time the velocity arrow swings round, and the change in velocity points **towards the centre**. So the acceleration points towards the centre; it is called **centripetal acceleration**.



Its size is

$$a = \frac{v^2}{r} = r\omega^2$$

(the two forms agree because $v = r\omega$). On the fan blade tip: $a = 0.55 \times 16.34^2 = 147$ m s^{-2}, about fifteen times g.

Uniform circular motion in one sentence: the speed is constant and the size of the acceleration is constant, but the velocity changes direction and the acceleration is always at right angles to the velocity (towards the centre). Speed, angular speed, period, kinetic energy and the size of the acceleration stay the same; velocity, momentum, acceleration (as a vector), displacement and the direction of the resultant force all change.

The force that makes it happen

By Newton's second law, an acceleration needs a resultant force in the same direction. So circular motion needs a **resultant force towards the centre**, of size

$$F = \frac{mv^2}{r} = mr\omega^2$$

This is often called the **centripetal force**, but it is **not an extra force**. It is the name for the resultant of the real forces acting, whatever they are:

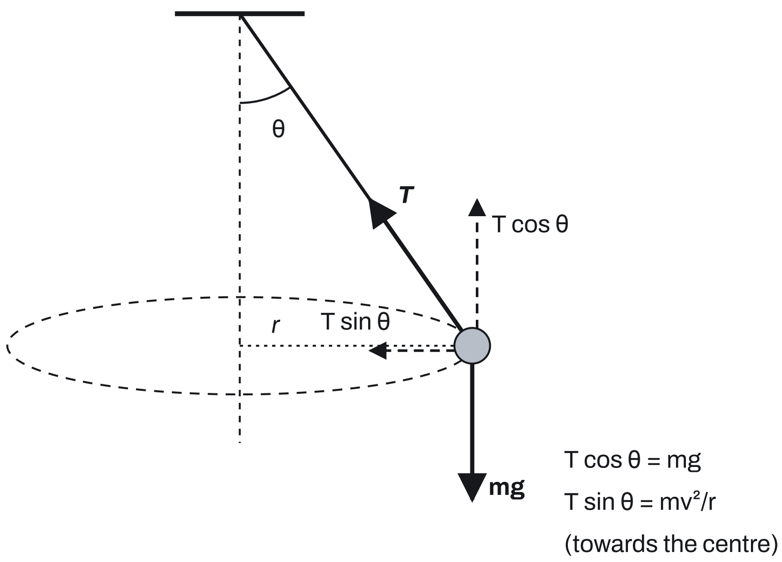
- a stone whirled on a string on a smooth table: the tension;
- a car going round a flat bend: sideways friction from the road;
- the Moon round the Earth: gravity;
- an electron round a nucleus, or a charged particle in a magnetic field: the electric or magnetic force.

Why does such a force give a circle, and not some other path? The force is always **at right angles to the velocity**, so it has no component along the motion. It does no work, so the speed never changes; it only turns the velocity. Because the speed stays the same and the force keeps the **same size**, the object turns through equal angles in equal times: a circle at constant angular speed. (A constant-sized force at right angles to the motion is the key idea. A force that is just "constant", like the weight of a thrown ball, gives a parabola.)

If the force suddenly vanishes (the string snaps), there is no longer any resultant force, so the object carries on in a straight line along the tangent at the point where it was released.

Horizontal circles with a slanting force

Often one force is tilted, and only part of it points to the centre. Examples are a mass swinging round on a string at an angle (a **conical pendulum**), a ball rolling round inside a bowl or cone, or a plane banking to turn.



There are only **two** forces: the weight and the tension (or the normal contact force, for a bowl or cone). Resolve the slanting one:

- **Vertically**, nothing accelerates: $T\cos\theta = mg$.
- **Horizontally**, the component towards the centre **is** the resultant force: $T\sin\theta = \frac{mv^2}{r}$.

Dividing the second by the first gets rid of T and m:

$$\tan\theta = \frac{v^2}{rg}$$

so the angle depends only on the speed and radius, not on the mass. For a ball inside a cone whose wall makes angle α with the horizontal, the contact force is at right angles to the wall, so it makes angle α with the vertical and the same algebra gives $\frac{v^2}{r} = g\tan\alpha$. The wall's angle is fixed, so the centripetal acceleration is fixed: go faster and the ball must ride up to a **bigger radius**, with $r \propto v^2$ (not $r \propto v$).

Objects on a spinning planet

A person standing on the Equator moves in a circle once per day. Two forces act: the weight mg (down) and the normal contact force N from the ground (up). Their resultant must point to the centre and equal $mr\omega^2$, so N is slightly **less** than mg:

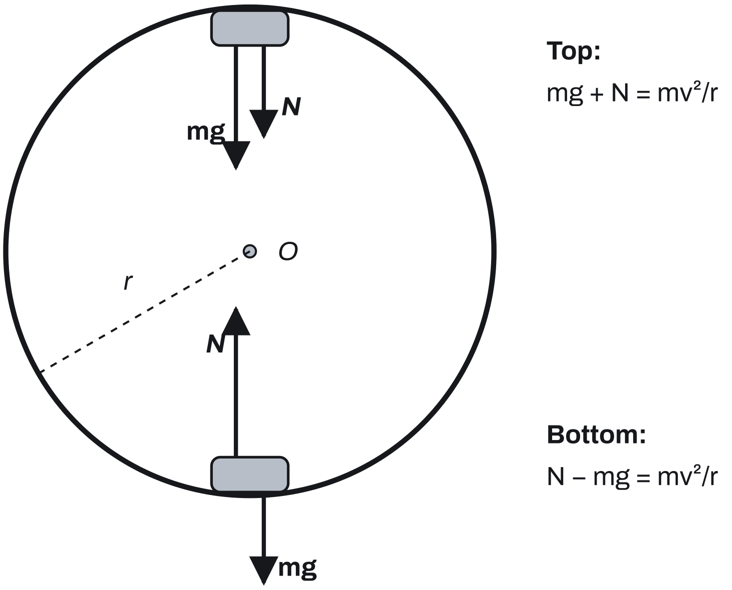
$$mg - N = mr\omega^2$$

For a planet of radius 4.6×10^6 m that spins once every 20 hours, $\omega = \frac{2\pi}{20 \times 3600} = 8.73 \times 10^{-5}$ rad s^{-1}, and at the Equator $a = r\omega^2 = 0.0350$ m s^{-2}. A 55 kg person there (taking $g = 9.81$ m s^{-2} for this planet) needs a resultant force of only 1.93 N, tiny next to their weight of about 540 N.

Away from the Equator the person moves in a **smaller** circle, parallel to the Equator, centred on the axis. At latitude ϕ its radius is $R\cos\phi$ (at 40° on this planet, 3.52×10^6 m), and the resultant force points towards the **axis** (at right angles to it), not towards the centre of the planet. The period is still one rotation of the planet, so ω is the same everywhere on the planet, but $v = (R\cos\phi)\,\omega$ is smaller. The weight still points at the planet's centre, so for the two forces to add up to a resultant pointing at the axis, the contact force from the ground cannot point exactly away from the centre: it points up and very slightly towards the axis.

Vertical circles

In a vertical circle (a loop-the-loop, a bucket swung over your head) gravity speeds the object up on the way down and slows it on the way up, so the speed is **not** constant. At any point, though, the resultant force towards the centre is still $\frac{mv^2}{r}$ for the speed there.



- **At the top**, both the weight and the contact force point down, towards the centre: $mg + N = \frac{mv^2}{r}$.
- **At the bottom**, the contact force points up (towards the centre) and the weight down: $N - mg = \frac{mv^2}{r}$, so the track pushes hardest here.

The car stays in contact at the top only if $N \geqslant 0$, which needs $\frac{v^2}{r} \geqslant g$: the centripetal acceleration at the top must be at least g. If it is less, gravity alone would pull the car into a tighter curve than the track, so it falls away. Find the speed at the top from energy: $\frac{1}{2}mv_{\text{top}}^2 = \frac{1}{2}mv_{\text{bottom}}^2 - mg(2r)$. The mass cancels in every step, so a heavier car with the same starting speed behaves the same.

Orbits and charged particles

When a satellite, moon or planet moves in a circular orbit, **gravity is the only force**, so gravity is the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2, \qquad \omega = \frac{2\pi}{T}$$

The gravitational force is always towards the planet's centre, at right angles to the velocity and (at constant r) of constant size, so the path is a circle. Newton's law of gravitation, and results such as $T^2 \propto r^3$ and geostationary orbits, are in Gravitational fields.

Charged particles work the same way. An electron orbiting a nucleus is held by the electric force; a charged particle moving at right angles to a magnetic field feels a force BQv at right angles to its velocity. In each case, write "this force $= \frac{mv^2}{r}$" and solve. The magnetic force never changes the speed, so the particle moves at constant speed round the circle, and the centripetal acceleration at any point is at right angles to its velocity, towards the centre.

Seen side-on

Watch a point going round a circle of radius r at angular speed ω from the side (or its shadow on a screen). If the angle is $\theta = \omega t$, measured from the line through the centre at right angles to the screen, the shadow's displacement from the centre line is $x = r \sin \omega t$. The shadow oscillates with amplitude r and period $T = \frac{2\pi}{\omega}$, the same period as the circular motion. This link is how simple harmonic motion is described in the Oscillations topic.

Key facts and formulas

"Given" means it is on the data sheet in the exam. "Learn it" means you must remember it. Nothing in this topic is on the data sheet.

Formula	Status
Angle in radians $\theta = \frac{s}{r}$; arc length $s = r\theta$	Learn it
2π rad $= 360^\circ$; 1 rad $= 57.3^\circ$	Learn it
Angular speed $\omega = \frac{\Delta\theta}{\Delta t}$	Learn it
$\omega = \frac{2\pi}{T} = 2\pi f$	Learn it
$v = r\omega$; also $v = \frac{2\pi r}{T}$	Learn it
Centripetal acceleration $a = r\omega^2 = \frac{v^2}{r}$ (also $a = v\omega$)	Learn it
Centripetal (resultant) force $F = mr\omega^2 = \frac{mv^2}{r}$	Learn it
Conical pendulum or cone: $T\cos\theta = mg$, $T\sin\theta = \frac{mv^2}{r}$, so $\tan\theta = \frac{v^2}{rg}$	Learn it
Vertical circle: top $mg + N = \frac{mv^2}{r}$; bottom $N - mg = \frac{mv^2}{r}$	Learn it
Circle at latitude ϕ on a planet of radius R: radius $R\cos\phi$	Learn it
$g = 9.81$ m s^{-2}	Given
$v^2 = u^2 + 2as$ (from Kinematics)	Given

How to do it

In the Paper 4 questions we have tagged for this topic (25 different questions, 2021–2025; 11 more appear on two papers), the types came up this often. 20 of the 25 questions mix two or more types, so the counts add up to more than 25.

Question type	Questions
Describe and explain: definitions, directions, why the path is a circle	14
Radians, angular speed, period and $v = r\omega$	11
Centripetal acceleration and force	10
Orbits: gravity provides the centripetal force	10
Charged particles moving in circles	5
Horizontal circles with a slanting force	3
Comparing points or objects	3
Vertical circles	1
Circular motion seen side-on	1

1. Describe and explain: definitions, directions, why the path is a circle (14 questions)

Learn these, in meaning:

- **Radian**: the angle at the centre of a circle when the arc length equals the radius.
- **Uniform circular motion**: constant speed and constant-sized acceleration; the velocity is along the tangent and the acceleration is at right angles to it, towards the centre.
- **Centripetal acceleration**: the acceleration at right angles to the velocity (towards the centre) that makes an object move in a circle.
- **Why the path is circular**: the force is at right angles to the velocity and has constant size; it provides the centripetal acceleration (and does no work, so the speed is constant).

Variations you will meet:

- **Draw arrows** on a diagram: velocity along the tangent in the direction of travel; acceleration (and resultant force) along the radius towards the centre. For a point on the rim of a disc turning anticlockwise, if the point is at the "3 o'clock" position, v points up the page and a points left.
- **Draw and label the forces** on an object in a horizontal circle: usually just two (weight, and tension or normal contact force). **Never** add a separate "centripetal force" arrow.
- **Explain how the forces give a centripetal acceleration**: the vertical component of the slanting force balances the weight, so the resultant is the horizontal component, which points to the centre.
- **Name quantities that vary** in uniform circular motion: velocity, momentum, displacement, acceleration (direction), resultant force (direction). Speed, angular speed and kinetic energy do not.
- **Why a force is negligible**: compare the centripetal acceleration with g. If $a \ll 9.81$ m s^{-2}, the resultant force is tiny compared with the weight, so the supporting force barely changes during the rotation.
- **A charged particle's path**: sketch an arc of a circle that curves towards the side the force pushes; a faster particle (or smaller charge-to-mass ratio) curves more gently (bigger radius).

2. Radians, angular speed, period and $v = r\omega$ (11 questions)

1. **Get** ω from what you are given: $\omega = \dfrac{2\pi}{T}$ from a period, $\omega = \dfrac{v}{r}$ from a speed, or angle ÷ time.
2. **Convert** revolutions or hours first: 1 rev $= 2\pi$ rad; 24 h $= 86\,400$ s; rpm × $\frac{2\pi}{60}$ = rad s^{-1}.
3. **Use** $v = r\omega$ or $s = r\theta$ with r in metres, measured from the **axis of rotation**.

For example, a wheel of radius 0.30 m rolling at 9.0 m s^{-1} spins at $\omega = \frac{9.0}{0.30} = 30$ rad s^{-1}, so $T = \frac{2\pi}{30} = 0.209$ s.

Variations you will meet:

- **The radius is not the obvious length.** A lump stuck on a disc a little way in from the rim has $r =$ disc radius minus that distance. A point at latitude ϕ has $r = R\cos\phi$.
- **Angle from arc length**: $\theta = \dfrac{s}{r}$, in rad. **Arc length from angle**: $s = r\theta$. With a clock hand or a slow wheel, you may first need $\theta = \omega t$.
- **Chains, belts and cogs**: the chain moves the same distance (and speed) at the rim of each cog, so $r_1\theta_1 = r_2\theta_2$. Cogs fixed to the same axle share ω.
- **"Explain, without calculation"** a change in a chain drive: go one link at a time. What fixes the first ω (for a bicycle, the road speed fixes the rear wheel's ω)? Then chain speed $=$ (radius of that cog) $\times$ ω, and the other cog's ω $=$ chain speed ÷ its radius. End by saying what happens to the ω asked about.
- **Don't confuse angular speed with speed.** The units tell you: rad s^{-1} for ω, m s^{-1} for v.

3. Centripetal acceleration and force (10 questions)

1. **Find** v **or** ω first (type 2).
2. **Acceleration**: $a = \dfrac{v^2}{r}$ or $a = r\omega^2$.
3. **Force**: $F = ma = \dfrac{mv^2}{r}$. This is the **resultant** force towards the centre.
4. **Find a real force** from it if asked: for example the friction on a car, or the tension in a string.

Variations you will meet:

- **Maximum speed without sliding**: the largest friction force gives the largest v: $v_{\text{max}} = \sqrt{\dfrac{F_{\text{max}} r}{m}}$.
- **Period from acceleration**: $a = r\omega^2$ and $\omega = \frac{2\pi}{T}$ give $T = 2\pi\sqrt{\dfrac{r}{a}}$.
- **Given the force, find the speed or period**: $v = \sqrt{\dfrac{Fr}{m}}$, then $T = \dfrac{2\pi r}{v}$.
- **Time for part of a lap**: distance round the arc ÷ speed.
- **On a rotating planet**: the contact force is $N = mg - mr\omega^2$; the force per unit mass from the ground is $g - r\omega^2$.

4. Orbits: gravity provides the centripetal force (10 questions)

1. **Say it**: "the gravitational force provides the centripetal force" (the mark scheme wants the reason, not just the equation).
2. **Write** $\dfrac{GMm}{r^2} = mr\omega^2$ (or $= \dfrac{mv^2}{r}$) with $\omega = \dfrac{2\pi}{T}$ (or $v = \dfrac{2\pi r}{T}$).
3. **Cancel** m and rearrange for what is asked (r, T, v or M). Use r from the **centre** of the planet: radius of planet + height.
4. **Substitute** with T in seconds.

Variations you will meet:

- **"Explain why the orbit is circular"**: gravity is the only force; it is always at right angles to the velocity and has constant size (constant r), so it gives a centripetal acceleration of constant size.
- **Stars orbiting each other**: each star feels the same gravitational force from the other, but each moves round the common centre with its **own** radius. Use the **separation** d in the gravity law, $F = \dfrac{GM_AM_B}{d^2}$, but each star's **own radius** in $\dfrac{mv^2}{r}$ or $mr\omega^2$, with $R_A + R_B = d$. For example, if $R_B = 3R_A$ then $4R_A = d$, so $R_A = \dfrac{d}{4}$ and $R_B = \dfrac{3d}{4}$. They have the same period and ω, so the one with the bigger radius has the bigger $v = r\omega$ and bigger $a = r\omega^2$ (here three times as big). Find the period from one star: $F = M_AR_A\omega^2$, then $T = \dfrac{2\pi}{\omega}$.
- **Straight-line graphs**: $\dfrac{GMm}{r^2} = mr\omega^2$ rearranges to $r^3 = \dfrac{GM}{\omega^2}$, so a graph of r^3 against $\dfrac{1}{\omega^2}$ is a straight line through the origin with gradient GM. Rearrange the equation into the form of the graph's axes before reading the gradient.
- **An orbit that stays above one point** on the planet has the same period as the planet's rotation.
- **Angular speed of an orbit**: $\omega = \frac{2\pi}{T}$, in rad s^{-1}, not m s^{-1}.
- The derivations of $T^2 \propto r^3$, geostationary conditions and orbital energy are covered in Gravitational fields.

5. Charged particles moving in circles (5 questions)

1. **Identify the force**: electric (Coulomb) for orbiting charges, BQv for a charge moving at right angles to a magnetic field.
2. **Set it equal to** $\dfrac{mv^2}{r}$ (or $mr\omega^2$) and solve for v, r or T.
3. **Period**: $T = \dfrac{2\pi r}{v}$ or $T = \dfrac{2\pi}{\omega}$.

Variations you will meet:

- **The resultant force changes slightly at the same radius** (for example another charge nearby partly cancels the attraction to the centre): a smaller resultant force at the same r means a smaller $\dfrac{mv^2}{r}$, so a lower speed $v = \sqrt{\dfrac{Fr}{m}}$ and a longer period $T = \dfrac{2\pi r}{v}$. A larger force gives the opposite.

- **Two particles orbiting their common centre** (like two equal masses): the separation is **twice** the orbit radius. Use the separation in the force law, but the orbit radius in $\frac{mv^2}{r}$.
- **Entering a magnetic field and leaving after a half-circle**: the gap between the paths going in and coming out is the diameter, $2r$.
- **"Show that the period does not depend on the radius"** in a magnetic field: from $BQv = \frac{mv^2}{r}$, $v = \frac{BQr}{m}$; then $T = \frac{2\pi r}{v} = \frac{2\pi m}{BQ}$. A faster particle moves in a bigger circle but takes the same time.

6. Horizontal circles with a slanting force (3 questions)

1. **Draw the two forces**: weight down; tension along the string (or contact force at right angles to the surface).
2. **Resolve the slanting force**: vertical component $=$ weight; horizontal component $=$ resultant $= \frac{mv^2}{r}$.
3. **Geometry**: radius $= L\sin\theta$ for a string of length L at θ to the vertical.
4. **Divide** the two equations if you want v without the tension: $\frac{v^2}{r} = g\tan\theta$.

Variations you will meet:

- **The tension or contact force is bigger than the weight**, because its vertical component alone equals the weight. A spring that stretches more when the mass swings round is showing this.
- **Spring instead of string**: find the tension first, then the extension from Hooke's law, $F = kx$, using the change in length.
- **Speed up and the radius changes**: for a cone, the angle is fixed, so $a = g\tan\alpha$ is fixed and $r \propto v^2$.
- **Centripetal acceleration from the forces**: $a = \frac{T\sin\theta}{m}$ (Newton's second law), then $T = 2\pi\sqrt{\frac{r}{a}}$ for the period.

7. Comparing points or objects (3 questions)

Decide first what is **shared**:

- **Points on the same rigid object** (a disc, a wheel): same ω and period. So $v = r\omega$ and $a = r\omega^2$ are both proportional to r. Mass makes no difference to any of them.
- **Two objects with the same maximum sideways force** (two cars on the same track, different paths): $\frac{mv^2}{r}$ is the same, so the centripetal acceleration is the same; $v \propto \sqrt{r}$; the time for a lap is $\frac{2\pi r}{v} \propto \sqrt{r}$, longer on the bigger circle.
- **Two stars orbiting a common centre**: same period, so the larger orbit has the larger speed and acceleration.

Fill in a "less / same / greater" table only after you have written which quantity is fixed.

8. Vertical circles (1 question)

1. **Energy** gives the speed at the top: $v_{\text{top}}^2 = v_{\text{bottom}}^2 - 2g(2r)$.
2. **At the top**, the centripetal acceleration is $\frac{v^2}{r}$; the car stays in contact if this is **at least** g.
3. **The contact force** at the top is $N = m\left(\frac{v^2}{r} - g\right)$; at the bottom $N = m\left(\frac{v^2}{r} + g\right)$.

As the car climbs from bottom to top it slows, so its centripetal acceleration **decreases**. Mass cancels throughout, so changing the mass alone changes nothing.

9. Circular motion seen side-on (1 question)

1. Displacement of the shadow: $x = r\sin\theta$ with $\theta = \omega t$, so $x = r\sin\omega t$.
2. Amplitude $=$ radius of the circle (half the diameter); period $= \frac{2\pi}{\omega}$.
3. The shadow's acceleration is greatest when the shadow is at an **end** of its path (the point is at the edge of the circle as seen side-on). There the whole centripetal acceleration lies along the screen, so the maximum acceleration is $r\omega^2$: the same as the centripetal acceleration, and the same as $a_0 = \omega^2 x_0$ with amplitude $x_0 = r$. It points back towards the centre of the shadow's path. Why this is simple harmonic motion is in Oscillations.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

The drum of a washing machine has radius 0.25 m and spins at 1000 revolutions per minute. A sock of mass 0.050 kg is pressed against the inside of the drum.

(a) Calculate the angular speed of the drum. **[2]**

(b) Calculate the speed of the sock. **[1]**

(c) Calculate the centripetal acceleration of the sock and the resultant force on it. **[3]**

(d) Calculate the angle, in degrees, that the drum turns through in 0.010 s. **[2]**

Solution

(a) 1000 rev min^{-1} $= \dfrac{1000}{60} = 16.7$ rev s^{-1}, so $\omega = \dfrac{2\pi \times 1000}{60}$ **M1** $= 105$ rad s^{-1}. **A1**

(b) $v = r\omega = 0.25 \times 104.7 = 26.2$ m s^{-1}. **A1**

(c) $a = r\omega^2 = 0.25 \times 104.7^2$ **M1** $= 2.74 \times 10^3$ m s^{-2}. **A1** $F = ma = 0.050 \times 2742 = 137$ N, towards the centre of the drum (provided by the contact force from the drum wall). **A1**

(d) $\theta = \omega t = 104.7 \times 0.010 = 1.047$ rad **M1**; $1.047 \times \dfrac{180^\circ}{\pi} = 60^\circ$. **A1**

Example 2

A ball of mass 0.40 kg hangs from a light string of length 0.80 m. The ball moves in a horizontal circle at constant speed with the string at 35° to the vertical.

(a) Explain how the forces on the ball cause it to move in a horizontal circle. **[2]**

(b) Calculate the tension in the string. **[2]**

(c) Calculate the radius of the circle and the speed of the ball. **[4]**

(d) Calculate the period of the motion. **[2]**

Solution

(a) The vertical component of the tension balances the weight, so there is no vertical resultant force. **B1** The horizontal component of the tension is the resultant force; it is always towards the centre of the circle, at right angles to the velocity, and gives the centripetal acceleration. **B1**

(b) $T \cos 35^\circ = mg$ **M1**, so $T = \dfrac{0.40 \times 9.81}{\cos 35^\circ} = 4.79$ N. **A1**

(c) $r = 0.80 \sin 35^\circ = 0.459$ m. **A1** $T \sin 35^\circ = \dfrac{mv^2}{r}$ **M1**, which with (b) gives $\dfrac{v^2}{r} = g \tan 35^\circ = 6.87$ m s^{-2}. **M1** $v = \sqrt{6.87 \times 0.459} = 1.78$ m s^{-1}. **A1**

(d) Period $= \dfrac{2\pi r}{v} = \dfrac{2\pi \times 0.459}{1.775}$ **M1** $= 1.62$ s. **A1**

Example 3

A satellite of mass 850 kg moves in a circular orbit of radius 7.2×10^6 m around a planet. The gravitational force on the satellite is 6540 N.

(a) Explain why the satellite moves in a circle at constant speed. **[2]**

(b) Calculate the speed of the satellite. **[2]**

(c) Calculate the period of the orbit and the angular speed of the satellite. Give a unit with the angular speed. **[3]**

Solution

(a) The gravitational force is always at right angles to the satellite's velocity and has constant size, so it does no work (constant speed) **B1** and provides the centripetal acceleration towards the centre of the planet. **B1**

(b) $F = \dfrac{mv^2}{r}$, so $v = \sqrt{\dfrac{6540 \times 7.2 \times 10^6}{850}}$ **M1** $= 7.44 \times 10^3$ m s^{-1}. **A1**

(c) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi \times 7.2 \times 10^6}{7443}$ **M1** $= 6.08 \times 10^3$ s (about 101 minutes). **A1** $\omega = \dfrac{v}{r} = 1.03 \times 10^{-3}$ rad s^{-1}. **A1**

Example 4

A roller-coaster car of mass 480 kg enters a vertical loop of radius 9.0 m. Its speed at the bottom of the loop is 22 m s^{-1}. Friction and air resistance are negligible.

(a) Calculate the speed of the car at the top of the loop. **[2]**

(b) Show that the track must still push on the car at the top of the loop, and calculate the size of this force. **[3]**

(c) Calculate the force exerted on the car by the track at the bottom of the loop. **[2]**

(d) Calculate the least speed at the bottom for which the car stays in contact with the track at the top. **[2]**

Solution

(a) Energy: $\frac{1}{2}mv^2 = \frac{1}{2}m(22)^2 - mg(2 \times 9.0)$ **M1**, so $v^2 = 484 - 353.2 = 130.8$ and $v = 11.4$ m s^{-1}. **A1**

(b) Centripetal acceleration at the top $= \dfrac{v^2}{r} = \dfrac{130.8}{9.0} = 14.5$ m s^{-2}. **M1** This is more than g, so the weight alone is not enough: the track must push down too. **A1** $mg + N = \dfrac{mv^2}{r}$, so $N = 480 \times (14.54 - 9.81) = 2.27 \times 10^3$ N, downwards. **A1**

(c) $N - mg = \dfrac{mv^2}{r}$ **M1**, so $N = 480 \times \left(\dfrac{22^2}{9.0} + 9.81\right) = 3.05 \times 10^4$ N, upwards. **A1**

(d) At the least speed, $N = 0$ at the top, so $\dfrac{v_{\text{top}}^2}{r} = g$ and $v_{\text{top}}^2 = gr$. **M1** Energy: $v_{\text{bottom}}^2 = gr + 4gr = 5gr$, so $v_{\text{bottom}} = \sqrt{5 \times 9.81 \times 9.0} = 21.0$ m s^{-1}. **A1** (The mass cancels, so this holds for any car.)

Common mistakes

- **Drawing "centripetal force" as an extra force.** Examiner reports on the cone (2025), the swinging spring (2023) and the loop-the-loop (2021) all found candidates adding it as a third force, or treating it as something that "balances" the weight. The centripetal force **is** the resultant of the forces you drew.
- **Resolving the weight instead of the slanting force.** In a conical pendulum the tension (or contact force) is the force to resolve: vertical component = weight, horizontal component = resultant. Resolving the weight leads to wrong numbers later.

- **Describing uniform circular motion wrongly.** "The velocity is constant and its direction changes" contradicts itself; say the **speed** is constant. Quoting $a = \frac{v^2}{r}$ is not a description: say the acceleration is at right angles to the velocity, towards the centre.
- **"The force is constant."** For circular motion the force must have constant **size** and always be at right angles to the velocity; its direction changes all the time. Examiners do not accept just "constant".
- **Mixing up angular speed and speed.** Reports in 2022 and 2023 found candidates confusing the two, working out a speed when an angular speed was asked for, or the other way round. Check the unit asked for: rad s^{-1} or m s^{-1}.
- **Assuming ω stays the same when something changes.** When a ball in a cone speeds up, the resultant force (fixed by the cone's angle) stays the same, so a is fixed and $r \propto v^2$, not $r \propto v$.
- **Treating an object on the ground as if it were in orbit.** An object on the Equator has a contact force as well as its weight; the contact force is slightly less than the weight, and the period is one day.
- **Thinking a heavier car in a loop behaves differently.** The 2021 report found many candidates expecting a greater centripetal acceleration for a greater mass. Mass cancels: the speed at the top and the centripetal acceleration are the same for any mass.
- **Forgetting that the speed changes in a vertical circle.** Find the speed at the top with energy first; the car is slower there, so its centripetal acceleration is smaller.
- **Using the wrong radius or time.** Measure r from the axis (subtract any gap from the rim; use $R\cos\phi$ at a latitude). Convert hours and minutes to seconds, and square π and T where the formula needs it.

How to get the marks

- **"Define the radian"**: "the angle subtended at the centre of a circle by an arc equal in length to the radius". Both "arc" and "radius" are needed.
- **Show the formula with numbers.** Mark schemes give a **C1** for the right equation ($v = r\omega$, $a = \frac{v^2}{r}$, $\omega = \frac{2\pi}{T}$) with values substituted (shown as **M1** in the examples above), then **A1** for the answer.
- **"Show that"**: write every step with the numbers, and give the answer to one more significant figure than the value printed. Don't work backwards from the value you are shown.
- **Explain with forces**: name each real force, say which component balances the weight and which component is the resultant towards the centre. "Provides the centripetal force" is the phrase the mark scheme looks for.
- **Arrows**: start on the object; velocity along the tangent, acceleration towards the centre; label each one as asked (V, A, weight, contact force).
- **Units**: rad s^{-1} for angular speed; the radian is written "rad". Give a unit whenever the question says "give a unit".
- **Tables of comparisons**: one tick per row. Work out which quantity is shared (usually ω) before ticking.
- **"Explain, without calculation"**: give the chain of cause and effect in words, ending in the quantity asked about.

Quick check

1. Convert 135° to radians, and find the arc length it cuts off on a circle of radius 0.60 m.
2. A turntable spins at 40 revolutions per minute. Calculate its angular speed and the speed of a point 0.12 m from the centre.
3. A 2.2 kg puck on a smooth horizontal table is tied to a peg by a string and moves in a circle of radius 0.90 m at 3.0 m s^{-1}. Calculate the tension. What would it be if the speed were doubled?
4. Points P and Q are on a spinning disc, 4.5 cm and 9.0 cm from its centre. Compare their angular speeds, speeds and centripetal accelerations.
5. A cyclist rides round a horizontal circular track of radius 25 m at 8.0 m s^{-1}. Calculate her centripetal acceleration and angular speed, and name the force that provides the centripetal force.

Answers

1. $135 \times \frac{\pi}{180} = \frac{3\pi}{4} = 2.36$ rad. Arc $= r\theta = 0.60 \times 2.356 = 1.41$ m.
2. $\omega = \frac{40 \times 2\pi}{60} = 4.19$ rad s^{-1}. $v = r\omega = 0.12 \times 4.189 = 0.503$ m s^{-1}.
3. $T = \frac{mv^2}{r} = \frac{2.2 \times 3.0^2}{0.90} = 22$ N. Doubling v multiplies v^2 by 4: 88 N.
4. Same angular speed (same rigid disc). Q has twice the speed ($v = r\omega$) and twice the centripetal acceleration ($a = r\omega^2$).
5. $a = \frac{v^2}{r} = \frac{8.0^2}{25} = 2.56$ m s^{-2}. $\omega = \frac{v}{r} = 0.32$ rad s^{-1}. Sideways friction from the track on the tyres.

Past questions to try

- **9702/43/M/J/23 Q2**: a mass on a spring swinging in a horizontal circle; components of the tension, Hooke's law and the period.
- **9702/42/O/N/24 Q1**: speed and centripetal acceleration of a point on the rim of a spinning wheel.
- **9702/43/O/N/21 Q1**: describing uniform circular motion, then two cars on different paths round a track.
- **9702/42/O/N/21 Q1**: a toy car in a vertical loop, using energy and the centripetal acceleration at the top.