Cambridge AS & A Level · Physics 9702 · Notes

Nuclear physics A2

Read online at **pastopic.com/cambridge/physics-9702/notes/nuclear-physics**

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Read first: **Particle physics**

What you need to know

By the end of this topic you should be able to:

1. **Understand that energy and mass are equivalent**, as $E = mc^2$ says, and recall and use this equation.
2. **Write simple nuclear reactions as nuclear equations**, such as ${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \to {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$.
3. **Define and use mass defect and binding energy.**
4. **Sketch how binding energy per nucleon varies with nucleon number.**
5. **Explain what nuclear fusion and nuclear fission are.**
6. **Explain why binding energy per nucleon matters in nuclear reactions**, fusion and fission included.
7. **Calculate the energy released in a nuclear reaction** with $E = c^2 \Delta m$.
8. **Understand that fluctuations in count rate are evidence** that radioactive decay is random.
9. **Understand that radioactive decay is both spontaneous and random.**
10. **Define activity and decay constant**, and recall and use $A = \lambda N$.
11. **Define half-life.**
12. **Use** $\lambda = \dfrac{0.693}{t_{1/2}}$**.**
13. **Understand that radioactive decay is exponential**, and sketch and use $x = x_0 e^{-\lambda t}$, where x can be the activity, the number of undecayed nuclei or the received count rate.

Nucleons, proton number, nucleon number, isotopes and the particles emitted in α, β^-, β^+ and γ decay come from Particle physics (AS). The unified atomic mass unit u and the Avogadro constant N_A are on the data sheet.

Understand it

Mass and energy are the same thing

Einstein's equation says that mass is a form of energy:

$$E = mc^2$$

Any change in energy ΔE comes with a change in mass $\Delta m = \dfrac{\Delta E}{c^2}$. Because $c^2 = 9.0 \times 10^{16}$ m² s⁻² is enormous, a tiny mass is a huge energy: 1.0 g turned completely into energy gives $1.0 \times 10^{-3} \times (3.00 \times 10^8)^2 = 9.0 \times 10^{13}$ J. In chemical reactions the mass change is far too small to notice; in nuclear reactions it is easily measured.

In a nuclear reaction **mass–energy is conserved, not mass on its own**. If a reaction releases energy (as kinetic energy of the products and as gamma photons), the products have **less mass** than what you started with, by $\Delta m = \dfrac{E}{c^2}$.

Nuclear masses are given in unified atomic mass units: $1\ \mathrm{u} = 1.66 \times 10^{-27}$ kg. With the data-sheet values, 1 u of mass is equivalent to $1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 = 1.49 \times 10^{-10}$ J, about 934 MeV. Energies of single nuclei are usually given in MeV: $1\ \mathrm{MeV} = 1.60 \times 10^{-13}$ J.

Nuclear equations

A nucleus is written ${}^{A}_{Z}\mathrm{X}$: A is the **nucleon number** (protons + neutrons) and Z the **proton number**. In every nuclear reaction both are conserved:

- the **top numbers** (nucleon number) add up to the same total on both sides;
- the **bottom numbers** (charge, in units of e) add up to the same total on both sides.

The particles you need, written the same way:

Particle	Symbol
alpha particle (helium nucleus)	${}^{4}_{2}\alpha$ or ${}^{4}_{2}\mathrm{He}$
beta-minus particle (electron)	${}^{0}_{-1}\beta$ or ${}^{0}_{-1}\mathrm{e}$
beta-plus particle (positron)	${}^{0}_{+1}\beta$ or ${}^{0}_{+1}\mathrm{e}$
neutron, proton	${}^{1}_{0}\mathrm{n}$, ${}^{1}_{1}\mathrm{p}$ (or ${}^{1}_{1}\mathrm{H}$)
gamma photon	${}^{0}_{0}\gamma$
electron neutrino, antineutrino	${}^{0}_{0}\nu$, ${}^{0}_{0}\bar{\nu}$

For example, radium-226 decays by alpha emission: ${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\alpha$ ($226 = 222 + 4$ and $88 = 86 + 2$). In β^- decay a neutron becomes a proton, so Z goes **up by 1** and an antineutrino is also emitted; in β^+ decay Z goes **down by 1** with a neutrino. A doesn't change in either. A gamma photon changes neither number.

A nuclide with **too many neutrons for its number of protons** (compare it with a stable isotope of the same element, which has fewer neutrons) is unstable, and tends to decay by β^- emission.

Mass defect and binding energy

A nucleus has **less mass than the protons and neutrons that make it**, measured separately. The difference is the **mass defect**:

$$\Delta m = Zm_p + (A - Z)m_n - m_{\text{nucleus}}$$

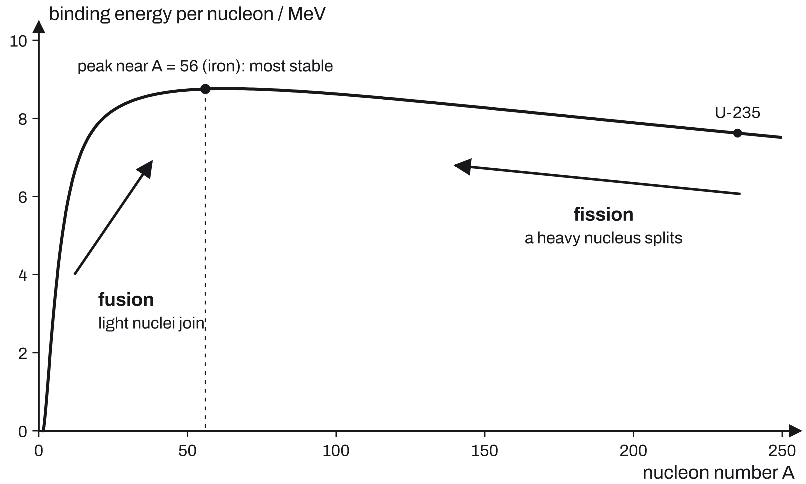
The **binding energy** of a nucleus is the **minimum energy needed to separate all its nucleons to infinity** (so that they no longer interact). Pulling the nucleons apart takes this energy in, so the separated nucleons have more mass than the nucleus by exactly $\dfrac{\text{binding energy}}{c^2}$:

$$\text{binding energy} = \Delta m\, c^2$$

Binding energy is energy you must **supply**, not energy the nucleus "has". A nucleus with a bigger binding energy **per nucleon** has its nucleons more tightly held: it is more stable.

$$\text{binding energy per nucleon} = \frac{\text{binding energy}}{A}$$

The binding energy per nucleon curve



What to learn about its shape (each point is usually a mark on a sketch):

- it **starts at zero** at $A = 1$ (a single nucleon has nothing binding it) and rises **very steeply**;
- it has **one peak**, near $A = 56$ (iron), at about 8.8 MeV per nucleon: the most stable nuclei;
- to the right of the peak it **falls gently** all the way to $A \approx 250$, and **never returns to zero**.

Fusion and fission

Nuclear fusion: two **light** nuclei **join** to form one heavier nucleus. For example, ${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \to {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$. The nuclei are both positive, so they must collide very fast to get close enough to join: fusion needs **very high temperatures**, as in the core of a star.

Nuclear fission: one **heavy** nucleus **splits** into two smaller nuclei of similar size, usually with a few neutrons. It is usually started by the nucleus **absorbing a neutron**: ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \to {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n}$.

Why both release energy. Energy is released when the products have a **greater total binding energy** than the starting nuclei: the products' nucleons are more tightly held, the products have less mass, and the missing mass appears as kinetic energy (and photons). On the curve:

- **fusion** takes nuclei on the steep left side and makes a nucleus **further up** the curve, nearer the peak;
- **fission** takes a nucleus on the far right and makes two nuclei **further up** the gentle slope, nearer the peak.

Either way, the **binding energy per nucleon increases**. The same idea explains why a heavy nucleus such as polonium-212 releases energy in **alpha decay**: the daughter has a smaller nucleon number and so a higher binding energy per nucleon.

Energy released, in two equivalent ways:

$$E = (\text{mass before} - \text{mass after})\,c^2 = (\text{total binding energy after}) - (\text{total binding energy before})$$

Free neutrons and protons have zero binding energy. Given binding energy **per nucleon**, multiply each by its nucleon number first. For ${}^{210}\mathrm{Po} \to {}^{206}\mathrm{Pb} + {}^{4}\mathrm{He}$ with binding energies per nucleon 7.83, 7.88 and 7.07 MeV: $E = 206 \times 7.88 + 4 \times 7.07 - 210 \times 7.83 = 7.26$ MeV.

Radioactive decay is spontaneous and random

Radioactive decay is the **spontaneous** emission of ionising radiation (an alpha or beta particle, or a gamma photon) from an **unstable nucleus**.

- **Spontaneous**: the decay is **not affected by external factors** such as temperature, pressure or chemical combination. Nothing outside the nucleus makes it decay.
- **Random**: you **cannot predict when a particular nucleus will decay**, or **which nucleus will decay next**. Every undecayed nucleus of the isotope has the **same, constant probability of decaying per unit time**, however long it has already existed.

Random does **not** mean that everything is unpredictable. With huge numbers of nuclei, the fraction that decays each second is very predictable: that is why half-life is a fixed property of an isotope.

Evidence for randomness: a detector near a steady source shows a count rate that **fluctuates** about an average from one short interval to the next. On a graph of count rate (or $\ln$ of it) against time, the points are scattered about a smooth line.

Activity, decay constant and half-life

The **activity** A of a sample is the **number of nuclei that decay per unit time**. Its unit is the **becquerel**: 1 Bq $= 1$ decay per second.

The **decay constant** λ is the **probability that a nucleus decays per unit time** (unit s^{-1}, or min^{-1}, h^{-1} ...). If each of N undecayed nuclei has probability λ of decaying each second, the number decaying each second is

$$A = \lambda N$$

N is the number of **undecayed** nuclei, counted, not a mass. From a mass: number of nuclei $= \dfrac{\text{mass}}{\text{mass of one nucleus}} = \dfrac{\text{mass in kg}}{A_r \times 1.66 \times 10^{-27}}$, or $\dfrac{\text{mass in g}}{A_r} \times N_A$. So 1.0 g of a nuclide of nucleon number 60 holds $\dfrac{1.0 \times 10^{-3}}{60 \times 1.66 \times 10^{-27}} = 1.00 \times 10^{22}$ nuclei. For A in Bq, λ must be in s^{-1}.

The **half-life** $t_{1/2}$ is the **time taken for the activity of a sample to halve** (or, equally, the time for the number of **undecayed** nuclei of that isotope to halve).

Why decay is exponential

Each nucleus has the same probability of decay per unit time, so the number decaying per second is proportional to the number still there:

$$\frac{dN}{dt} = -\lambda N$$

The rate of decrease of N is proportional to N itself. As nuclei decay, fewer are left, so fewer decay in the next second: N falls quickly at first and then more and more slowly. A quantity whose rate of change is proportional to itself varies **exponentially**:

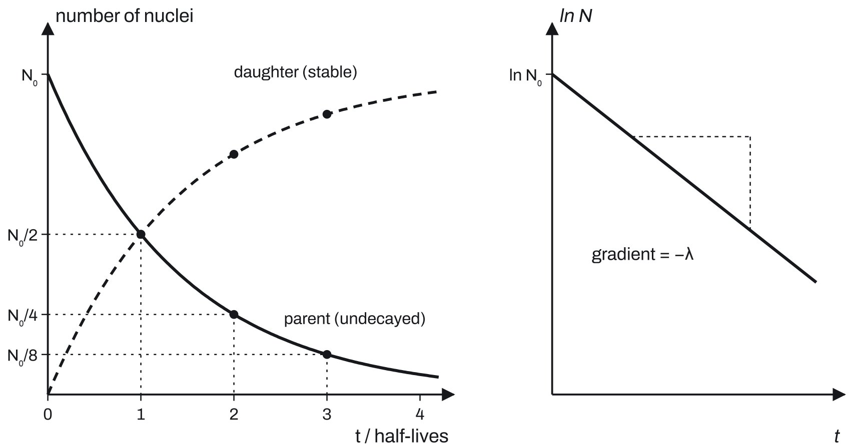
$$N = N_0 e^{-\lambda t}$$

Because $A = \lambda N$ and the received count rate is a fixed fraction of A, **activity and count rate follow the same curve**: $x = x_0 e^{-\lambda t}$ for any of them.

Half-life and decay constant. After one half-life, $N = \frac{1}{2}N_0$, so $e^{-\lambda t_{1/2}} = \frac{1}{2}$, giving $\lambda t_{1/2} = \ln 2$:

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{t_{1/2}}$$

After n half-lives the fraction left is $\left(\frac{1}{2}\right)^n$: after 3 half-lives, $\frac{1}{8}$. After 4 half-lives $\frac{1}{16}$ is left and $\frac{15}{16}$ has decayed, so the ratio decayed : undecayed is 15.



What the graphs tell you:

- N **against** t: an exponential decay curve. The **gradient** is $\dfrac{dN}{dt} = -A$: its size is the **activity**.
- **Daughter against** t (when the daughter is stable): the mirror image, rising from zero towards N_0, since parent + daughter $= N_0$ at all times.
- A **against** N: $A = \lambda N$ is a straight line **through the origin** with gradient λ.
- $\ln N$ **(or** $\ln A$**) against** t: taking logs of $N = N_0 e^{-\lambda t}$ gives $\ln N = \ln N_0 - \lambda t$: a **straight line** with gradient $-\lambda$ and intercept $\ln N_0$. The size of the gradient is the decay constant.

Count rate and activity

A detector near a sample records a **count rate** that is **less than the activity**, because:

- the radiation goes out in **all directions**, and only a small fraction heads towards the detector;
- some radiation is **absorbed** or scattered in the sample itself, in the air or in the detector's window;
- some radiation that reaches the detector **isn't registered** (it passes through without ionising, or arrives at the same moment as another particle).

Background radiation adds to the count, so it can't explain a count **lower** than the activity. If a sample's activity is **higher** than expected after a time, a likely reason is that the **daughter nuclide is also radioactive**.

Key facts and formulas

"Given" means it is on the data sheet printed in the paper (the decay formulas are on the Paper 4 sheet). "Learn it" means you must remember it.

Formula	Status
Mass–energy $E = mc^2$; energy released $E = c^2 \Delta m$	Learn it
Mass defect $\Delta m = Zm_p + (A - Z)m_n - m_{\text{nucleus}}$; binding energy $= \Delta m\, c^2$	Learn it
Binding energy per nucleon $= \dfrac{\text{binding energy}}{A}$	Learn it
Energy released $=$ total binding energy after $-$ total binding energy before	Learn it
Nucleon number and proton number conserved in nuclear equations	Learn it
Activity $A = \lambda N$	Learn it
Radioactive decay $x = x_0 e^{-\lambda t}$	Given
Decay constant $\lambda = \dfrac{0.693}{t_{1/2}}$	Given
$\ln x = \ln x_0 - \lambda t$; fraction left after n half-lives $= \left(\frac{1}{2}\right)^n$	Learn it
Number of nuclei $N = \dfrac{\text{mass}}{A_r \times 1\ \text{u}} = \dfrac{\text{mass in g}}{A_r} N_A$	Learn it
$1\ \text{eV} = 1.60 \times 10^{-19}$ J, so $1\ \text{MeV} = 1.60 \times 10^{-13}$ J	Learn it

Formula	Status
$c = 3.00 \times 10^8$ m s^{-1}, $1\ \mathrm{u} = 1.66 \times 10^{-27}$ kg, $e = 1.60 \times 10^{-19}$ C, $N_A = 6.02 \times 10^{23}$ mol^{-1}, $m_p = 1.67 \times 10^{-27}$ kg	Given

How to do it

In the Paper 4 questions we have tagged for this topic (24 different questions, 2021–2025; 9 more appear on two papers), the types came up this often. 23 of the 24 questions mix two or more types, so the counts add up to more than 24.

Question type	Questions
Definitions and the nature of decay	17
Decay calculations: λ, half-life, $A = \lambda N$	15
Decay graphs: sketching and reading	10
Mass defect, binding energy and energy released	10
Nuclear equations and the particles emitted	7
The binding energy per nucleon curve, fusion and fission	5
Count rate and activity	3

1. Definitions and the nature of decay (17 questions)

Learn these, in meaning; each bold phrase is usually a mark. Don't use the word you are defining ("decay is when a nucleus decays" earns nothing).

- **Activity**: the **number of nuclear decays** (disintegrations) **per unit time**.
- **Decay constant**: the **probability** that a nucleus decays **per unit time**.
- **Half-life**: the time for the **activity** to **halve** (or for the number of **undecayed** nuclei to halve; not "for the nuclei to halve").
- **Random**: you **cannot predict when a particular nucleus** will decay (or which will decay next).
- **Spontaneous**: **not affected by external or environmental factors**.
- **Radioactive decay**: the **spontaneous emission of ionising radiation** from an **unstable nucleus**.
- **Mass defect**: the **difference** between the **total mass of the separate nucleons** and the **mass of the nucleus**.
- **Binding energy**: the (minimum) **energy needed to separate all the nucleons** of a nucleus **to infinity**.
- **Fusion**: two **light nuclei join** to form one **heavier nucleus**. **Fission**: a **heavy nucleus splits** into two smaller nuclei.

Variations you will meet:

- **Evidence for randomness**: **fluctuations** in the measured count rate (or the scatter of points about a smooth line on a graph).
- **"Explain why the activity decreases, with reference to the random nature of decay"**: every undecayed nucleus has the **same probability of decay per unit time**; as time passes there are **fewer undecayed nuclei**, so fewer decay in each interval.
- **"Explain why decay is exponential"**: the activity is proportional to the number of undecayed nuclei, and the activity is the rate of decrease of that number, so the rate of change of N is proportional to N.

2. Decay calculations: λ, half-life, $A = \lambda N$ (15 questions)

1. **Decay constant from half-life**: $\lambda = \dfrac{\ln 2}{t_{1/2}}$. Convert the half-life to **seconds** if the activity is in Bq (1 year $=$ $365 \times 24 \times 3600$ s).
2. **Number of nuclei from a mass**: $N = \dfrac{\text{mass in kg}}{A_r \times 1.66 \times 10^{-27}}$ (or $\dfrac{\text{mass in g}}{A_r} \times N_A$).
3. **Activity**: $A = \lambda N$. Or the reverse: $N = \dfrac{A}{\lambda}$, then the mass.
4. **Later values**: $x = x_0 e^{-\lambda t}$ for N, A or count rate. **To find a time**, take logs: $t = \dfrac{1}{\lambda} \ln \dfrac{x_0}{x}$. **To find λ from two readings**, $\lambda = \dfrac{1}{t} \ln \dfrac{x_0}{x}$.

Variations you will meet:

- **A whole number of half-lives**: if decayed : undecayed $= 7$, then $\frac{1}{8}$ is left: 3 half-lives. If it is 31, $\frac{1}{32}$ is left: 5 half-lives.
- **The nucleon number from mass and number**: mass of one nucleus $= \dfrac{\text{mass}}{N}$; nucleon number $= \dfrac{\text{mass of one nucleus}}{1.66 \times 10^{-27}}$, given as a **whole number**.
- **Two samples with different half-lives**: write $A = A_0 e^{-\lambda t}$ for each, set them equal and solve for t. Read each half-life off a graph as the time for the activity to halve **from any point**, not half the time axis. In each case $N_0 = \dfrac{A_0}{\lambda}$.
- **Power from a radioactive source**: power $=$ activity $\times$ energy per decay (in J). It falls exponentially with the activity. Keep extra figures in λ when finding a time, or the answer drifts.
- **Energy released over a time interval**: number decayed $= N_1 - N_2$ (from $N = N_0 e^{-\lambda t}$ at each time), times the energy per decay.
- **Rate of photon production** when each decay gives two photons: $2\lambda N$, with $\lambda = \dfrac{\ln 2}{T}$ and $N = nN_A$ for amount n mol.
- **"Show that the fraction left after 1.70 half-lives is …"**: $\dfrac{N}{N_0} = e^{-\ln 2 \times 1.70} = 0.31$. Calculate; don't read it off a graph.

3. Decay graphs: sketching and reading (10 questions)

1. **Sketching N, A or count rate against** t: a curve from $(0, x_0)$ with a **negative gradient that gets less steep**, passing through $\frac{1}{2}x_0$ at $t_{1/2}$, $\frac{1}{4}x_0$ at $2t_{1/2}$, $\frac{1}{8}x_0$ at $3t_{1/2}$, and never reaching zero or turning up.
2. **A stable daughter against** t: a curve **rising** from $(0, 0)$ through $\frac{1}{2}N_0$, $\frac{3}{4}N_0$, $\frac{7}{8}N_0$ at $1, 2, 3$ half-lives, flattening towards N_0. Don't draw the decay curve by mistake.
3. **Gradients**: the size of the gradient of an N–t graph is the **activity**; the gradient of an A–N graph (a straight line through the origin) is the **decay constant**.
4. $\ln$ **graphs**: $\ln x = \ln x_0 - \lambda t$, so the size of the gradient is λ. Take two points far apart **on your line**: $\lambda = \frac{\Delta(\ln x)}{\Delta t}$ (in size), with the unit **per** the time unit (s^{-1}, min^{-1}). Then $t_{1/2} = \frac{\ln 2}{\lambda}$.

Variations you will meet:

- **"Show that λ is the magnitude of the gradient"**: start from $x = x_0 e^{-\lambda t}$ and take natural logs, then compare with $y = mx + c$.
- **Axis labels like** $\ln(M/10^{-16}\ \mathrm{g})$: read the values of $\ln$ straight off the axis; don't put the 10^{-16} into the gradient (it only shifts the line up or down).
- **Conclusions from a graph of parent and daughter**: the total stays constant; the daughter is stable; read off the half-life (where the curves cross, if the daughter starts at zero); N_0; and so $A_0 = \lambda N_0$.
- **Scattered points about the line**: evidence that decay is random.

4. Mass defect, binding energy and energy released (10 questions)

1. **Mass defect**: $\Delta m = Zm_p + (A - Z)m_n - m_{\mathrm{nucleus}}$, in u. Keep all the decimal places given; the answer is a small difference of big numbers.
2. **Convert** to kg: $\times 1.66 \times 10^{-27}$.
3. **Binding energy** $= \Delta m\, c^2$ in J; divide by 1.60×10^{-13} for MeV. **Per nucleon**: divide by A.
4. **Energy released in a reaction**: $\Delta m =$ (total mass before) $-$ (total mass after), then $E = c^2 \Delta m$. Or, from mass defects or binding energies: (total after) $-$ (total before). The answer must be **positive** for energy released.

Variations you will meet:

- **Binding energy per nucleon data**: multiply each by its nucleon number first, then subtract; don't subtract the per-nucleon values directly.
- **Per mole**: multiply the energy of one reaction by the number of reactions. Check how many reactions: if **two** deuterium nuclei are used per reaction, 1.00 mol of deuterium gives 0.500 mol of reactions.
- **A star or a power source**: reactions per second $= \frac{\text{power}}{\text{energy per reaction}}$; mass of product per second $=$ that rate $\times$ the mass of one product nucleus. A star of power 3.8×10^{26} W with 4.3×10^{-12} J per reaction does 8.8×10^{37} reactions per second. The **mass lost** per second is $\frac{\text{power}}{c^2}$: here 4.2×10^{9} kg s^{-1}.
- **A missing mass**: work backwards: $\Delta m = \frac{E}{c^2}$ in u, then add it to the products' masses.

- **"How does the mass of the products compare?"**: energy is **released**, so (mass–energy being conserved) the products have **less mass**.
- **Where the energy goes in alpha decay**: kinetic energy of the alpha particle, **and** of the recoiling daughter nucleus, and any gamma photon. So the alpha's kinetic energy is less than the total energy released.
- **From binding energies of products to one of the reactants**: total binding energy of the reactant $=$ total of the products $-$ energy released per reaction; then divide by its nucleon number.

5. Nuclear equations and the particles emitted (7 questions)

1. Write each particle with its nucleon and proton numbers (table in "Nuclear equations").
2. **Balance the top numbers**, then **the bottom numbers**, to find the missing values or the number of neutrons.
3. Name the particles: β^+ is a **positron**; with it comes a **neutrino**; with β^-, an **antineutrino**.

Variations you will meet:

- **Number of neutrons in fission**: ${}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + x\,{}^{1}_{0}\mathrm{n}$: $1 + 235 = 141 + 92 + x$, so $x = 3$.
- **A particle with no mass defect** in a fusion equation, with top number 1 and bottom number 0: a neutron.
- **Why an isotope is unstable**: it has **too many neutrons** for its number of protons (more than a stable isotope of the same element).

6. The binding energy per nucleon curve, fusion and fission (5 questions)

1. **Sketch**: start at (or very near) zero at $A = 1$; **steep** rise; **single peak near** $A = 56$; **gentle** fall to $A = 250$, staying well above zero.
2. **Mark nuclei**: one that undergoes **fusion** at small A (near the left end); one that undergoes **fission** or **alpha decay** at large A (well right of the peak).
3. **Explain energy release**: binding energy per nucleon is **greatest for intermediate nucleon numbers**; fusion (light nuclei) and fission (heavy nuclei) both make products with a **higher binding energy per nucleon**, so energy is released.

Differences between fission and fusion (three are usually wanted): fission is of **large** nuclei, fusion of **small** ones; fission is one nucleus **splitting** into two, fusion is two nuclei **joining** into one; fission is usually **started by a neutron**, fusion needs a **very high temperature**.

7. Count rate and activity (3 questions)

State that the count rate is **less than** the activity and give reasons from "Count rate and activity" above. Randomness, background radiation and a radioactive daughter are **not** reasons for a smaller count rate.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **M1** method mark: for a correct method, even if the numbers slip.
- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

Sodium-24 ($^{24}_{11}\mathrm{Na}$) is radioactive, with a half-life of 15.0 hours. A sample contains 4.0×10^{-9} kg of sodium-24.

(a) Define decay constant. **[2]**

(b) Show that the decay constant of sodium-24 is 1.28×10^{-5} s^{-1}. **[1]**

(c) Calculate the activity of the sample. **[3]**

(d) Calculate the time, in hours, for the activity to fall to 5.0% of its initial value. **[2]**

Solution

(a) The probability that a nucleus decays **M1** per unit time. **A1**

(b) $\lambda = \dfrac{\ln 2}{15.0 \times 3600} = 1.28 \times 10^{-5}$ s^{-1}. **A1**

(c) $N = \dfrac{4.0 \times 10^{-9}}{24 \times 1.66 \times 10^{-27}} = 1.004 \times 10^{17}$ **C1** $A = \lambda N$ **C1** $= 1.284 \times 10^{-5} \times 1.004 \times 10^{17} = 1.29 \times 10^{12}$ Bq. **A1**

(d) $0.050 = e^{-\lambda t}$, so $t = \dfrac{\ln 20}{1.284 \times 10^{-5}}$ **C1** $= 2.33 \times 10^{5}$ s $= 64.8$ hours. **A1**

Example 2

A sample contains a single radioactive isotope that decays to a stable product. Its activity is 240 Bq at $t = 0$ and 90 Bq at $t = 30$ min.

(a) Explain why the activity of the sample decreases with time. **[2]**

(b) Determine the decay constant, in min^{-1}, and the half-life, in min. **[3]**

(c) Calculate the activity at $t = 60$ min. **[1]**

(d) A student plots $\ln(A/\mathrm{Bq})$ against t. Describe the graph and give the values at $t = 0$ and $t = 30$ min. **[2]**

Solution

(a) Every undecayed nucleus has the same probability of decay per unit time; **M1** as time goes on there are fewer undecayed nuclei, so fewer decay per unit time. **A1**

(b) $90 = 240e^{-30\lambda}$ **C1**, so $\lambda = \dfrac{1}{30}\ln\dfrac{240}{90} = 0.0327$ min⁻¹. **A1** $t_{1/2} = \dfrac{0.693}{0.0327} = 21.2$ min. **A1**

(c) In each 30 min the activity is multiplied by $\frac{90}{240}$, so $A = 240 \times \left(\frac{90}{240}\right)^2 = 33.8$ Bq (the same as $240e^{-60\lambda}$ with λ unrounded). **A1**

(d) A straight line with a negative gradient of size λ **B1**, from $\ln 240 = 5.48$ at $t = 0$ to $\ln 90 = 4.50$ at $t = 30$ min. **B1**

Example 3

The mass of a nitrogen-14 nucleus ($^{14}_{7}\mathrm{N}$) is $13.999\,234$ u. The mass of a proton is $1.007\,276$ u and of a neutron $1.008\,665$ u.

(a) State what is meant by the binding energy of a nucleus. **[2]**

(b) Calculate, for nitrogen-14: (i) the mass defect, in kg **[2]** (ii) the binding energy, in J and in MeV **[2]** (iii) the binding energy per nucleon, in MeV. **[1]**

Solution

(a) The energy needed to separate all the nucleons of the nucleus **M1** to infinity. **A1**

(b)(i) $\Delta m = (7 \times 1.007\,276 + 7 \times 1.008\,665) - 13.999\,234 = 0.112\,353$ u **C1** $= 0.112\,353 \times 1.66 \times 10^{-27} = 1.87 \times 10^{-28}$ kg. **A1**

(ii) $E = \Delta m\, c^2 = 1.87 \times 10^{-28} \times (3.00 \times 10^8)^2 = 1.68 \times 10^{-11}$ J **C1** $= \dfrac{1.68 \times 10^{-11}}{1.60 \times 10^{-13}} = 104.9$ MeV, about 105 MeV. **A1**

(iii) $\dfrac{104.9}{14} = 7.49$ MeV. **A1**

Example 4

A uranium-235 nucleus absorbs a neutron and undergoes fission:

$$^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + x\,{}^{1}_{0}\mathrm{n}$$

The binding energies per nucleon are: uranium-235, 7.59 MeV; barium-141, 8.33 MeV; krypton-92, 8.51 MeV.

(a) Determine x. **[1]**

(b) Calculate the energy, in MeV, released in one fission. **[2]**

(c) Calculate the energy, in J, released when 1.0 g of uranium-235 undergoes fission in this way. **[2]**

(d) Explain, with reference to the variation of binding energy per nucleon with nucleon number, why fission of uranium-235 releases energy. **[2]**

Solution

(a) $1 + 235 = 141 + 92 + x$, so $x = 3$. **A1**

(b) $E = (141 \times 8.33 + 92 \times 8.51) - 235 \times 7.59$ **C1** $= 1957.45 - 1783.65 = 174$ MeV. **A1**

(c) $N = \dfrac{1.0 \times 10^{-3}}{235 \times 1.66 \times 10^{-27}} = 2.56 \times 10^{21}$ **C1** $E = 2.56 \times 10^{21} \times 174 \times 1.60 \times 10^{-13} = 7.13 \times 10^{10}$ J. **A1**

(d) Binding energy per nucleon is greatest for intermediate nucleon numbers (near $A = 56$); uranium-235 is far to the right of the peak. **B1** The two products have smaller nucleon numbers and a higher binding energy per nucleon, so the total binding energy increases and energy is released. **B1**

Common mistakes

- **Vague "random".** In November 2021 and March 2025 many wrote that "decay is unpredictable" without saying **what**: it is the **time at which a particular nucleus** decays. Decay of a large sample is very predictable. Some also swapped the meanings of random and spontaneous (November 2021), or used the word being defined (March 2021); say **nucleus** in a definition of radioactive decay (November 2022, June 2022).
- **Half-life as "the time for the nuclei to halve".** June 2023 and March 2025: the number of nuclei often doesn't change at all (they just become daughter nuclei). Use "the time for the **activity** to halve", or say **undecayed** nuclei.
- **Decay constant without "per unit time"**, or confused with half-life (November 2021).
- N **from a mass.** March 2021 and November 2021: N confused with mass or moles, and answers out by 1000 from mixing grams and kilograms. Also (March 2023) years not converted to seconds while giving the activity in Bq, and λ rounded too early, so the final answer was not accurate to 3 significant figures.
- **Power from a source** found as energy ÷ half-life (March 2023). Power is activity × energy per decay.
- **Log graphs.** November 2022: the minus sign lost when showing the gradient is $-\lambda$; the 10^{-16} in the axis label put into the calculation; logs taken of values that were already logs. March 2022: the half-life taken as half the time axis.
- **Binding energy curve drawn wrongly** (June 2022): straight lines through the origin, or lines falling from a positive intercept. It rises steeply to one peak near 56 and then falls gently.
- **"Binding energy decreases" in fusion or fission** (June 2022). It **increases**; that is where the energy comes from. And "energy to split the nucleons" is too vague for binding energy: say **separate** the nucleons **to infinity**.
- **Per-nucleon values used as totals** (March 2025), or a **negative** energy released from subtracting the wrong way round. Multiply by nucleon numbers, then give the energy released as positive.
- **One reaction instead of a mole** (June 2022): count how many nuclei each reaction uses.

- **"Mass is conserved"** (November 2022). Mass–energy is conserved; when energy is released, the products have less mass.
- **The daughter's curve** (June 2023): many drew the decay curve instead of the growth curve for the stable product.
- **Count rate and activity** (June 2023, November 2021, March 2021): the count rate is always **less** than the activity; randomness, background radiation and a radioactive daughter are wrong reasons for this. A radioactive daughter can make the activity **higher** than expected.
- **Energy of the recoil** (March 2022): the alpha particle's energy is less than the energy released because the daughter nucleus recoils; it is nothing to do with randomness.

How to get the marks

- **Definitions**: every bold phrase in type 1 is a mark. Quantities, not units ("per unit time", not "per second").
- **Calculations**: the **C1** marks are for the right equation with the numbers in, so write $A = \lambda N$ or $E = \Delta m\, c^2$ before substituting. Convert u to kg, MeV to J, years or hours to seconds.
- **"Give a unit"**: Bq for activity, s^{-1} (or min^{-1}, matching the time) for λ, J or MeV for energy, kg or u for mass.
- **"Show that"**: show every substitution and give the answer to more significant figures than printed.
- **Nucleon number**: a whole number.
- **Graph sketches**: start and end points, the curve's key points (halves, quarters), a gradient that keeps getting less steep, and no turning up or touching zero. On a binding-energy sketch: shape, peak position, and steep left / gentle right are marked separately.
- **Reading $\ln$ graphs**: use a large triangle on the line, not data points off it.
- **"Explain, with reference to the curve"**: name where the nuclei are on the curve **and** say the binding energy per nucleon **increases**.
- **"Suggest one advantage and one disadvantage"**: explain each one's consequence (a shorter half-life means the power falls sooner), not just restate the data given.

Quick check

1. Thorium-234 ($^{234}_{90}\mathrm{Th}$) decays by β^- emission. State the nucleon number and proton number of the nucleus formed, and name the other particle emitted.
2. An isotope has a half-life of 8.0 days. Calculate the fraction of its nuclei left undecayed after 20 days.
3. In the fusion reaction $^2_1\mathrm{H} + ^3_1\mathrm{H} \rightarrow ^4_2\mathrm{He} + ^1_0\mathrm{n}$, the masses are: $^2_1\mathrm{H}$ $2.013\,553$ u, $^3_1\mathrm{H}$ $3.015\,501$ u, $^4_2\mathrm{He}$ $4.001\,506$ u, neutron $1.008\,665$ u. Calculate the energy released, in J and in MeV.
4. A sealed source of cobalt-60 (half-life 5.27 years) has an activity of 5.0×10^6 Bq. Calculate the mass of cobalt-60 in it.
5. A detector 20 cm from a small radioactive source records a count rate much smaller than the source's activity. Give two reasons.

Answers

1. ${}^{234}_{90}\mathrm{Th} \rightarrow {}^{234}_{91}\mathrm{Pa} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$: nucleon number 234, proton number 91; an (electron) antineutrino is also emitted.
2. $\dfrac{N}{N_0} = e^{-\ln 2 \times 20/8.0} = \left(\frac{1}{2}\right)^{2.5} = 0.177$.
3. $\Delta m = (2.013\,553 + 3.015\,501) - (4.001\,506 + 1.008\,665) = 0.018\,883$ u. $E = 0.018\,883 \times 1.66 \times 10^{-27} \times (3.00 \times 10^{8})^{2} = 2.82 \times 10^{-12}$ J $= 17.6$ MeV. That is about 3.5 MeV per nucleon taking part, several times more than the 0.74 MeV per nucleon in the fission of Example 4.
4. $\lambda = \dfrac{0.693}{5.27 \times 365 \times 24 \times 3600} = 4.17 \times 10^{-9}$ s^{-1}. $N = \dfrac{A}{\lambda} = \dfrac{5.0 \times 10^{6}}{4.17 \times 10^{-9}} = 1.199 \times 10^{15}$. Mass $=$ $1.199 \times 10^{15} \times 60 \times 1.66 \times 10^{-27} = 1.19 \times 10^{-10}$ kg.
5. Any two: the radiation is emitted in all directions, so only a small fraction travels towards the detector; some is absorbed in the source, the air or the detector's window; some that reaches the detector is not registered.

Past questions to try

- **9702/42/F/M/22 Q9**: an alpha-decay equation, λ from a graph of N against t, the daughter's growth curve, and energy released between two times.
- **9702/43/O/N/23 Q9**: sketching the binding energy per nucleon curve, marking nuclei on it, and a binding energy per nucleon from fission data.
- **9702/42/F/M/23 Q8**: a plutonium power source: number of nuclei, activity, power output and how long it lasts.
- **9702/43/M/J/22 Q8**: binding energy, the curve, why fusion releases energy, and the energy from a mole of deuterium.