Cambridge AS & A Level · Physics 9702 · Notes

Oscillations A2

Read online at **pastopic.com/cambridge/physics-9702/notes/oscillations**

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Read first: **Motion in a circle**, **Waves**

What you need to know

By the end of this topic you should be able to:

1. **Use the words that describe an oscillation**: displacement, amplitude, period, frequency, angular frequency and phase difference. Write the period in terms of the frequency ($T = \frac{1}{f}$) and in terms of the angular frequency ($T = \frac{2\pi}{\omega}$).
2. **Say what simple harmonic motion (s.h.m.) is**: motion in which the acceleration is proportional to the displacement from a fixed point and always directed in the opposite direction, back towards that point.
3. **Use** $a = -\omega^2 x$, and recall and use $x = x_0 \sin \omega t$ as a solution of it.
4. **Use the velocity equations** $v = v_0 \cos \omega t$ and $v = \pm \omega \sqrt{x_0^2 - x^2}$.
5. **Read and explain graphs** of displacement, velocity and acceleration in s.h.m., against time or against displacement.
6. **Describe how kinetic and potential energy swap** during s.h.m. while their total stays the same.
7. **Recall and use** $E = \frac{1}{2} m \omega^2 x_0^2$ for the total energy of an oscillating system.
8. **Explain damping**: a resistive force acting on an oscillating system takes energy away and damps the oscillations.
9. **Use the terms light, critical and heavy damping**, and sketch displacement–time graphs for each.
10. **Explain resonance**: when a system is forced to oscillate, its amplitude is a maximum when the driving frequency equals its natural frequency.

You need radians and angular speed from Motion in a circle, and period, frequency and phase difference from Waves. You do **not** need to learn the formulas for the period of a pendulum or of a mass on a spring: when a question needs one, it gives you the acceleration in the form $a = -(\text{constant})\, x$.

Understand it

Describing an oscillation

An oscillation is a motion that repeats itself, backwards and forwards about an **equilibrium position** (the place where the object would rest if left alone): a pendulum bob, a mass bouncing on a spring, a block floating in water.

- **Displacement** x: the distance from the equilibrium position, in a stated direction, so it can be positive or negative.
- **Amplitude** x_0: the **maximum** displacement. It is measured from the equilibrium position, so it is **half** the distance between the two ends of the motion.
- **Period** T: the time for one complete oscillation (there and back again).
- **Frequency** f: the number of oscillations per unit time. $f = \frac{1}{T}$, in hertz (Hz).
- **Angular frequency** ω: $\omega = 2\pi f = \frac{2\pi}{T}$, in rad s^{-1}. One oscillation is treated as one turn of 2π radians, just as for motion in a circle.
- **Phase difference** ϕ: how far one oscillation is ahead of another, as an angle. If one reaches its maximum a time Δt after the other, $\phi = \frac{\Delta t}{T} \times 2\pi$ rad (or $\times 360^\circ$). Half a cycle apart is π rad; a quarter of a cycle is $\frac{\pi}{2}$ rad.

What makes motion simple harmonic

When an object is pulled away from its equilibrium position, a **restoring force** pushes it back. In **simple harmonic motion**, this force, and so the acceleration, is:

1. **proportional to the displacement** (twice as far away, twice the acceleration), and
2. **always in the opposite direction** to the displacement, towards the equilibrium position.

Both ideas are in one equation, the **defining equation of s.h.m.**:

$$a = -\omega^2 x$$

a is the acceleration, x the displacement **from the equilibrium position**, and ω the angular frequency. ω^2 is a constant, so $a \propto x$; the **minus sign** shows that a is always in the opposite direction to x.

Real examples: a mass on a spring, a pendulum swinging through a small angle, liquid sloshing in a U-tube, a floating block bobbing up and down. For each, a question gives you an equation such as $a = -\left(\frac{2k}{m}\right) x$ or $a = -\left(\frac{g}{L}\right) x$. Compare it with $a = -\omega^2 x$: the bracket is a positive constant, so the motion is s.h.m., and the bracket **equals** ω^2 (not ω).

The period does not depend on the amplitude. ω is fixed by the system (the spring and the mass, or the length of the pendulum), not by how far you pull it. Pull it twice as far and it moves twice as fast, taking the same time for each oscillation.

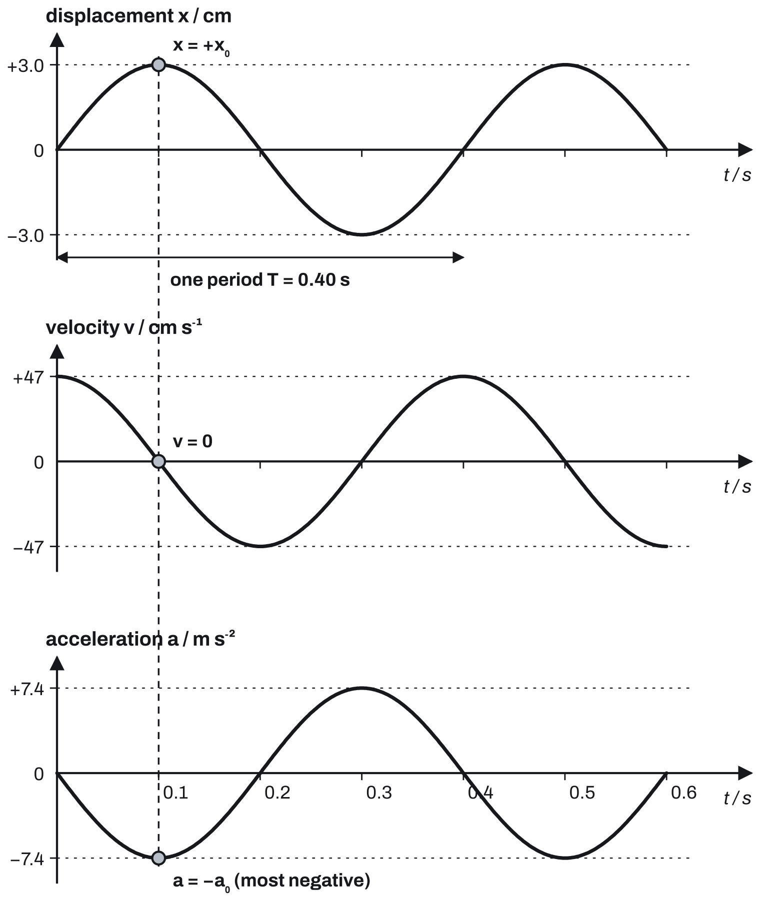
Displacement, velocity and acceleration against time

A solution of $a = -\omega^2 x$ is

$$x = x_0 \sin \omega t,$$

which starts at the equilibrium position ($x = 0$ when $t = 0$) moving in the positive direction. The velocity and acceleration follow from it:

$$v = v_0 \cos \omega t \quad \text{with } v_0 = \omega x_0, \qquad a = -\omega^2 x = -a_0 \sin \omega t \quad \text{with } a_0 = \omega^2 x_0.$$



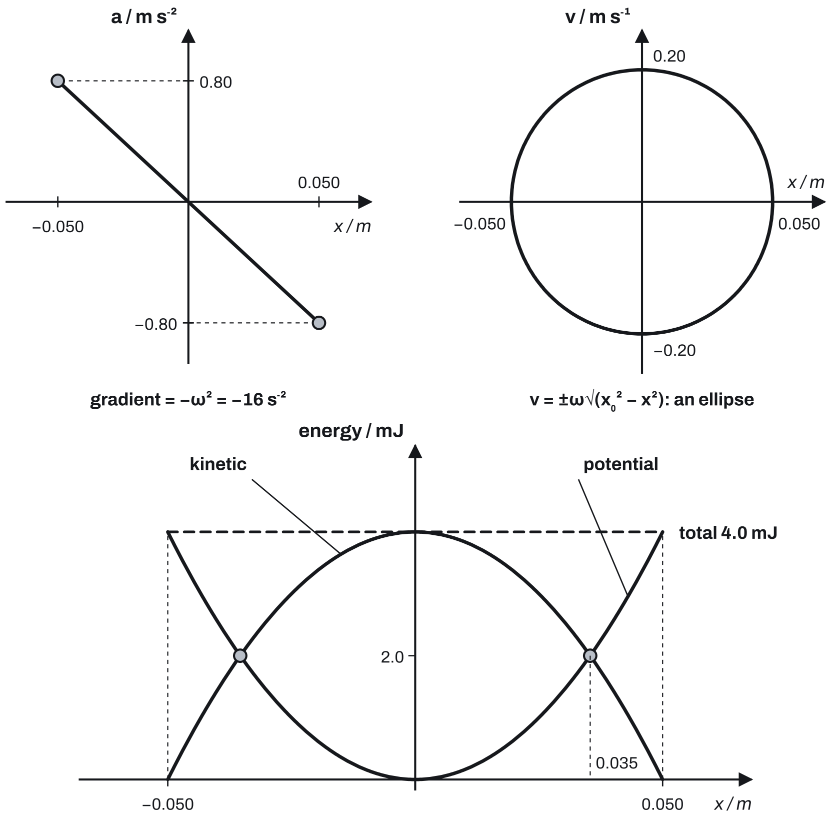
Read the graphs together:

- At the **ends** of the motion ($x = \pm x_0$), the object stops for an instant, so $v = 0$, and the acceleration is **largest** ($\mp a_0$), pointing back to the middle.
- At the **equilibrium position** ($x = 0$), the speed is **largest** (v_0) and the acceleration is **zero**.
- The velocity is a **quarter of a cycle** ($\frac{\pi}{2}$ rad) ahead of the displacement, and the acceleration is **half a cycle** (π rad) out of phase with the displacement.
- The velocity is the gradient of the displacement–time graph, and the acceleration is the gradient of the velocity–time graph.
- If the timing starts somewhere else, only the starting point changes. Released from rest at maximum displacement, $x = x_0 \cos \omega t$; released at the lowest point, the displacement–time graph starts at a minimum.

- The object passes through the equilibrium position **twice** each cycle, once each way. Two times when it passes **in the same direction** are one whole period apart.

Graphs against displacement

Plotting against x instead of t gives three shapes worth knowing. The diagram uses a 0.20 kg mass with amplitude 0.050 m and $\omega = 4.0$ rad s^{-1}, so $a_0 = 4.0^2 \times 0.050 = 0.80$ m s^{-2}, $v_0 = 4.0 \times 0.050 = 0.20$ m s^{-1}, and total energy $\frac{1}{2} \times 0.20 \times 4.0^2 \times 0.050^2 = 4.0 \times 10^{-3}$ J.



- **Acceleration against displacement** is a **straight line through the origin** (so $a \propto x$) with a **negative gradient** (so a is opposite to x). That is exactly what makes it s.h.m. The gradient is $-\omega^2$, so $\omega = \sqrt{\text{magnitude of the gradient}} = \sqrt{\frac{a_0}{x_0}}$. The line **stops at** $\pm x_0$: the object never goes further.
- **Velocity against displacement** is a **closed loop** (an ellipse) round the origin, from $v = \pm\,\omega\sqrt{x_0^2 - x^2}$. It meets the x-axis at $\pm x_0$ (at rest at the ends) and the v-axis at $\pm v_0$ (fastest in the middle). The ratio $\frac{v_0}{x_0}$ gives ω. For the same ω and half the amplitude, both intercepts halve.
- **Energy against displacement** comes next.

Energy in s.h.m.

As the object oscillates, energy moves back and forth between **kinetic energy** (of its motion) and **potential energy** (stored in the spring, or gravitational, or both):

- at the **equilibrium position**, kinetic energy is a **maximum** and potential energy is **zero**;

- at **maximum displacement**, kinetic energy is **zero** and potential energy is a **maximum**;
- at every point in between, **kinetic + potential = total energy**, which is **constant** if there is no damping.

The total energy equals the maximum kinetic energy, $\frac{1}{2}mv_0^2$, and $v_0 = \omega x_0$, so

$$E = \tfrac{1}{2}m\omega^2 x_0^2.$$

At displacement x, the kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2)$ and the potential energy is $\frac{1}{2}m\omega^2x^2$. So on an energy–displacement graph the potential energy is a **U-shaped parabola** and the kinetic energy an **upside-down U**, both reaching the total at the ends of their range. They are equal (each half the total) at $x = \frac{x_0}{\sqrt{2}} \approx 0.71x_0$, **not** at half the amplitude, because the energies depend on x^2.

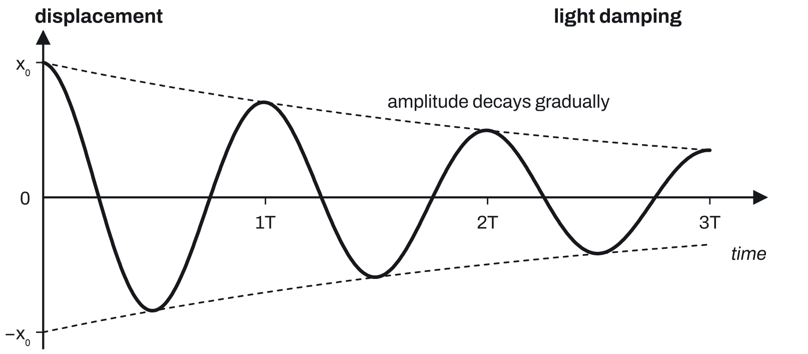
Against **time**, the kinetic energy rises and falls **twice** in each period (it is a maximum each time the object passes through the middle, whichever way it is going), so the energy–time graph repeats every $\frac{T}{2}$.

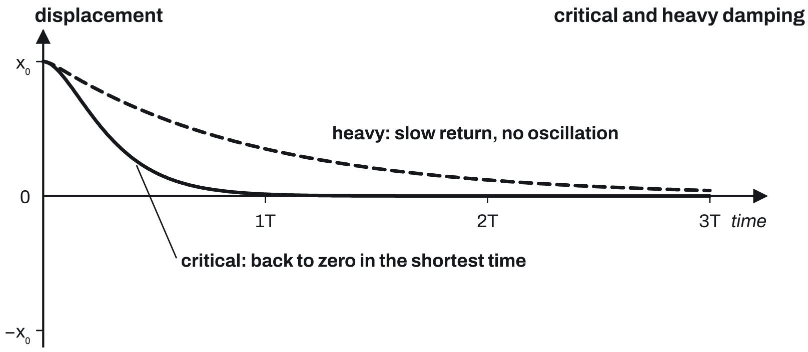
For a mass bouncing on a vertical spring, "the potential energy of the oscillations" is the elastic and gravitational potential energy **together**. Use $E = \frac{1}{2}m\omega^2x_0^2$ for the whole system, not mgh alone.

$E \propto x_0^2$: doubling the amplitude gives **four times** the energy. And if the energy stays constant while ω changes (for example, a pendulum whose string is slowly lengthened, so ω falls), then $\omega^2x_0^2$ stays constant, so the amplitude must **increase**.

Damping

In practice every oscillation dies away. **Damping** is the **loss of energy** from an oscillating system caused by a **resistive force** (air resistance, friction, drag in a liquid, or the force from induced currents), which always acts against the motion. The energy goes to thermal energy in the surroundings. Because $E \propto x_0^2$, losing energy means the amplitude falls.





- **Light damping**: the system **still oscillates**, with about the same period, but the amplitude **gradually** decreases. The motion is still (damped) simple harmonic.
- **Critical damping**: the system returns to equilibrium in the **shortest possible time** without oscillating (it does not pass through $x = 0$). Car suspensions are designed to be close to this.
- **Heavy damping**: the resistive force is so large that the system does not oscillate at all, and creeps back to equilibrium **slowly**, taking much longer than with critical damping.

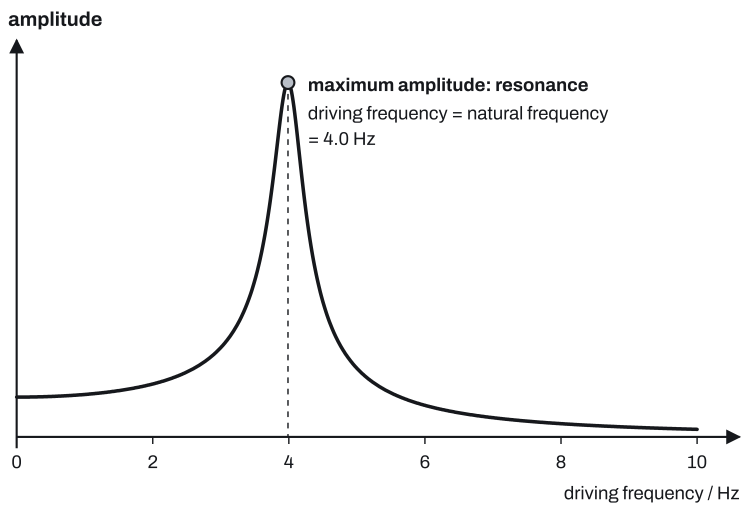
To sketch them from maximum displacement at $t = 0$: light damping crosses the axis every half period with peaks getting smaller; critical and heavy damping stay on one side of the axis, both with the displacement and the gradient getting smaller, critical reaching zero within about one period, heavy still well away from zero after a few periods.

Energy lost each cycle. If the energy falls by a fixed fraction every cycle, multiply by the fraction left once for each cycle. A system losing 5.0% of its energy each cycle keeps 0.95 of it per cycle, so after 4 cycles it keeps $0.95^4 = 0.815$ of the original energy and has lost 18.5%. If instead the **amplitude** falls by 5.0% each cycle, the energy keeps 0.95^2 per cycle, because $E \propto x_0^2$.

Magnetic damping. A magnet oscillating in a coil connected to a resistor induces an e.m.f. in the coil, which drives a current; the current heats the resistor, and that thermal energy comes from the energy of the oscillations. A larger resistance means a smaller current, so energy is dissipated at a **lower rate** and the damping is lighter.

Forced oscillations and resonance

Left alone after a push, a system oscillates at its own **natural frequency** f_0. A **forced oscillation** is one where a periodic driving force keeps pushing it at the **driving frequency**, and the system then oscillates at the driving frequency.



The driver feeds energy in most effectively when it pushes in step with the natural oscillation.

Resonance is the maximum amplitude of oscillation, which happens when the **driving frequency equals the natural frequency**. The driver then pushes in step with the motion every cycle, so it transfers energy most effectively.

When a forced oscillation settles to a **constant amplitude**, the energy supplied by the driver each cycle equals the energy removed by resistive forces each cycle. If there were no resistive forces, the energy, and the amplitude, would keep growing.

Key facts and formulas

"Given" means it is on the Paper 4 data sheet. "Learn it" means you must remember it.

Formula	Status
$a = -\omega^2 x$	Given
$v = v_0 \cos \omega t$	Given
$v = \pm\,\omega\sqrt{x_0^2 - x^2}$	Given
$x = x_0 \sin \omega t$ (the same form is on the data sheet, but only as the a.c. formula)	Learn it
$T = \dfrac{1}{f}, \omega = 2\pi f = \dfrac{2\pi}{T}$	Learn it
Maximum speed $v_0 = \omega x_0$; maximum acceleration $a_0 = \omega^2 x_0$	Learn it
Total energy $E = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2}mv_0^2$	Learn it
Kinetic energy at x: $\frac{1}{2}m\omega^2(x_0^2 - x^2)$; potential energy at x: $\frac{1}{2}m\omega^2 x^2$	Learn it
Phase difference $\phi = \dfrac{\Delta t}{T} \times 2\pi$ rad	Learn it
Definition of s.h.m.: acceleration proportional to displacement from a fixed point and in the opposite direction	Learn it
Resonance: maximum amplitude when driving frequency = natural frequency	Learn it

How to do it

In the Paper 4 questions we have tagged for this topic (26 different questions, 2021–2025; 8 more appear on two papers), the types came up this often. 25 of the 26 questions mix two or more types, so the counts add up to more than 26.

Question type	Questions
Period, frequency and angular frequency	25

Question type	Questions
Amplitude, maximum speed and acceleration, and equations of motion	16
Showing or stating that motion is simple harmonic	15
Energy in simple harmonic motion	15
Damping	10
Sketching graphs of simple harmonic motion	9
Resonance	3
Phase difference	1

1. Period, frequency and angular frequency (25 questions)

1. **Find whichever of T, f or ω the information gives you**, then convert with $f = \frac{1}{T}$ and $\omega = \frac{2\pi}{T} = 2\pi f$.
2. Give ω in rad s^{-1}, f in Hz and T in s, to 2 or 3 significant figures.

Where the information comes from:

- **A displacement–time (or velocity–time) graph**: measure several whole cycles and divide, for example 3 cycles in 1.2 s gives $T = 0.40$ s. Check you are measuring a **whole** cycle (peak to peak), not half.
- **Two times of passing through equilibrium in the same direction** are one period apart. If a time interval covers more than one cycle, count the cycles: 3.5 cycles in 2.8 s gives $T = 0.80$ s.
- **An energy–time graph**: the kinetic energy peaks twice per cycle, so the period of the oscillation is **twice** the time between kinetic energy peaks.
- **An acceleration–displacement graph**: $\omega^2 =$ magnitude of the gradient $= \frac{a_0}{x_0}$. Read the end point of the line, convert cm to m if the axes differ, and **take the square root**.
- **An equation**: in $x = 0.40 \sin 3.0\pi t$ or $v = 0.72 \cos 20t$, the number multiplying t is ω. In a given $a = -(\ldots)x$, the bracket is ω^2.
- **Maximum speed and amplitude**: $\omega = \frac{v_0}{x_0}$. **Maximum acceleration and amplitude**: $\omega = \sqrt{\frac{a_0}{x_0}}$.
- **Energy**: rearrange $E = \frac{1}{2} m\omega^2 x_0^2$ for ω (mass in kg, amplitude in m).

Variations you will meet:

- **"Show that ω is 2.5 rad s^{-1}"**: write the equation, substitute the numbers you read, and give the answer to one more significant figure than the value printed.
- **A constant from a given relation**, such as $\omega^2 = \frac{k}{L}$ for a pendulum: $k = \omega^2 L$, with units from the equation (here m s^{-2}).
- **"Explain the effect on the period of a smaller amplitude"**: no effect, because ω depends only on the constant in $a = -\omega^2 x$ (the spring constant and the mass), not on the amplitude.

2. Amplitude, maximum speed and acceleration, and equations of motion (16 questions)

1. **Amplitude**: read the largest displacement from the equilibrium position. If the graph shows a **position** (height h above the floor, or length L from a wall), the equilibrium position is the **middle** of the range and the amplitude is **half the range**: between 3.0 cm and 9.0 cm the amplitude is 3.0 cm about 6.0 cm.
2. **Maximum speed** $v_0 = \omega x_0$; **maximum acceleration** $a_0 = \omega^2 x_0$. Use the amplitude in metres for answers in m s^{-1} or m s^{-2}.
3. **Speed at a displacement** x: $v = \omega\sqrt{x_0^2 - x^2}$.
4. **An equation for x, v or a in terms of t**: put in your numbers, for example $x = 1.5\sin(12t)$ with x in cm, or $v = 0.30\cos(12t)$ in m s^{-1}. Use $\sin$ if timing starts at equilibrium, $\cos$ if it starts at the maximum displacement.

Variations you will meet:

- **Amplitude from a velocity equation**: $v = 0.30\cos 12t$ gives $v_0 = 0.30$ m s^{-1} and $\omega = 12$ rad s^{-1}, so $x_0 = \frac{0.30}{12} = 0.025$ m. Then $v = \pm 12\sqrt{0.025^2 - x^2}$.
- **Amplitude from $v = \pm\,\omega\sqrt{c - x^2}$**: the number c under the root is x_0^2, so $x_0 = \sqrt{c}$.
- **Resultant force on release**: with springs, the resultant force is the **difference** of the spring forces, each $k\times$ extension (use the **extension**, not the length). Then $a_0 = \frac{F}{m}$ and $\omega = \sqrt{\frac{a_0}{x_0}}$.
- **Answers in symbols** (for example amplitude $2P$, maximum acceleration $5Q$): $\omega = \sqrt{\frac{5Q}{2P}}$, $v_0 = 2P\omega = \sqrt{10PQ}$, $E = \frac{1}{2}m\omega^2(2P)^2 = 5mPQ$. Use only the letters the question allows.
- **An object resting on a vibrating platform** stays in contact only while its downward acceleration at the top is no more than g. The largest amplitude is where $a_0 = \omega^2 x_0 = g$, so $x_0 = \frac{g}{\omega^2}$. Above it, contact is lost first at the **top** of the oscillation, where the downward acceleration is greatest.
- **"State three conclusions from these graphs"**: any three of: the motion is s.h.m. (straight a–x line through the origin), the amplitude, the equilibrium position, ω, T or f, the maximum speed, the total energy (maximum kinetic energy), the mass ($m = \frac{2E}{v_0^2}$), the maximum momentum (mv_0). Give **numbers** with units when it says "quantitative".

3. Showing or stating that motion is simple harmonic (15 questions)

1. **Definition**: acceleration is **proportional to displacement** (from a fixed point) **and** in the **opposite direction** to the displacement (towards the fixed point). Two ideas, two marks.
2. **Defining equation**: $a = -\omega^2 x$, naming every symbol: a acceleration, x displacement **from the equilibrium position**, ω angular frequency.
3. **"Explain how the graph shows s.h.m."**: link each feature to each idea:
   - the line is **straight and passes through the origin**, so a is proportional to x;
   - the gradient is **negative**, so a is in the opposite direction to x.
4. **"Explain how the expression shows s.h.m."**, for example $a = -\left(\frac{2k}{m}\right)x$: the bracket is a **constant**, so $a \propto x$; the **minus sign** shows a is opposite to x.

Variations you will meet:

- **"State the significance of the minus sign"**: only one idea, that the acceleration is in the opposite direction to the displacement (towards the equilibrium position).
- **Force instead of acceleration**, such as $F = -kx$: $F = ma$, so $a = -\frac{k}{m}x$ with $\frac{k}{m}$ constant; hence s.h.m. with $\omega = \sqrt{\frac{k}{m}}$.
- **Direction of the resultant force** at a marked position: always **towards the equilibrium position** (for a pendulum, at right angles to the string, towards the lowest point).

4. Energy in simple harmonic motion (15 questions)

1. **Total energy**: $E = \frac{1}{2}m\omega^2 x_0^2$ (or $\frac{1}{2}mv_0^2$). Mass in kg, amplitude in m. **Square both ω and x_0.**
2. **Kinetic energy at** x: $\frac{1}{2}m\omega^2(x_0^2 - x^2)$; potential energy $=$ total $-$ kinetic.
3. **Rearrange** for what is asked: $m = \frac{2E}{\omega^2 x_0^2}$, $\omega = \sqrt{\frac{2E}{mx_0^2}}$.

Variations you will meet:

- **"Describe the interchange of kinetic and potential energy"**: kinetic is maximum and potential zero at the equilibrium position; kinetic is zero and potential maximum at maximum displacement; the sum stays constant. Say "displacement", not "amplitude", for the position.
- **Identify lines on an energy–displacement graph**: horizontal = total; U-shape (zero in the middle) = potential; upside-down U = kinetic.
- **Amplitude from the total distance travelled**: in one oscillation the object travels $4x_0$ (out, back, out the other way, back).
- **Where kinetic energy equals potential energy**: at $x = \frac{x_0}{\sqrt{2}}$, about $0.71x_0$.
- **Energy lost between two amplitudes** in damped motion: $\Delta E = \frac{1}{2}m\omega^2(x_1^2 - x_2^2)$, using **both** amplitudes.
- **Energy constant, ω changes**: $\omega^2 x_0^2$ is constant, so a smaller ω means a larger amplitude.

5. Damping (10 questions)

1. **Meaning**: **loss of energy** of the oscillations **due to resistive forces**. Both halves are needed.
2. **Type**: light (still oscillates, amplitude decreases gradually), critical (back to equilibrium in the shortest time, no oscillation), heavy (no oscillation, slow return).
3. **Reason for "light"**: the system **continues to oscillate** for several cycles. "Energy is lost slowly" is not enough, since that is also true of heavy damping.

Variations you will meet:

- **Energy after n cycles**, losing a fraction p of the energy each cycle: energy left $= E_0(1-p)^n$, and energy lost $= E_0 - E_0(1-p)^n$. Never multiply p by n.
- **Amplitude falls by a fraction p each cycle**: amplitude left $= x_0(1-p)^n$, energy left $= E_0(1-p)^{2n}$.

- **Sketching heavy or critical damping** from maximum displacement: start at x_0, stay on one side of the axis, and make both the displacement and the gradient decrease continuously. Critical reaches zero within about one period.
- **Damping starting part-way through** (a blower switched off, a ball lowered into oil): the amplitude is constant before and decreases after; the period stays about the same.
- **Magnetic damping**: moving magnet cuts the flux linking the coil, so an e.m.f. is induced; the e.m.f. drives a current; the current dissipates thermal energy in the resistor; that energy comes from the oscillations. With a larger resistance the current, and the rate of energy loss, is smaller, so the oscillations last longer.

6. Sketching graphs of simple harmonic motion (9 questions)

1. **Decide the shape** from the "Understand it" section: sine or cosine curve against time; straight line through the origin with negative gradient for a against x; ellipse for v against x; parabolas for energy against x.
2. **Mark the key values** from earlier parts: $\pm x_0$, $\pm v_0$, $\pm a_0$, T, the total energy.
3. **Get the start right**: at maximum displacement, at equilibrium, or at the lowest point (a minimum on a height–time graph).

Variations you will meet:

- **Against position instead of displacement** (height h, length L): shift everything so the centre is at the equilibrium position. Kinetic energy against h is an upside-down U, zero at the two ends of the motion and maximum at the equilibrium position; potential energy is a U with its minimum there.
- **v against x for a smaller amplitude, same ω**: a smaller ellipse, with both intercepts scaled down by the same factor.
- **a against x**: a line segment, **ending** at $(\pm x_0, \mp a_0)$. Don't draw a loop (that is v against x), and don't run it past the amplitude.
- **Labelling the scales** of a displacement–time graph from an equation: mark the amplitude on both the positive and negative halves, and use the whole period, not half, for one cycle.

7. Resonance (3 questions)

1. **Meaning**: the amplitude of a forced oscillation is a **maximum** when the **driving frequency equals the natural frequency**.
2. **Sketch** of amplitude against driving frequency: small non-zero amplitude at low frequency, a **single peak** at the natural frequency, falling off either side.

Variations you will meet:

- **Natural frequency from a free-oscillation graph** first (for example $T = 0.20$ s gives 5.0 Hz), then mark the peak there.
- **Constant amplitude at a fixed driving frequency** shows there are resistive forces: the driver supplies energy every cycle, and since the energy of the oscillations is not increasing, the same energy must be dissipated.

8. Phase difference (1 question)

1. Find the time Δt between matching points (two peaks, or two zero crossings going the same way) of the two oscillations, and the period T.
2. $\phi = \frac{\Delta t}{T} \times 2\pi$ rad. Give a unit (rad or °). A phase difference is an **angle**, not a time.

For example, with $T = 0.60$ s and peaks 0.15 s apart, $\phi = \frac{0.15}{0.60} \times 2\pi = \frac{\pi}{2} = 1.57$ rad.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A trolley of mass 0.20 kg is attached to springs and oscillates horizontally. The graph of its acceleration a against its displacement x is a straight line from $(-0.050\ \mathrm{m},\ 0.80\ \mathrm{m\ s^{-2}})$ to $(0.050\ \mathrm{m},\ -0.80\ \mathrm{m\ s^{-2}})$, as in the top left of the second diagram.

(a) Explain how the graph shows that the motion is simple harmonic. **[2]**

(b) Determine the period of the oscillations. **[3]**

(c) Calculate the maximum speed of the trolley. **[2]**

(d) Calculate the total energy of the oscillations. **[2]**

(e) Calculate the speed of the trolley when its displacement is 0.030 m. **[2]**

Solution

(a) The line is straight and passes through the origin, so a is proportional to x **B1**. The gradient is negative, so a is always in the opposite direction to x **B1**.

(b) $a_0 = \omega^2 x_0$, so $\omega = \sqrt{\dfrac{0.80}{0.050}} = \sqrt{16} = 4.0$ rad s^{-1} **C1** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.0}$ **C1** $T = 1.57$ s **A1**

(c) $v_0 = \omega x_0 = 4.0 \times 0.050$ **C1** $v_0 = 0.20$ m s^{-1} **A1**

(d) $E = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2} \times 0.20 \times 4.0^2 \times 0.050^2$ **C1** $E = 4.0 \times 10^{-3}$ J **A1**

(e) $v = \omega\sqrt{x_0^2 - x^2} = 4.0\sqrt{0.050^2 - 0.030^2} = 4.0 \times 0.040$ **C1** $v = 0.16$ m s^{-1} **A1**

Example 2

A point on a vibrating strip moves with simple harmonic motion of amplitude 3.0 cm and period 0.40 s. At time $t = 0$ it passes through its equilibrium position, moving in the positive direction.

(a) Calculate the angular frequency. **[2]**

(b) Write an equation for the displacement x, in cm, in terms of t, in s. **[1]**

(c) Calculate the maximum acceleration. **[2]**

(d) Determine the displacement and the speed of the point at $t = 0.050$ s. **[3]**

(e) A second point oscillates with the same period, but reaches its maximum displacement 0.10 s after the first. Calculate the phase difference between them. **[2]**

Solution

(a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.40}$ **C1** $\omega = 15.7$ rad s^{-1} **A1**

(b) $x = 3.0\sin(15.7t)$ **B1**

(c) $a_0 = \omega^2 x_0 = 15.71^2 \times 0.030$ **C1** $a_0 = 7.40$ m s^{-2} **A1**

(d) $\omega t = 15.71 \times 0.050 = 0.785$ rad (a calculator in **radians**) **C1** $x = 3.0\sin 0.785 = 2.12$ cm **A1** $v = v_0 \cos\omega t = (15.71 \times 0.030)\cos 0.785 = 0.471 \times 0.707 = 0.333$ m s^{-1} **A1** (Check: $v = \omega\sqrt{x_0^2 - x^2} = 15.71\sqrt{3.0^2 - 2.12^2} = 33.3$ cm s^{-1}.)

(e) $\phi = \frac{\Delta t}{T} \times 2\pi = \frac{0.10}{0.40} \times 2\pi$ **C1** $\phi = 1.57$ rad (a quarter of a cycle, 90°) **A1**

Example 3

A glider on an air track is held between two springs. It is displaced and released, and oscillates with light damping. The amplitude decreases by 4.0% of its value at the start of each cycle.

(a) State what is meant by damping. **[2]**

(b) State how the motion shows that the damping is light. **[1]**

(c) Determine the fraction of the initial energy of the oscillations that remains after 5 complete cycles. **[3]**

(d) The natural frequency of the glider is 1.6 Hz. A vibrator is attached to one spring, and its frequency is slowly increased from 0.5 Hz to 3.0 Hz. Describe and explain how the amplitude of the glider's oscillations changes. **[3]**

(e) At one fixed frequency of the vibrator, the amplitude of the glider stays constant. Explain, in terms of energy, why this shows that resistive forces act on the glider. **[2]**

Solution

(a) Loss of energy of the oscillations **B1** due to resistive forces **B1**.

(b) The glider continues to oscillate (for many cycles) while the amplitude decreases gradually **B1**.

(c) Amplitude after 5 cycles $= 0.96^5 x_0 = 0.815 x_0$ **C1** Energy $\propto$ amplitude^2, so the fraction left $= 0.96^{10}$ **C1** Fraction $= 0.66$ **A1**

(d) The amplitude increases to a maximum at 1.6 Hz, then decreases **B1**. The maximum is resonance **B1**: the driving frequency equals the natural frequency, so the vibrator transfers energy most effectively **B1**.

(e) The vibrator supplies energy to the glider every cycle **B1**. The amplitude, and so the energy of the oscillations, is constant, so the same energy must be removed each cycle by resistive forces **B1**.

Example 4

A bead of mass 0.50 g sits loosely on a horizontal tray. A motor makes the tray vibrate vertically with simple harmonic motion of fixed amplitude 2.0 mm, and the frequency of the motor is slowly raised from zero.

(a) Determine the highest frequency at which the bead stays in contact with the tray throughout each oscillation. **[3]**

(b) Calculate the maximum kinetic energy of the bead at this frequency. **[2]**

(c) Explain why the contact force on the bead is smaller when the tray is at the top of its motion than when it is at the bottom. **[2]**

Solution

(a) The bead stays in contact as long as the tray's greatest downward acceleration is no more than g: $\omega^2 x_0 = 9.81$ **C1** $\omega = \sqrt{\dfrac{9.81}{2.0 \times 10^{-3}}} = 70.0$ rad s^{-1} **C1** $f = \dfrac{\omega}{2\pi} = 11$ Hz **A1**

(b) $E_{\mathrm{k,max}} = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2} \times 5.0 \times 10^{-4} \times 70.04^2 \times (2.0 \times 10^{-3})^2$ **C1** $E_{\mathrm{k,max}} = 4.9 \times 10^{-6}$ J **A1**

(c) At the top the acceleration is downwards, so the resultant force on the bead is downwards and the contact force is less than its weight: $m(g - a_0)$ **B1**. At the bottom the acceleration is upwards, so the contact force is more than the weight: $m(g + a_0)$ **B1**. (If a_0 exceeded g, the contact force at the top would have to be negative, so the bead leaves the tray there first.)

Common mistakes

- **Giving the definition when asked to explain a graph or an expression.** The June 2021 and November 2021 reports saw many learned definitions of s.h.m. that never linked the features of the given line or equation to them. Say which feature shows which idea.
- **Leaving out the origin.** The June 2022 report says many forgot that the line passes through the origin, which is what shows **proportionality**; a straight line alone is not enough.

- **Two ideas for the minus sign.** The March 2021 report says candidates gave both properties of s.h.m. when asked for the one shown by the minus sign: it only shows that the acceleration is opposite to the displacement.
- **Setting the bracket equal to ω.** In $a = -\left(\frac{2k}{m}\right)x$ the bracket is ω^2 (June 2021 report). Also remember to square T when using $\omega = \frac{2\pi}{T}$ in a rearrangement.
- **Using $a = -\omega^2 x$ as if it were $a = -\omega x$, or inverting the gradient** (November 2021 report). $\omega^2 = \frac{a_0}{x_0}$, not $\frac{x_0}{a_0}$.
- **Forgetting to convert cm to m.** The June 2021, November 2021 and June 2023 reports all show power-of-ten errors, such as a maximum acceleration 100 times too big from an amplitude left in cm.
- **Energy slips.** The November 2022 report lists not squaring the amplitude, not converting it to metres, and rounding too early. Keep unrounded values between parts.
- **Using only gravitational (or only elastic) potential energy** for a mass on a vertical spring (November 2022 and June 2023 reports). The potential energy of the oscillations includes both; use $E = \frac{1}{2}m\omega^2 x_0^2$.
- **Mixing up "amplitude" and "displacement"** (June 2023 report). The amplitude is the maximum displacement; it does not change during an undamped oscillation.
- **A loop for a against x, or a line running past the amplitude** (June 2023 report). Acceleration against displacement is a straight line segment ending at $\pm x_0$.
- **A wave instead of a loop for v against position** (November 2021 report). Velocity against displacement is a closed loop, limited to the range of the motion.
- **Kinetic energy equal to potential energy at half the amplitude.** The November 2021 report says only the strongest realised the energies depend on x^2, so they are equal at $\frac{x_0}{\sqrt{2}}$, more than half the amplitude.
- **Only half the damping definition.** The March 2022 report saw either "loss of energy" or "resistive forces", rarely both.
- **Weak reason for "light".** The March 2023 report says "energy is lost slowly" or "it takes time to stop" also fits heavy damping; the reason must be that the oscillations continue.
- **Damping arithmetic.** The March 2023 report saw $6 \times 8\%$, 0.08^6, or no subtraction from the total; the March 2022 report saw only one amplitude used for an energy loss. Use $(1-p)^n$, and square for amplitudes.
- **Thinking damped motion stops being s.h.m.** The November 2021 report notes that after damping starts the motion is damped s.h.m., with about the same period.
- **Confusing resonance with damping** (June 2022 report), and saying a forced system has the same amplitude at all frequencies (June 2021 report).
- **Treating a period as half a cycle, or a phase difference as a time** (June 2022 report). One period is a whole cycle; phase difference is an angle in rad or degrees.
- **"The amplitude stays the same because the energy does"** when the pendulum length changes (June 2022 report). $E = \frac{1}{2}m\omega^2 x_0^2$: if ω falls with E constant, x_0 rises.
- **Too few significant figures.** The March 2023 report says many gave one significant figure; match the data (2 or 3 s.f.) and don't leave answers as surds (June 2022 report).

How to get the marks

- **Definitions** (one mark per idea):
  - Simple harmonic motion: acceleration proportional to displacement (from a fixed point); acceleration in the opposite direction to displacement (towards the fixed point).
  - Defining equation: $a = -\omega^2 x$, with a acceleration, x displacement from the equilibrium position, ω angular frequency.
  - Frequency: number of oscillations per unit time (not "per second").
  - Damping: loss of energy of the oscillations; due to resistive forces.
  - Resonance: maximum amplitude of oscillation; when the driving frequency equals the natural frequency.
- **"Explain how Fig. X shows s.h.m."**: "straight line through the origin, so $a \propto x$" and "negative gradient, so a is opposite to x". One mark each.
- **Calculations (C1 then A1)**: write the equation ($a_0 = \omega^2 x_0$, $E = \frac{1}{2} m\omega^2 x_0^2$), substitute in SI units, then give the answer with its unit. Method marks are given for the right equation with your values, even after an earlier slip.
- **"Show that"**: write the equation, substitute your readings, and give a value to more significant figures than the one printed.
- **"Determine" from a graph**: show the values you read (for example "$a_0 = 0.80$ m s^{-2} at $x_0 = 0.050$ m").
- **"Describe the interchange of energy"**: where kinetic energy is maximum and potential zero (equilibrium), where kinetic is zero and potential maximum (maximum displacement), and that their sum is constant. Numbers are not needed.
- **"State three conclusions"**: three different quantities, each with a value and unit if "quantitative" is asked. Period, frequency and angular frequency count as one.
- **Sketches**: marks for the right shape, the right start, the right period, and key values (peaks, intercepts) at the right places. Draw the a–x line with a ruler and stop it at the amplitude.
- **"Suggest, with a reason" / "State and explain"**: the effect (increases, decreases, no change) earns nothing without the reason that follows from an equation or a physical idea.
- **Significant figures**: 2 or 3, matching the data. Use radians on your calculator for $\sin\omega t$ and $\cos\omega t$.

Quick check

1. A body oscillates with simple harmonic motion of frequency 2.5 Hz and amplitude 4.0 cm. Calculate its angular frequency, its maximum speed and its maximum acceleration.
2. The velocity v of an oscillating mass is given by $v = \pm 8.0\sqrt{0.0036 - x^2}$, with v in m s^{-1} and x in m. Find the amplitude, the period, and the speed when $x = 0.030$ m.
3. The amplitude of a lightly damped oscillation decreases by 10% of its value at the start of each cycle. What fraction of the original energy remains after 2 cycles?
4. At what displacement, as a fraction of the amplitude, are the kinetic energy and the potential energy of an undamped simple harmonic oscillator equal?

5. Two pendulums each have a period of 1.2 s. One reaches its maximum displacement 0.15 s after the other. Calculate their phase difference in radians.

Answers

1. $\omega = 2\pi \times 2.5 = 15.7$ rad s⁻¹. $v_0 = \omega x_0 = 15.71 \times 0.040 = 0.628$ m s⁻¹. $a_0 = \omega^2 x_0 = 15.71^2 \times 0.040 = 9.87$ m s⁻².
2. $x_0^2 = 0.0036$, so $x_0 = 0.060$ m. $\omega = 8.0$ rad s⁻¹, so $T = \dfrac{2\pi}{8.0} = 0.785$ s. $v = 8.0\sqrt{0.0036 - 0.0009} = 8.0 \times 0.0520 = 0.416$ m s⁻¹.
3. Energy $\propto$ amplitude², so each cycle keeps $0.90^2 = 0.81$ of the energy. After 2 cycles: $0.90^4 = 0.656$ of the original energy.
4. Kinetic energy $\frac{1}{2}m\omega^2(x_0^2 - x^2)$ equals potential energy $\frac{1}{2}m\omega^2 x^2$ when $x^2 = \frac{x_0^2}{2}$, so $x = \dfrac{x_0}{\sqrt{2}} = 0.707x_0$.
5. $\phi = \dfrac{0.15}{1.2} \times 2\pi = \dfrac{\pi}{4} = 0.785$ rad (an eighth of a cycle, 45°).

Past questions to try

- **9702/43/O/N/22 Q3**: the defining equation, amplitude and ω from a velocity–displacement loop, mass from energy, and a damped loop.
- **9702/42/F/M/23 Q3**: energy–displacement lines, the period from the energy, and energy lost after several damped cycles.
- **9702/42/F/M/24 Q3**: ω from the energy, and the largest amplitude before an object leaves a vibrating platform.
- **9702/42/M/J/22 Q4**: resonance, the period from an equation, labelling graph scales and a phase difference.