Cambridge AS & A Level · Physics (9702) · Notes

Superposition AS

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Read first: Waves

What you need to know

By the end of this topic you should be able to:

1. **Explain and use the principle of superposition**: where waves meet, the resultant displacement is the sum of their displacements.
2. **Describe experiments that show stationary waves** with microwaves, a stretched string and an air column (end corrections are ignored).
3. **Explain how a stationary wave forms**, using graphs of the two waves and their sum, and **identify its nodes and antinodes**.
4. **Find the wavelength** from the positions of the nodes or antinodes of a stationary wave.
5. **Explain what diffraction is.**
6. **Describe experiments that show diffraction**, such as water waves in a ripple tank, including how the width of the gap compared with the wavelength changes the amount of spreading.
7. **Understand the terms interference and coherence.**
8. **Describe experiments that show two-source interference** with water waves in a ripple tank, sound, light and microwaves.
9. **State the conditions needed** to see two-source interference fringes.
10. **Recall and use** $\lambda = \dfrac{ax}{D}$ for double-slit interference of light.
11. **Recall and use** $d \sin\theta = n\lambda$ for a diffraction grating.
12. **Describe how a diffraction grating is used to measure the wavelength of light** (the spectrometer itself is not needed).

Wave words (displacement, amplitude, phase difference, wavelength), $v = f\lambda$, intensity $\propto$ amplitude^2 and the electromagnetic spectrum are in the Waves note; this note uses them without explaining them again.

Understand it

The principle of superposition

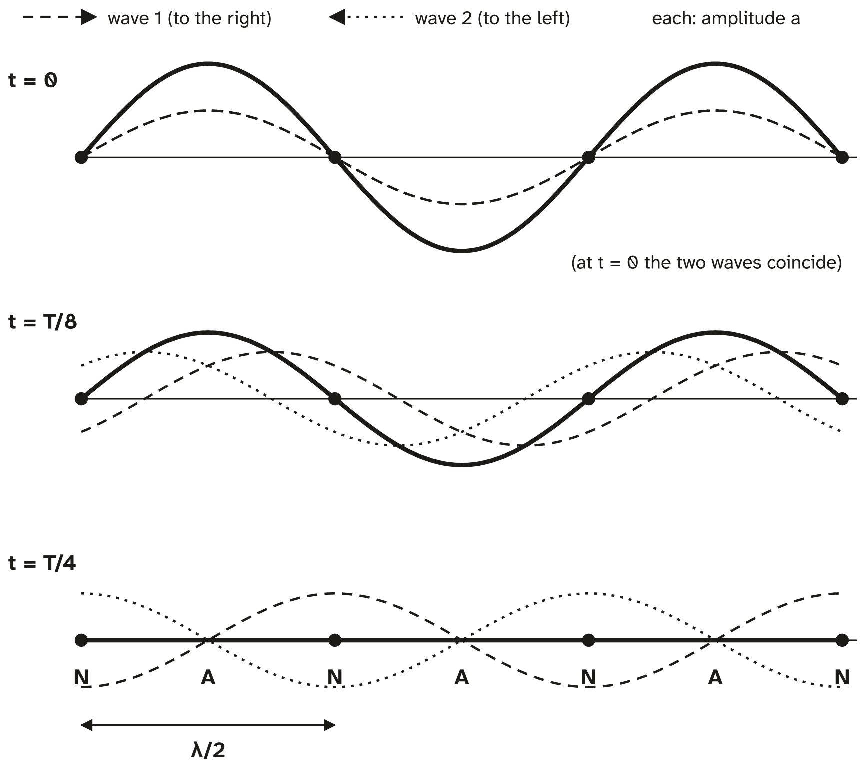
When two or more waves of the same type meet at a point, the **resultant displacement** at that point is the **sum of the displacements** of the individual waves at that instant. Displacements have a sign: if one wave gives $+4.0$ mm and the other -2.5 mm at the same moment, the resultant is $+1.5$ mm. After they pass through each other, each wave carries on unchanged.

- It applies to **any** waves of the same type that overlap (two sound waves, two light waves). They do **not** need the same frequency, amplitude or direction.
- It adds **displacements**, not amplitudes or intensities. Amplitudes only add simply in two special cases:
  - **in phase** (phase difference 0°): resultant amplitude $A_1 + A_2$ (**constructive** interference);
  - **in antiphase** (180°): resultant amplitude $|A_1 - A_2|$ (**destructive** interference).
- With amplitudes 5.0 mm and 3.0 mm, the resultant amplitude is 8.0 mm in phase and 2.0 mm in antiphase. Intensity goes as amplitude squared, so the loudest point is $\left(\frac{8.0}{2.0}\right)^2 = 16$ times as intense as the quietest.
- Two **equal** waves in phase give amplitude $2A$ and so **four** times the intensity of one wave; in antiphase they cancel completely.

Stationary waves

A **stationary (standing) wave** forms when two progressive waves of the same type, with the **same frequency** (so the same wavelength and speed) and similar amplitude, travel **in opposite directions** through the same place and superpose. Usually one wave is a reflection of the other: a wave travels to a fixed end, a wall or a metal sheet, bounces back, and the incident and reflected waves overlap.

The graphical method: draw both waves at one instant and add their displacements point by point. Repeat a short time later, after each wave has moved a little in its own direction.



Thick line = sum of the displacements. Nodes (N) never move;
the sum's size at the antinodes (A) swings between +2a and −2a.

Some points are always in antiphase between the two waves, so their displacement is **always zero**: these are **nodes**. Midway between nodes the two waves are always in phase, so the oscillation has its **largest amplitude**: these are **antinodes**.

- Nodes are $\frac{\lambda}{2}$ apart; antinodes are $\frac{\lambda}{2}$ apart; a node and the next antinode are $\frac{\lambda}{4}$ apart. λ is the wavelength of the progressive waves that made the pattern.
- All points between two neighbouring nodes (one "loop") oscillate **in phase**, with **different amplitudes**. Points in **neighbouring** loops are in **antiphase** (180°).
- Every point oscillates with the same frequency as the progressive waves. Twice in each cycle the whole string is straight (every displacement zero at once): a quarter of a cycle after each position of largest displacement.
- A stationary wave **does not transfer energy** along it, and the wave profile does not move along.

	Progressive wave	Stationary wave
Energy	transferred along the wave	not transferred
Amplitude	the same at every point	zero at nodes, largest at antinodes
Phase	changes steadily along the wave	same within a loop, 180° between loops
Wavelength	distance between neighbouring points in phase	twice the node-to-node distance

Showing stationary waves

Stretched string. A vibration generator drives one end of a string that passes over a pulley to a hanging mass (or is fixed at a wall). The wave reflects at the far end. At certain frequencies a steady pattern of loops appears; a strobe light makes it easy to see. Both ends are nodes, so a whole number of half-wavelengths fits the length L:

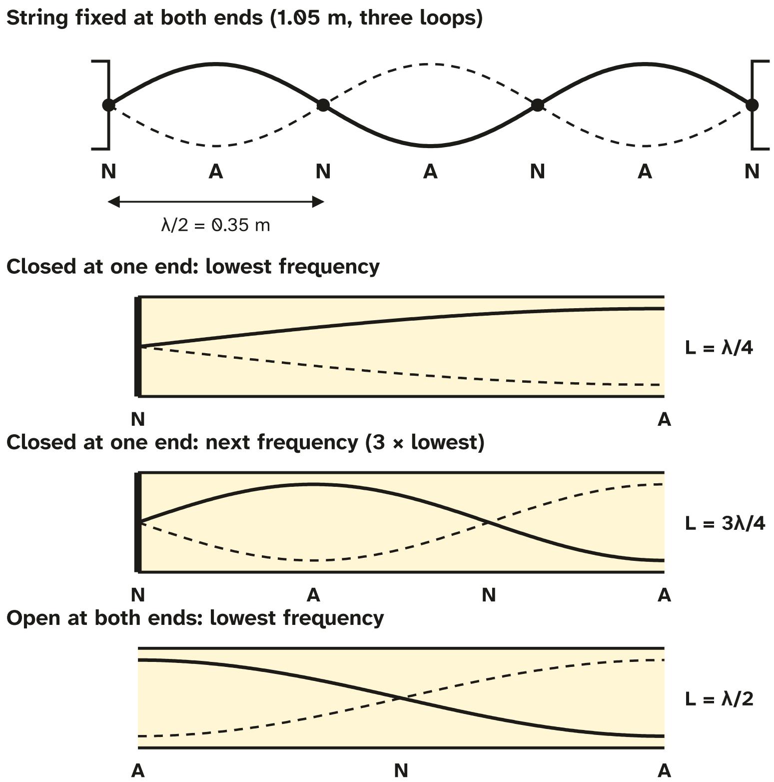
$$L = \frac{\lambda}{2},\ \lambda,\ \frac{3\lambda}{2},\ \ldots \qquad \left(L = \tfrac{n\lambda}{2}\right)$$

The wave speed on a given string stays the same, so $f \propto \frac{1}{\lambda}$: going from 3 loops to 4 loops multiplies the frequency by $\frac{4}{3}$.

Microwaves. A microwave transmitter faces a flat metal sheet, which **reflects** the waves back. A small receiver (probe) moved along the line between them detects maxima (antinodes) and minima (nodes). Adjacent minima are $\frac{\lambda}{2}$ apart, so measuring across several minima gives λ. For nodes 1.15 cm apart, $\lambda = 2.3$ cm. The metal sheet is a node.

Air column (sound). A loudspeaker at the open end of a tube sends sound in; it reflects at the other end. At certain frequencies or tube lengths the sound becomes much **louder** (resonance): a stationary wave has formed. Moving a microphone connected to an oscilloscope along the tube shows the pattern, and fine powder in a horizontal tube collects in heaps at the **nodes**, where the air does not move. Sound in air is longitudinal, but the displacement pattern of nodes and antinodes is the same idea.

- A **closed end** is a **displacement node** (the air cannot move into the wall).
- An **open end** is a **displacement antinode** (ignoring end corrections).

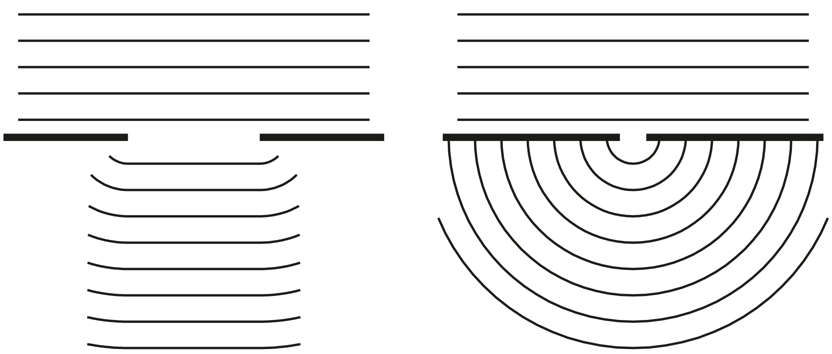


Arrangement	Ends	Lengths that fit	Frequencies
String fixed at both ends	N and N	$L = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$	$f_0, 2f_0, 3f_0, \ldots$
Tube open at both ends	A and A	$L = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$	$f_0, 2f_0, 3f_0, \ldots$
Tube closed at one end	N and A	$L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$	$f_0, 3f_0, 5f_0, \ldots$

For a tube 0.34 m long closed at one end, with sound at 340 m s^{-1}, the lowest frequency has $\lambda = 4 \times 0.34 = 1.36$ m, so $f_0 = \frac{340}{1.36} = 250$ Hz; the next stationary wave is at 750 Hz. The same tube open at both ends has $\lambda = 0.68$ m and $f_0 = 500$ Hz. The longest wavelength a closed tube can hold is $4L$.

Diffraction

Diffraction is the **spreading** of a wave as it passes through a gap or past the edge of an obstacle. The wavelength, frequency and speed do not change; only the direction of travel spreads out.



gap much wider than λ: little spreading   gap about λ: a lot of spreading

The ripple tank shows it well: a vibrating bar makes straight ripples, which meet a barrier with a gap. The spreading is greatest when the gap is about the **same size as the wavelength**, and small when the gap is much wider. Ripples of wavelength 1.3 cm spread widely through a 1.7 cm gap but hardly at all through a 9.5 cm gap. To get more spreading, make the gap narrower or the wavelength longer (a lower frequency in the same water).

This is why sound (wavelength around a metre) is heard round the corner of a building and through doorways, while light (under a micrometre) is not; and why long radio waves reach a valley behind a hill that short microwaves do not.

Interference and coherence

Interference is the superposition of waves from two (or more) sources to give a pattern of maxima and minima that stays in the same place.

Two sources are **coherent** if they have a **constant phase difference**. (This needs the same frequency, but "same frequency" alone is not the definition.) Coherent sources need not be in phase and need not have the same amplitude.

At a point a distance r_1 from one source and r_2 from the other, the **path difference** is $r_2 - r_1$. For sources that emit **in phase**:

- **maximum** (constructive) where the path difference is a whole number of wavelengths: $0, \lambda, 2\lambda, \ldots = n\lambda$;
- **minimum** (destructive) where it is an odd number of half-wavelengths: $\frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots = (n + \frac{1}{2})\lambda$.

Path difference turns into phase difference as $\frac{\text{path difference}}{\lambda} \times 360^\circ$. If the sources emit in **antiphase**, add the extra 180°: then a path difference of $n\lambda$ gives a **minimum** and $(n + \frac{1}{2})\lambda$ a **maximum**.

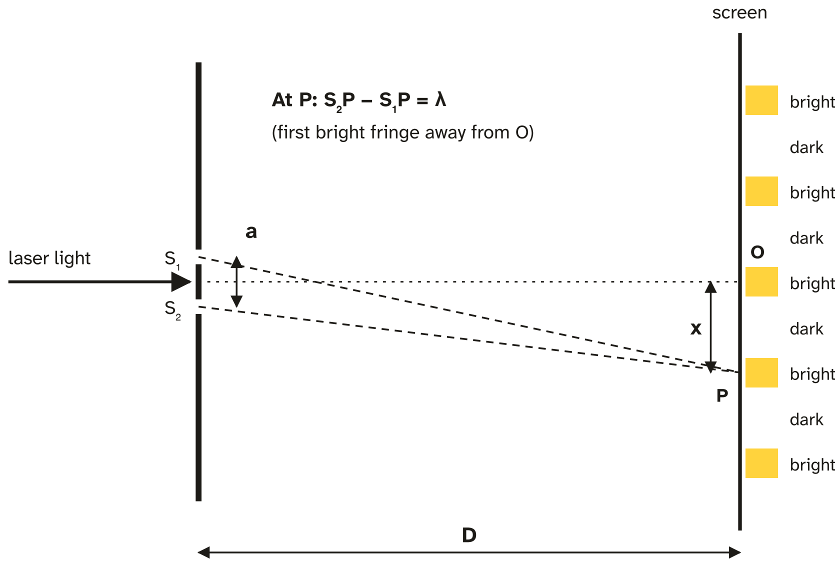
Conditions to see fringes.

- The sources must be **coherent**, otherwise the pattern shifts about too quickly to see. Two separate lamps or lasers are not coherent; that is why a single laser lights **both** slits.
- The waves must **overlap**: the slits must be narrow enough for the light to diffract and spread over the same region.
- For clear dark fringes the two waves should have **similar amplitudes** at the screen; unequal amplitudes leave the minima not quite dark.
- For light, the slit separation must be small and the screen far enough away for the fringes to be spaced out enough to see.

Demonstrations.

- **Ripple tank**: two dippers on the same vibrating bar (so coherent). Lines of still water (minima) spread out between lines of large ripples (maxima).
- **Sound**: two loudspeakers connected to the **same** signal generator. Walking along a line in front of them, you hear the sound rise and fall.
- **Microwaves**: one transmitter aimed at a metal sheet with two slits, or two horns from one source; a receiver moved along a line finds maxima and minima.
- **Light**: a laser shone on a double slit makes bright and dark fringes on a distant screen.

Double-slit interference: $\lambda = \dfrac{ax}{D}$



When the screen is far away compared with the slit separation ($D \gg a$), the fringes are evenly spaced and

$$\lambda = \frac{ax}{D} \qquad \text{so} \qquad x = \frac{\lambda D}{a}$$

where a is the slit separation (centre to centre), D the distance from the slits to the screen and x the **fringe separation**: the distance between the centres of neighbouring bright fringes (or of neighbouring dark fringes).

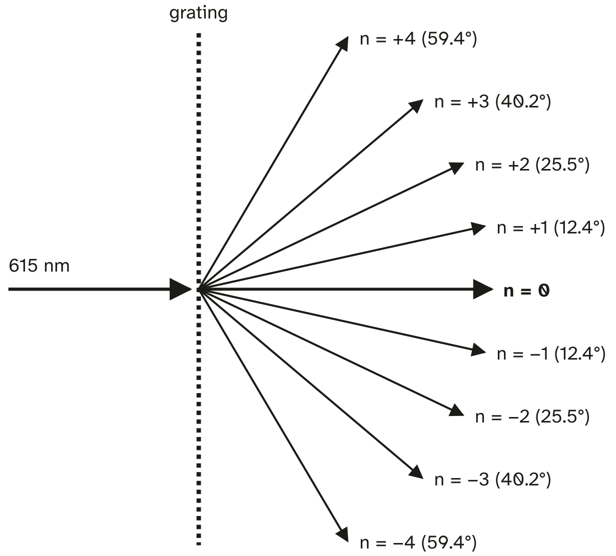
- For $\lambda = 580$ nm, $a = 0.45$ mm and $D = 2.6$ m: $x = \frac{580\times10^{-9}\times2.6}{0.45\times10^{-3}} = 3.4 \times 10^{-3}$ m $= 3.4$ mm.
- Fringes spread out (larger x) with a **longer wavelength** (red rather than blue), a **larger** D, or a **smaller** a. x is proportional to λ and to D, and inversely proportional to a.
- Measure across many fringes and divide: from the centre of the 1st bright fringe to the centre of the 6th there are **5** fringe separations.
- The central fringe O has path difference 0. The first dark fringe on either side has path difference $\frac{\lambda}{2}$, the second $\frac{3\lambda}{2}$; the nth bright fringe away from the centre has $n\lambda$.
- Brighter light (more intensity) makes the bright fringes brighter, while the dark fringes stay dark and the spacing stays the same. If one slit lets through less light, the bright fringes get dimmer and the dark fringes are no longer fully dark.
- Covering one slit removes the pattern: at the centre the amplitude halves, so the intensity falls to a **quarter**.

The diffraction grating: $d \sin\theta = n\lambda$

A diffraction grating is a plate with a very large number of equally spaced, very narrow parallel slits (lines). Light diffracts at every slit, and the waves from all of them superpose. In most directions they cancel. They reinforce only where the path difference between neighbouring slits is a whole number of wavelengths, which gives sharp, bright maxima at angles θ where

$$d\sin\theta = n\lambda$$

- d is the spacing between neighbouring lines. A grating with N lines per metre has $d = \frac{1}{N}$: 350 lines per mm is 3.5×10^5 lines per m, so $d = 2.86 \times 10^{-6}$ m.
- n is the **order**: $n = 0$ is straight through, then $n = 1, 2, 3, \ldots$ on **both** sides.
- θ is measured from the **straight-through** direction, not from the grating surface and not between two maxima on opposite sides.
- $\sin\theta$ cannot be more than 1, so the highest order is the largest whole number $n \leq \frac{d}{\lambda}$ (round **down**). The total number of maxima is $2n_{\max} + 1$.
- The angles depend only on λ and d. Moving the screen further away spreads the spots on the screen, but not the angles; the number of lines lit or the total number of lines on the grating does not change the angles either.
- White light: the zero order is white (all wavelengths go straight through); each other order is a spectrum, **violet nearest the centre** and red furthest out (shorter λ, smaller θ).



Angles are θ, measured from the straight-through direction (n = 0).

Measuring the wavelength of light. Shine the light normally onto a grating of known lines per mm. Measure the angle between the nth-order maxima on the two sides and halve it, so you don't have to find the exact straight-through position, and a small misalignment averages out. Work out d from the lines per mm, then $\lambda = \frac{d\sin\theta}{n}$. Using several orders and plotting $\sin\theta$ against n gives a straight line through the origin with gradient $\frac{\lambda}{d}$. A grating gives brighter, sharper, more widely spaced maxima than a double slit, so the angles can be measured accurately.

Key facts and formulas

"Given" means it is on the data sheet in the exam. "Learn it" means you must remember it.

Formula	Status
Superposition: resultant displacement = sum of the individual displacements	Learn it
In phase: amplitude $A_1 + A_2$; antiphase: amplitude $\lvert A_1 - A_2 \rvert$	Learn it
$v = f\lambda$ (and $c = f\lambda$)	Learn it
Speed of light in free space $c = 3.00 \times 10^8$ m s^{-1}	Given
Stationary wave: node to node $= \frac{\lambda}{2}$, node to antinode $= \frac{\lambda}{4}$	Learn it
String fixed at both ends or tube open at both ends: $L = \frac{n\lambda}{2}$	Learn it
Tube closed at one end: $L = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$	Learn it
Maximum: path difference $n\lambda$; minimum: $(n + \frac{1}{2})\lambda$ (sources in phase)	Learn it
Phase difference $= \frac{\text{path difference}}{\lambda} \times 360^\circ$	Learn it
Double slit: $\lambda = \dfrac{ax}{D}$	Learn it
Diffraction grating: $d \sin\theta = n\lambda$, with $d = \frac{1}{\text{lines per metre}}$	Learn it
Highest order: largest whole number $n \leq \frac{d}{\lambda}$; total maxima $2n_{\text{max}} + 1$	Learn it
Intensity $\propto$ (amplitude)^2	Learn it

Prefixes that appear here: nm $= 10^{-9}$ m, μm $= 10^{-6}$ m, mm $= 10^{-3}$ m, GHz $= 10^9$ Hz, and "lines per mm" $\times 10^3$ = lines per m.

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (188 different questions, 2021–2025; 4 more appear on two papers), the types came up this often. 11 of the 188 questions mix two or more types, so the counts add up to more than 188. Several Paper 2 questions start with parts on the Waves topic.

Question type	Questions
Stationary waves (strings, air columns, microwaves)	58
Two-source interference, coherence and $\lambda = \frac{ax}{D}$	50
The diffraction grating	37
Diffraction at a gap	34
The principle of superposition and adding two waves	21

1. Stationary waves (58 questions)

1. **Draw or picture the pattern**, marking N and A. Fixed ends and closed ends are nodes; open ends are antinodes; a reflecting metal sheet is a node.

2. **Count the half-wavelengths** (loops) that fit the length, or use the spacing: node to node $\frac{\lambda}{2}$, node to antinode $\frac{\lambda}{4}$.
3. **Use** $v = f\lambda$ with the wavelength of the progressive waves.

Variations you will meet:

- **"Explain how the stationary wave is formed"**: the wave from the source is **reflected** at the fixed end, sheet or closed end; the incident and reflected waves travel in **opposite directions** and **superpose** (interfere). Nodes are where they always cancel (amplitude zero); antinodes are where the amplitude is largest.
- **Conditions**: the two waves must be the same type and have the same frequency (so the same wavelength and speed), travel in **opposite** directions, and have similar amplitudes. "Coherent" and "same direction" are traps.
- **Wavelength from a detector**: count the gaps, not the points. From the 1st minimum to the 5th minimum is $4 \times \frac{\lambda}{2} = 2\lambda$. From a maximum to the next minimum is $\frac{\lambda}{4}$.
- **Microwave frequency**: $f = \frac{c}{\lambda}$ with $c = 3.00 \times 10^8$ m s^{-1}; give it in GHz if asked. Region: millimetres to centimetres is microwaves.
- **Next frequency**: the speed stays the same. A string with k loops goes to $k+1$ loops: $f \times \frac{k+1}{k}$. A closed tube only allows $f_0, 3f_0, 5f_0$: from $L = \frac{\lambda}{4}$ the next is $3f_0$, not $2f_0$.
- **Number of nodes or antinodes after a frequency change**: $f \times 2$ means $\lambda \div 2$, so twice as many loops (3 loops become 6, with 7 nodes counting both fixed ends).
- **Phase difference between two points**: 0° in the same loop, 180° in neighbouring loops, 0° again two loops apart. Never "$\frac{\Delta x}{\lambda} \times 360^\circ$" (that is for progressive waves).
- **Shape a time later**: in a stationary wave each point just goes up and down. After $\frac{T}{4}$ from maximum displacement the string is straight; after $\frac{T}{2}$ the shape is flipped; after T it is back. For t longer than T, take away whole periods first.
- **Distance moved by a point**: each period it moves $4\times$ its own amplitude (which is zero at a node).
- **Air column experiments**: raise a tube out of water, or move a piston, until the sound is loudest; the first loud position is $L = \frac{\lambda}{4}$, the next $\frac{3\lambda}{4}$, so they are $\frac{\lambda}{2}$ apart. Powder heaps are at nodes.
- **Amplitude against distance** for a tube: zero at a closed end, largest at an open end, following the half-loops in between.
- **Longest possible wavelength**: $4L$ for a tube closed at one end; $2L$ for a string or an open tube.

2. Two-source interference, coherence and $\lambda = \frac{ax}{D}$ (50 questions)

1. **Find** x (bright to bright, or dark to dark) from the data: distance across N bright fringes $\div(N-1)$.
2. **Convert** mm → m and nm → m.
3. $\lambda = \frac{ax}{D}$; rearrange for whichever is unknown.

Variations you will meet:

- **"Explain how the fringes are formed"**: light **diffracts** at each slit so the two beams overlap; the waves **superpose** (interfere); where the path difference is $n\lambda$ (phase difference 0°) there is a **bright** fringe, and where it is $(n+\frac{1}{2})\lambda$ (180°) there is a **dark** fringe.
- **Coherent**: a **constant phase difference**. Why two different lasers or lamps give no steady fringes: they are not coherent (for different frequencies, the phase difference keeps changing).
- **Conditions for observable fringes**: coherent sources, waves that overlap, similar amplitudes, and (for light) a small slit separation.
- **Path difference at a given fringe**: the nth dark fringe from the centre has $(n-\frac{1}{2})\lambda$; the nth bright one has $n\lambda$. Phase difference $= \frac{\text{path difference}}{\lambda} \times 360^\circ$.
- **Two sources, a detector on a line** (microwaves, sound): work out both distances (Pythagoras if needed), take the difference, divide by λ. Whole number → maximum; half number → minimum. If the sources are in antiphase or have a phase difference, include it.
- **Proportion questions**: $x \propto \frac{\lambda D}{a}$. Halve D and halve a: x unchanged. Change from red to blue: x smaller.
- **Sketches**: x against λ is a straight line through the origin; x against a is a curve falling as $\frac{1}{a}$ (doubling a halves x); x against D gives a straight line through the origin with gradient $\frac{\lambda}{a}$, so $a = \frac{\lambda}{\text{gradient}}$.
- **Changing intensity**: brighter bright fringes, dark fringes still dark, same separation. Less light from one slit: dark fringes lighter, bright fringes dimmer. Covering one slit: the centre falls to $\frac{1}{4}$ of its intensity (half the amplitude).
- **Narrower slits** (same separation): more diffraction, so the fringes cover a wider region but are dimmer; the separation is unchanged.
- **Microwaves or radio with $\lambda = \frac{ax}{D}$**: the same formula works; find a or x and name the region from λ.

3. The diffraction grating (37 questions)

1. **Get d in metres**: $d = \frac{1}{\text{lines per m}}$ (lines per mm $\times 10^3$ first).
2. **Get θ from the straight-through line.** If you are given the angle between the two nth-order maxima, halve it.
3. **Use $d\sin\theta = n\lambda$** and rearrange.

Variations you will meet:

- **Number of maxima**: n_{max} = whole-number part of $\frac{d}{\lambda}$; total $= 2n_{\text{max}} + 1$ (both sides plus the centre). If $\frac{d}{\lambda}$ is exactly a whole number, that order is at 90° and is not seen on a flat or semicircular screen.
- **Angle between two orders**: find both angles and subtract (a common wrong answer is just one of the angles).
- **Two wavelengths overlapping**: $n_1\lambda_1 = n_2\lambda_2$ at the same angle. For example the 4th order of λ_1 on the 3rd order of λ_2 gives $\lambda_1 = \frac{3}{4}\lambda_2$.
- **Graph questions**: $\sin\theta$ against λ for one order n has gradient $\frac{n}{d} = nN$; n against $\sin\theta$ has gradient $\frac{d}{\lambda}$. Put the stated order into your expression.
- **What changes the angles**: only λ and d (lines per mm). The screen distance changes the spacing on the screen but not the angles; more lines of the same spacing make brighter, sharper maxima.

- **Distance on a flat screen**: $y = L \tan\theta$ from the centre; the spread between the two nth orders is $2L\tan\theta$.
- **Same angles with a different colour**: longer λ needs bigger d (fewer lines per mm), and vice versa.
- **White light pattern**: white centre; spectra either side, violet on the inside.
- **"Describe the diffraction at the grating"**: the light spreads out as it passes through each slit, so the waves from the slits overlap and interfere.
- **Symbols**: d is the line spacing (not the grating-to-screen distance); n is the order (not the number of lines).

4. Diffraction at a gap (34 questions)

- **Definition**: a wave **spreads out** as it passes through a gap or around an edge (or obstacle).
- **More or less spreading**: most when the gap is about λ. To increase it: narrow the gap or lengthen the wavelength (lower frequency, since the speed in the same medium is fixed). Amplitude makes no difference.
- **After the gap**: the wavelength, frequency and speed are unchanged; wavefronts become curved, keeping the same spacing.
- **Same amount of diffraction**: keep $\frac{\lambda}{\text{gap}}$ the same; double λ means double the gap.
- **Which sound diffracts most through a doorway**: the one whose wavelength $\lambda = \frac{v}{f}$ is closest to the door width (the lowest frequency, usually).
- **Everyday cases**: sound heard round a corner; radio (long λ) reaching behind a mountain while microwaves (short λ) do not.
- **Ripple tank sketches**: straight wavefronts in, same spacing out; curved edges for a wide gap, near-semicircles for a narrow one. A slit that is tall and narrow spreads light **sideways** (across the narrow direction).

5. The principle of superposition and adding two waves (21 questions)

- **State the principle**: when two (or more) waves **meet** at a point, the resultant **displacement** is the **sum** of the individual displacements.
- **When does it apply?** Always, for waves of the same type that overlap. No need for the same frequency, amplitude or direction, or for coherence.
- **Resultant displacement from graphs**: read each wave's displacement at the given time (with sign) and add.
- **Resultant amplitude**: in phase $A_1 + A_2$, antiphase $|A_1 - A_2|$. Given both, solve: $A_1 + A_2 =$ max and $A_1 - A_2 =$ min.
- **Intensities**: turn intensities into amplitudes first ($A \propto \sqrt{I}$), add amplitudes, then square. Waves of intensity I and $9I$ have amplitudes in the ratio $1:3$, so the maximum is $(1+3)^2 = 16I$ and the minimum $(3-1)^2 = 4I$.
- **Zero resultant**: the two waves must be in antiphase and have the **same amplitude** at that point.

- **Phase difference between two waves at a point** from displacement–time graphs: time shift $\div T \times 360^\circ$.
- **Sketching the sum** of two waves on the same axes: add point by point. Two equal waves in antiphase give a flat line; equal and opposite shapes cancel.

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A string of length 1.05 m is fixed at one end to a wall and at the other end to a vibration generator, which acts as a node. At a frequency of 180 Hz a stationary wave with three loops forms on the string.

(a) Explain how the stationary wave is formed. **[2]**

(b) Calculate the wavelength. **[1]**

(c) Calculate the speed of the progressive waves on the string. **[2]**

(d) The frequency is slowly increased. Calculate the next frequency at which a stationary wave forms. **[2]**

(e) Point P is 0.22 m from the wall and point Q is 0.63 m from the wall. State the phase difference between their oscillations. **[1]**

Solution

(a) Waves travel along the string from the generator and are reflected at the wall. **B1** The incident and reflected waves, of the same frequency and speed, travel in opposite directions and superpose, giving nodes where they always cancel and antinodes of maximum amplitude between them. **B1**

(b) Three loops are $\frac{3\lambda}{2} = 1.05$ m, so $\lambda = 0.70$ m. **A1**

(c) $v = f\lambda$ **C1** $= 180 \times 0.70 = 126$ m s^{-1}. **A1**

(d) Next pattern: four loops, $\lambda = \frac{2\times1.05}{4} = 0.525$ m. **C1** $f = \frac{126}{0.525} = 240$ Hz. **A1**

(e) The nodes are at 0, 0.35 m, 0.70 m and 1.05 m, so P is in the first loop and Q in the second: phase difference 180°. **B1**

Example 2

Light from a laser is incident normally on a double slit of slit separation 0.28 mm. A pattern of bright and dark fringes forms on a screen 2.1 m from the slits. The distance from the centre of the first bright fringe to the centre of the sixth bright fringe is 21.0 mm.

(a) Explain how the pattern of bright and dark fringes is formed. **[3]**

(b) Calculate the wavelength of the light. **[3]**

(c) Calculate the path difference, in nm, for the light from the two slits meeting at the second dark fringe from the centre. **[1]**

(d) Calculate the frequency of the light. **[2]**

(e) One slit is made narrower so that less light passes through it. The slit separation is unchanged. Describe the change in the fringe pattern. **[2]**

Solution

(a) The light diffracts (spreads) at each slit, so light from the two slits overlaps. **B1** The waves superpose (interfere); where the path difference is a whole number of wavelengths (phase difference 0°) there is a bright fringe, **B1** and where it is an odd number of half-wavelengths (phase difference 180°) there is a dark fringe. **B1**

(b) $x = \frac{21.0}{5} = 4.2$ mm **C1** $\lambda = \frac{ax}{D} = \frac{0.28 \times 10^{-3} \times 4.2 \times 10^{-3}}{2.1}$ **C1** $= 5.6 \times 10^{-7}$ m. **A1**

(c) $1.5\lambda = 1.5 \times 560 = 840$ nm. **A1**

(d) $f = \frac{c}{\lambda}$ **C1** $= \frac{3.00 \times 10^8}{5.6 \times 10^{-7}} = 5.4 \times 10^{14}$ Hz. **A1**

(e) The fringe separation is unchanged. **B1** The waves from the two slits now have different amplitudes, so the dark fringes are no longer completely dark and the bright fringes are dimmer. **B1**

Example 3

A loudspeaker emitting sound of frequency 640 Hz is held at the open end of a tube. A piston inside the tube is slowly moved away from the open end. The sound becomes loud when the distance from the open end to the piston is 12.8 cm and again when it is 38.4 cm. End corrections can be ignored.

(a) Explain why the sound is loud at these positions. **[2]**

(b) Calculate the wavelength and the speed of the sound. **[3]**

(c) With the piston at 38.4 cm, calculate the lowest frequency that forms a stationary wave in the tube. **[2]**

(d) A small microphone is moved into the tube at 640 Hz. State the distance from the open end to the first position where the sound is quietest. **[1]**

Solution

(a) The sound reflects at the piston, and the incident and reflected waves superpose to form a stationary wave. **B1** It is loud when the length fits the pattern with a node at the piston and an antinode at the open end. **B1**

(b) The two lengths are $\frac{\lambda}{4}$ and $\frac{3\lambda}{4}$, so they differ by $\frac{\lambda}{2}$: $\lambda = 2(0.384 - 0.128) = 0.512$ m. **C1** **A1** $v = f\lambda = 640 \times 0.512 = 328$ m s^{-1}. **A1**

(c) Lowest frequency: $L = \frac{\lambda}{4}$, so $\lambda = 4 \times 0.384 = 1.536$ m. **C1** $f = \frac{327.7}{1.536} = 213$ Hz (a third of 640 Hz). **A1**

(d) The open end is an antinode, and the nearest node is a quarter of a wavelength in: $\frac{\lambda}{4} = \frac{0.512}{4} = 0.128$ m. **B1**

Example 4

A parallel beam of light of wavelength 615 nm is incident normally on a diffraction grating with 350 lines per millimetre.

(a) Calculate the line spacing d. **[1]**

(b) Calculate the angle of the first-order maximum. **[2]**

(c) Determine the total number of maxima produced. **[3]**

(d) Calculate the angle between the two second-order maxima. **[1]**

(e) The light is replaced by light of another wavelength. Its fourth-order maxima are at the same angles as the third-order maxima of the 615 nm light. Calculate this wavelength. **[2]**

Solution

(a) $d = \dfrac{1}{350 \times 10^3} = 2.86 \times 10^{-6}$ m. **A1**

(b) $\sin\theta = \dfrac{n\lambda}{d} = \dfrac{615 \times 10^{-9}}{2.86 \times 10^{-6}}$ **C1** $\theta = 12.4^\circ$. **A1**

(c) Largest order: $\sin\theta \le 1$, so $n \le \dfrac{d}{\lambda}$ **C1** $\dfrac{2.857 \times 10^{-6}}{615 \times 10^{-9}} = 4.6$, so $n_{\max} = 4$. **C1** Total $= 2 \times 4 + 1 = 9$ maxima. **A1**

(d) $\sin\theta_2 = \dfrac{2 \times 615 \times 10^{-9}}{2.857 \times 10^{-6}}$ gives $\theta_2 = 25.5^\circ$, so the angle between them is 51.0°. **A1**

(e) Same angle means the same $n\lambda$: $4\lambda' = 3 \times 615$ **C1** $\lambda' = 461$ nm. **A1**

Common mistakes

- **"Amplitudes add" in the principle of superposition.** Reports from March 2021, June 2021 and June 2022 all note candidates writing amplitude where displacement is needed. The principle adds **displacements** at a point.
- **Mixing up the conditions.** The June 2023 and March 2022 reports describe candidates confusing what superposition needs (just two waves of the same type meeting) with what a stationary wave needs (same frequency, opposite directions) and what a fringe pattern needs (coherence).
- **Half-explaining a stationary wave.** The June 2022 and March 2025 reports say answers often mentioned reflection but not where it happens, or superposition but not that the two waves are the incident and reflected ones travelling in opposite directions. Say all three: reflection at the named end, opposite directions, superposition.
- **"Maximum displacement" at antinodes.** The March 2025 report points out that antinodes have maximum **amplitude** and nodes zero **amplitude**; the displacement at an antinode is zero twice every cycle.
- **Node to antinode $= \frac{\lambda}{2}$.** The June 2023 report calls this a common misconception; it is $\frac{\lambda}{4}$. A node and the next antinode 0.21 m apart mean $\lambda = 0.84$ m, not 0.42 m.
- **Counting fringes or nodes instead of gaps.** Across 8 bright fringes there are 7 separations: 26.6 mm across 8 gives $x = 3.8$ mm, not 3.3 mm. The June 2023 report notes many could not get the fringe width from the data.
- **Taking x as bright-to-dark.** The June 2022 report says this doubled answers. x in $\lambda = \frac{ax}{D}$ is bright to bright (or dark to dark).
- **Powers of ten** with mm and nm. The March 2023 and June 2022 reports both mention them. Write every value in metres before substituting.
- **Using the full angle in $d \sin\theta = n\lambda$.** The November 2021 and June 2023 reports describe candidates using the angle **between** the two maxima. If the two second-order maxima are 46° apart, use $\theta = 23^\circ$.
- **Rounding up the highest order.** If $\frac{d}{\lambda} = 3.9$, the highest order is 3 (7 maxima), not 4. The June 2022 report explains that no maximum can be beyond 90°.
- **"Same phase difference" or "same frequency" for coherent.** The June 2022 report says neither earns the mark; the June 2023 report adds "same amplitude" and "same path difference" as wrong answers. Coherent means a **constant phase difference**.
- **Dark fringes "getting darker"** when the light is brighter. The June 2022 report calls this a common misconception; dark fringes stay dark, and the fringe separation stays the same (say so when comparing).
- **Air columns.** The November 2021 and June 2023 reports say candidates struggled to link the tube length to the wavelength and to find the longest wavelength a closed tube can hold ($4L$).
- **Sketching x against λ** so that it is zero at 400 nm. The March 2023 report notes this; $x \propto \lambda$, so the line would pass through the origin if extended. Use a ruler.

How to get the marks

- **Definitions** (use the key words):

- Principle of superposition: waves **meet**; resultant **displacement** $=$ **sum** of individual displacements.
- Diffraction: a wave **spreads** as it passes through a **gap** or around an **edge**.
- Coherent: **constant phase difference**.
- Interference: superposition of waves from coherent sources giving a fixed pattern of maxima and minima.

- **"Explain how a stationary wave forms"**: reflection (at the named place), two waves travelling in **opposite directions**, **superpose**; then nodes (zero amplitude) and antinodes (maximum amplitude) if asked.
- **"Explain the fringes"**: diffraction at each slit (so the light overlaps), superposition/interference, and the path or phase difference for bright ($n\lambda$, 0°) and dark ($(n+\frac{1}{2})\lambda$, 180°).
- **Calculations (C1 then A1)**: write the equation in symbols ($\lambda = \frac{ax}{D}$, $d\sin\theta = n\lambda$, $v = f\lambda$), substitute in metres, then the answer with a unit. A correct fringe width or d on its own often earns the first mark.
- **"Show that"**: every substitution written out, and the answer to one more significant figure than the value printed.
- **Grating answers**: a whole number of maxima (never a decimal), and angles in degrees to 2 or 3 significant figures.
- **"State and explain"**: the effect **and** the reason, for example "fringe separation increases: $x = \frac{\lambda D}{a}$ and λ is larger for red light".
- **Comparing patterns**: say what stays the **same** (separation, positions) as well as what changes (brightness).
- **Sketches**: stationary wave a time later drawn with the same nodes; straight lines with a ruler; for x against a, a curve that halves as a doubles; intensity against angle for a grating as narrow peaks at the correct angles, the central one tallest.
- **Multiple choice**: work it out first. Wrong options often use $\frac{\lambda}{2}$ for node to antinode, the full angle between two orders, the number of fringes instead of gaps, or "amplitude" for "displacement".

Quick check

1. At one instant, two waves passing through a point give displacements of $+3.0$ mm and -1.4 mm. State the resultant displacement. If these are the amplitudes of two waves of the same frequency, what are the largest and smallest possible resultant amplitudes?
2. A microwave probe is moved between a transmitter and a metal sheet. The distance from the 1st to the 7th minimum is 9.3 cm. Calculate the wavelength and the frequency of the microwaves.
3. In a double-slit experiment the fringe separation is 3.9 mm. The distance to the screen is doubled and the slit separation is made three times as large. Calculate the new fringe separation.
4. Light is incident normally on a grating with 250 lines per mm, and the second-order maximum is at 16.5°. Calculate the wavelength, and the total number of maxima.
5. Two coherent sources emit waves of wavelength 2.4 cm in antiphase. A point is 52.3 cm from one source and 58.3 cm from the other. State and explain whether there is a maximum or a minimum there.

Answers

1. $+3.0 + (-1.4) = +1.6$ mm. Largest amplitude (in phase) $3.0 + 1.4 = 4.4$ mm; smallest (antiphase) $3.0 - 1.4 = 1.6$ mm.
2. 1st to 7th minimum is 6 gaps of $\frac{\lambda}{2}$: $3\lambda = 9.3$ cm, so $\lambda = 3.1$ cm. $f = \dfrac{3.00 \times 10^8}{0.031} = 9.7 \times 10^9$ Hz (9.7 GHz).
3. $x \propto \dfrac{D}{a}$: $3.9 \times \dfrac{2}{3} = 2.6$ mm.
4. $d = \dfrac{1}{250 \times 10^3} = 4.0 \times 10^{-6}$ m; $\lambda = \dfrac{d \sin\theta}{n} = \dfrac{4.0 \times 10^{-6} \times \sin 16.5^\circ}{2} = 5.7 \times 10^{-7}$ m (568 nm). $\dfrac{d}{\lambda} = 7.04$, so $n_{\max} = 7$ and there are $2 \times 7 + 1 = 15$ maxima.
5. Path difference $58.3 - 52.3 = 6.0$ cm $= 2.5\lambda$, which alone would give 180°. The sources add another 180°, so the waves arrive in phase: a **maximum**.

Past questions to try

- **9702/22/F/M/25 Q4**: a stationary wave between a microwave device and a metal sheet, its wavelength and the node-to-antinode distance.
- **9702/22/M/J/22 Q5**: coherence, path and phase difference at a dark fringe, and fringe distances from $\lambda = \frac{ax}{D}$.
- **9702/22/M/J/24 Q6**: the number of bright fringes from a grating, and what happens with red light.
- **9702/22/O/N/21 Q5**: an air column in a tube raised out of water, and the distance between two resonance positions.