Cambridge AS & A Level · Physics (9702) · Notes

Work, energy and power AS

Read online at pastopic.com/cambridge/physics-9702/notes/work-energy-and-power

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Read first: Kinematics, Dynamics

What you need to know

By the end of this topic you should be able to:

1. **Understand work** and use work done = force × displacement in the direction of the force.
2. **State and use the principle of conservation of energy.**
3. **Understand efficiency**: the ratio of the useful energy output of a system to the total energy input.
4. **Use efficiency** to solve problems.
5. **Define power** as work done per unit time.
6. **Use** $P = \dfrac{W}{t}$**.**
7. **Derive** $P = Fv$ and use it.
8. **Derive** $\Delta E_P = mg\Delta h$ from $W = Fs$, for a uniform gravitational field.
9. **Use** $\Delta E_P = mg\Delta h$**.**
10. **Derive** $E_K = \frac{1}{2}mv^2$ from the equations of motion.
11. **Use** $E_K = \frac{1}{2}mv^2$**.**

The forms of energy and energy transfers from IGCSE are assumed. The equations of motion come from Kinematics, and $F = ma$ and momentum from Dynamics. Elastic potential energy belongs to Deformation of solids, but it often appears in the same questions as one more energy store.

Understand it

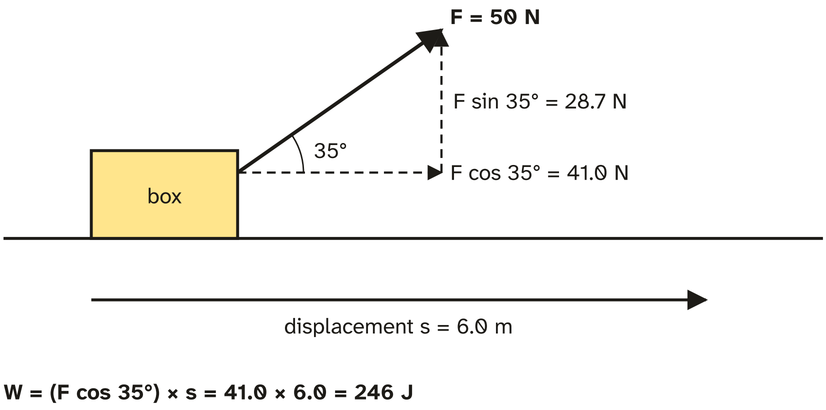
Work

A force **does work** when the object it acts on moves in the direction of the force:

$$W = Fs$$

where s is the displacement **in the direction of the force**. The unit is the joule: 1 J $= 1$ N m, the work done when a 1 N force moves its point of action 1 m along its line. Pushing a crate with 85 N for 3.2 m along the floor does $85 \times 3.2 = 272$ J of work. Work done is energy transferred: the 272 J goes from the person to the crate (and, through friction, to thermal energy).

When the force is at an angle to the displacement, only its component along the displacement does work.



$$W = Fs\cos\theta = 50 \times 6.0 \times \cos 35^\circ = 246\ \text{J}$$

where θ is the angle between the force and the displacement.

No work is done when:

- there is no displacement (holding a bag still, however tiring);
- the force is at right angles to the motion: $\cos 90^\circ = 0$ (the weight of a trolley rolling along a level floor, the normal contact force);
- an object slides at constant velocity with no forces along its motion (a puck on frictionless ice).

Work against a force. Moving an object against a resistive force f for a distance d transfers fd of energy, usually to thermal energy. Lifting an object at constant speed does work $mg\times$ (height) against gravity. If an object goes up and comes back down to the same level, the total work done **by gravity** is zero: negative on the way up, the same amount positive on the way down.

Work done by a gas. A gas at constant pressure p pushing a piston of area A through Δx does work $W = F\Delta x = pA\Delta x = p\Delta V$. An expanding gas does work, so its volume increases.

Gravitational potential energy: deriving $\Delta E_P = mg\Delta h$

Lift an object of mass m at constant speed through a height Δh in a **uniform** gravitational field (where g is the same everywhere, which is true close to the Earth's surface).

1. At constant speed the lifting force equals the weight: $F = mg$.
2. The displacement in the direction of the force is Δh.
3. Work done $= Fs = mg\Delta h$.
4. This work is stored as gravitational potential energy, so $\Delta E_P = mg\Delta h$.

Lifting a 3.5 kg box onto a shelf 1.4 m higher stores $3.5 \times 9.81 \times 1.4 = 48.1$ J. Only the **vertical** height counts: pushing the same box up a 4.0 m ramp to the same shelf gives it the same 48.1 J (more work is done only if there is friction). Moving an object horizontally doesn't change its E_P at all.

If a ball falls from 2.2 m and bounces to 1.5 m, its overall change in height is $2.2 - 1.5 = 0.7$ m, so its E_P is $mg \times 0.7$ lower at the end. The formula needs a uniform field; far from the Earth, g changes with height and you need the A Level formula instead.

Kinetic energy: deriving $E_K = \frac{1}{2}mv^2$

A constant resultant force F accelerates an object of mass m from rest to speed v over a displacement s.

1. Work done on it $W = Fs$.
2. Newton's second law: $F = ma$, so $W = mas$.
3. Equation of motion with $u = 0$: $v^2 = u^2 + 2as = 2as$, so $as = \dfrac{v^2}{2}$.
4. So $W = m \times \dfrac{v^2}{2} = \frac{1}{2}mv^2$. The work done becomes the kinetic energy, $E_K = \frac{1}{2}mv^2$.

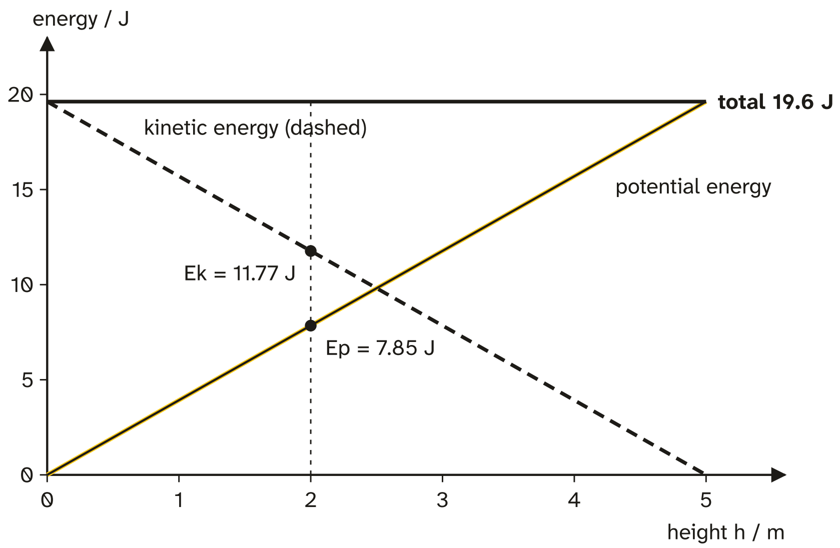
A 0.15 kg ball thrown at 26 m s^{-1} has $E_K = \frac{1}{2} \times 0.15 \times 26^2 = 50.7$ J.

- $E_K \propto v^2$: doubling the speed multiplies the kinetic energy by **4** (52 m s^{-1} gives 202.8 J). If the kinetic energy is multiplied by 4, the speed doubles.
- $E_K \propto m$ at the same speed.
- **From momentum**: $p = mv$, so $E_K = \dfrac{p^2}{2m}$. The ball's momentum is $0.15 \times 26 = 3.9$ N s, and $\frac{3.9^2}{2\times0.15} = 50.7$ J again.
- A **change** in kinetic energy is $\frac{1}{2}m(v^2 - u^2)$, not $\frac{1}{2}m(v-u)^2$.
- Kinetic energy is a scalar: direction doesn't matter, and it is never negative. At the top of a projectile's path the vertical velocity is zero but the horizontal velocity isn't, so its kinetic energy is $\frac{1}{2}mv_x^2$, not zero.

Conservation of energy

The principle of conservation of energy: energy cannot be created or destroyed; it can only be transferred from one form to another. The total energy of a closed system stays constant.

With no resistive forces, gravitational potential energy and kinetic energy simply swap. Drop a 0.40 kg ball from 5.0 m:



- At the top, $E_P = 0.40 \times 9.81 \times 5.0 = 19.6$ J and $E_K = 0$.
- At 2.0 m, it has fallen 3.0 m, so $E_K = 0.40 \times 9.81 \times 3.0 = 11.77$ J and $E_P = 7.85$ J. The total is still 19.6 J.
- Just before the ground, all 19.6 J is kinetic: $\frac{1}{2}mv^2 = mgh$, so $v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 5.0} = 9.90$ m s^{-1}. The mass cancels: without air resistance, every object dropped from 5.0 m lands at the same speed.

Both lines are straight because $E_P \propto h$. Plotted against **time** instead, E_K grows as t^2 (a curve), since the ball speeds up.

With resistive forces, some energy goes to thermal energy. Write an energy balance:

$$\text{energy at start} = \text{energy at end} + \text{work done against resistive forces}$$

A 25 kg sled starts from rest and slides 15 m down a slope, dropping 6.0 m vertically, against a friction force of 40 N.

- Loss of $E_P = 25 \times 9.81 \times 6.0 = 1471.5$ J.
- Work against friction $= 40 \times 15 = 600$ J (force × distance **along the slope**).
- Gain of $E_K = 1471.5 - 600 = 871.5$ J, so $v = \sqrt{\dfrac{2 \times 871.5}{25}} = 8.35$ m s^{-1}.

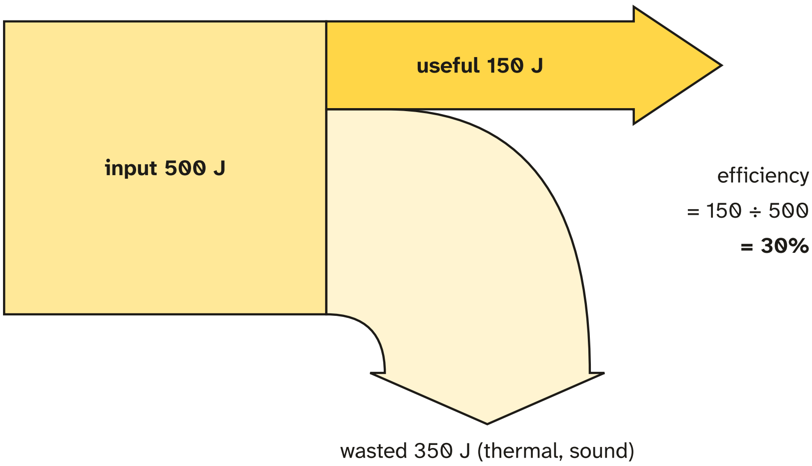
Energy changes to name:

- An object falling at **terminal (constant) velocity**: its E_K doesn't change, so all the E_P it loses goes to thermal energy (of the air and the object) through air resistance.
- A ball thrown upwards: E_K decreases as E_P increases.
- A rocket accelerating upwards: chemical energy decreases while E_P and E_K both increase.
- A bungee jumper still falling while the cord stretches has E_P, E_K and elastic potential energy all non-zero; at the lowest point $E_K = 0$.
- In an inelastic collision kinetic energy is not conserved, but the **total** energy still is: the lost E_K becomes thermal energy and sound.

Efficiency

No real machine turns all its input energy into the form you want. The rest is **wasted**, usually as thermal energy.

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} = \frac{\text{useful power output}}{\text{total power input}}$$



- Efficiency has **no unit**. It is a ratio (0.30) or a percentage (30%), always less than 1 for a real machine.
- Total input = useful output + wasted, so efficiency $= 1 - \dfrac{\text{wasted}}{\text{total input}}$. Here $1 - \frac{350}{500} = 0.30$.
- To find the **input** from the output, divide: a motor giving 45 W of useful power at 30% efficiency takes in $\frac{45}{0.30} = 150$ W. Multiplying instead is a common slip; the input must be the bigger number.
- Efficiencies in a chain multiply: a 90% motor driving an 80% pump is $0.90 \times 0.80 = 0.72$, i.e. 72%, efficient overall.
- For a machine lifting a load, the useful output is the gain in E_P, $mg\Delta h$. For a ramp used to raise a box, efficiency $= \dfrac{W\Delta h}{Pd}$ (useful work $W\Delta h$, work put in by the pull P along the slope length d).

Power

Power is the work done per unit time (or energy transferred per unit time):

$$P = \frac{W}{t}$$

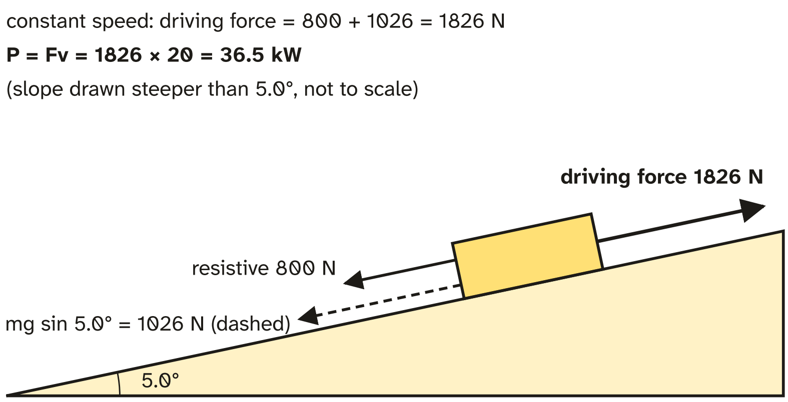
The unit is the watt: 1 W $= 1$ J s^{-1}. A 70 kg climber gaining 12 m of height in 20 s works against gravity at a rate of $\dfrac{70 \times 9.81 \times 12}{20} = 412$ W. Running up stairs gives the same E_P as walking up them, but in less time, so the power is greater.

Deriving $P = Fv$

A constant force F moves an object at constant velocity v in the direction of the force.

1. In a time t, the object moves $s = vt$.
2. Work done $W = Fs = Fvt$.
3. Power $P = \frac{W}{t} = \frac{Fvt}{t} = Fv$.

For a vehicle at **constant speed**, the driving force equals the total opposing force, so the engine's output power $= (\text{total opposing force}) \times v$. On a slope, the component of the weight down the slope, $mg\sin\theta$, is one of the opposing forces.



- Weight component down the slope: $1200 \times 9.81 \times \sin 5.0^\circ = 1026$ N.
- Driving force $= 800 + 1026 = 1826$ N, so $P = 1826 \times 20 = 36.5$ kW.
- On level road at the same speed, $P = 800 \times 20 = 16.0$ kW. The extra 20.5 kW is the rate of gain of E_P: $mg\sin\theta \times v$, which is $mg\times$ (height gained per second).
- Use the **driving** force of the engine in $P = Fv$, not the resultant force. When the car is accelerating, the engine's force is bigger than the resistance, and $P = F_{\text{engine}}v$ at that instant.
- At **constant power**, $F = \frac{P}{v}$: the faster the vehicle, the smaller the driving force. At 40 kW, the force is 4000 N at 10 m s^{-1} but only 1600 N at 25 m s^{-1}. That is why a vehicle has a top speed: when $\frac{P}{v}$ falls to the size of the resistive force, it stops accelerating.

Key facts and formulas

"Given" means it is on the data sheet in the exam. "Learn it" means you must remember it.

Formula	Status
Work done $W = Fs$, s = displacement in the direction of the force; $W = Fs\cos\theta$ at an angle θ (J)	Learn it
Work done by a gas at constant pressure $W = p\Delta V$	Learn it

Formula	Status
Change in gravitational potential energy $\Delta E_P = mg\Delta h$ (uniform field)	Learn it
Kinetic energy $E_K = \frac{1}{2}mv^2$; also $E_K = \dfrac{p^2}{2m}$	Learn it
Conservation of energy: energy is never created or destroyed; the total energy of a closed system is constant	Learn it
Efficiency $= \dfrac{\text{useful energy output}}{\text{total energy input}} = \dfrac{\text{useful power output}}{\text{total power input}}$ (no unit)	Learn it
Power $P = \dfrac{W}{t}$ (W = J s^{-1})	Learn it
$P = Fv$	Learn it
Component of weight down a slope $mg\sin\theta$	Learn it
$v^2 = u^2 + 2as$ (used to derive E_K)	Given
$g = 9.81$ m s^{-2}	Given

How to do it

In the Paper 1 and Paper 2 questions we have tagged for this topic (148 different questions, 2021–2025; 5 more appear on two papers), the types came up this often. 25 of the 148 questions mix two or more types, so the counts add up to more than 148.

Question type	Questions
Kinetic and gravitational potential energy	44
Power, $P = W/t$ and $P = Fv$	35
Efficiency	29
Conservation of energy with work against resistance	24
Definitions, derivations and units	21
Work done by a force	19
Energy changes and energy graphs	16

1. Kinetic and gravitational potential energy (44 questions)

1. **Convert units first**: g → kg, cm or mm → m, km h^{-1} → m s^{-1} ($\div 3.6$).

2. E_P: $mg\Delta h$ with the **vertical** change in height. E_K: $\frac{1}{2}mv^2$, squaring the speed.
3. **For a change** in kinetic energy, find each E_K and subtract: $\frac{1}{2}m(v^2 - u^2)$.
4. **To find a speed from an energy**, rearrange: $v = \sqrt{\frac{2E_K}{m}}$.

Variations you will meet:

- **Ratios** ("half the mass, twice the speed"): $E_K \propto mv^2$, so multiply the factors: $\frac{1}{2} \times 2^2 = 2$ times the kinetic energy. If E_K is multiplied by n at fixed mass, the speed is multiplied by $\sqrt{n}$.
- **Kinetic energy from momentum**: $E_K = \frac{p^2}{2m}$; a change from p_1 to p_2 is $\frac{p_2^2 - p_1^2}{2m}$. Or find m from E_K and p: $m = \frac{p^2}{2E_K}$.
- **Kinetic energy lost in a collision** (with momentum from Dynamics): total E_K before minus total E_K after; as a percentage, divide by the total before.
- **Projectile at its highest point**: only the horizontal velocity is left, $v_x = v\cos\theta$, so $E_K = E_0\cos^2\theta$ (at 60°, a quarter of the launch energy). v_x can come from range ÷ time of flight.
- **Stacking or lifting several things**: add each object's $mg\Delta h$, using how far **its centre** rises.
- **"Which two quantities are enough?"**: for ΔE_P, weight and change in height (mg is the weight).
- **Different planet**: $\Delta E_P = mg\Delta h$ with that planet's g; g can come from $\frac{\Delta E_P}{m\Delta h}$.
- **Order-of-magnitude estimates**: a 2 kg bag of flour dropped about 1.5 m loses about 30 J.
- **With a spring or cord** (Deformation of solids): elastic potential energy $\frac{1}{2}Fx = \frac{1}{2}kx^2$ is one more store; at the lowest point of a bungee jump, E_P lost $=$ elastic energy stored and $E_K = 0$.

2. Power, $P = W/t$ and $P = Fv$ (35 questions)

1. **Work or energy ÷ time**, with time in seconds (minutes × 60, hours × 3600).
2. **At constant velocity**, $P = Fv$ with F = the driving force = total opposing force.
3. **On a slope**, the opposing forces are the resistive force **plus** $mg\sin\theta$ (going up) or **minus** it (going down).
4. **Lifting at constant speed**: $P = mgv$, where v is the vertical speed; the time to lift through h is $\frac{mgh}{P}$.

Variations you will meet:

- **Resistive force depending on speed** (for example $F \propto v^2$): scale the force first, then $P = Fv$. With $F \propto v^2$, $P \propto v^3$, and the energy for a fixed distance ($W = Fd$) goes as v^2.
- **Thrust and resistance given at one instant**: the engine's power is thrust × speed, not resultant force × speed.
- **Force at an angle** to the motion: $P = Fv\cos\theta$.
- **Rate of loss of** E_P of a parachutist at constant speed: $\frac{mg\Delta h}{t} = mgv$, so $m = \frac{R}{gv}$.
- **Water pumps, hydroelectric stations, lifts and escalators**: find the mass per second (volume per second × density), then power $= \frac{m}{t}gh$; add any power needed against friction; then apply the efficiency.
- **Average power over an acceleration**: total work (gain in E_K + gain in E_P + work against resistance) ÷ time.
- **Constant power graph**: $F = \frac{P}{v}$, so F against v is a curve falling towards the axis (inverse proportion).

- **"State and explain the effect on the power"**: at the same speed, air resistance stays the same; extra mass going uphill means a bigger $mg\sin\theta$, so more power; going downhill, the weight component helps, so less power is needed. Once a car stops accelerating, it needs less power (no more gain in E_K).

3. Efficiency (29 questions)

1. **Decide what is useful**: the energy (or power) in the form you want. Everything else is wasted.
2. **Efficiency** $= \dfrac{\text{useful output}}{\text{total input}}$, with both in the same units (both energies or both powers).
3. **Rearrange** as needed: useful output $=$ efficiency × input; input $= \dfrac{\text{useful output}}{\text{efficiency}}$.

Variations you will meet:

- **Given wasted and useful**: input = useful + wasted. For useful rate X and wasted rate Y, efficiency $= \frac{X}{X+Y}$.
- **Given efficiency and wasted power**: wasted $= (1 - \text{efficiency})\times$ input, so find the input first.
- **"Which expression gives the efficiency?"**: useful output over total input; in symbols $\frac{P_{out}}{P_{in}}$ or $1 - \frac{Q}{E}$ for Q wasted out of E. Watch the chain diagrams: for chemical → thermal → electrical, the efficiency of the power station is electrical output ÷ chemical input.
- **Two devices with the same useful output**: the input is inversely proportional to efficiency, so a 50% device needs $\frac{10}{50} = \frac{1}{5}$ of the input of a 10% one.
- **Units**: efficiency has none.
- **Solar panels**: input power $=$ intensity × area; efficiency $=$ electrical output ÷ that.
- **A motor lifting a load**: useful output $= mgh$ (or mgv per second); input comes from the supply.

4. Conservation of energy with work against resistance (24 questions)

1. **List the energy at the start** (E_K, E_P, any elastic energy) and **at the end**.
2. **Write the balance**: start $=$ end $+$ work done against resistive forces ($f\times$ distance moved).
3. **Solve** for what's asked: a speed, a height, the work against friction, or the average resistive force ($= \frac{\text{work}}{\text{distance}}$).

Variations you will meet:

- **No resistance**: $\frac{1}{2}mv^2 = mg\Delta h$, so $v = \sqrt{2g\Delta h}$; mass cancels. On a slope of length L at angle θ, $\Delta h = L\sin\theta$, so $v = \sqrt{2gL\sin\theta}$.
- **Loop or track**: use the **drop** in height between the two points (for example start height minus height at the top of the loop).
- **Thrown down or up with a starting speed**: final E_K = initial E_K + $mg\Delta h$ (falling). Direction doesn't matter; energy is a scalar.
- **Pushed up a slope at constant speed**: work by the push $=$ gain in E_P + work against friction, so $FL = mgh + fL$.
- **A rough ramp with E_K at both ends**: work against friction $= \Delta E_P$ lost $-\Delta E_K$ gained.

- **Starting and ending at rest** (a trolley released that stops higher up): $mg(h_1 - h_2) = f \times d$.
- **Height a projectile reaches between two points with drag**: work against air resistance $=$ decrease in $(E_K + E_P)$.
- **Ratio of speeds with and without resistance**: $\frac{v_1}{v_2} = \sqrt{\frac{E_{K1}}{E_{K2}}}$.

5. Definitions, derivations and units (21 questions)

- **Work done**: force × displacement (distance moved) in the direction of the force. Say displacement or distance moved, not just "distance".
- **Power**: work done per unit time (or energy transferred per unit time). "Force × velocity" is a formula, not the definition; "work done per second" and "in one second" are not the definition either.
- **Efficiency**: useful energy output ÷ total energy input.
- **Conservation of energy**: the total energy of a closed system is constant; energy can't be created or destroyed.
- **Derive** $\Delta E_P = mg\Delta h$: work $= Fs$, the force needed is mg at constant speed, the distance is Δh. It needs a **uniform field** (constant g), and the definition of **work done**.
- **Derive** $E_K = \frac{1}{2}mv^2$: uses $W = Fs$, $F = ma$ and $v^2 = u^2 + 2as$ (Example 4). It doesn't need $E_K = Fs$ as a separate step; that is what you are proving.
- **Derive** $P = Fv$: $P = \frac{W}{t} = \frac{Fs}{t}$ and $\frac{s}{t} = v$. It uses displacement $=$ velocity × time.
- **Units**: 1 J $= 1$ N m $= 1$ kg m² s^{-2}; 1 W $= 1$ J s^{-1} $= 1$ kg m² s^{-3}. Energy per unit mass (J kg^{-1}) divided into energy per unit volume (J m^{-3}) gives a density.

6. Work done by a force (19 questions)

1. **Find the component of the force along the displacement**: $F\cos\theta$.
2. **Multiply by the displacement**: $W = Fs\cos\theta$.
3. **Add up** separate stages (pushed along, then lifted) to get the total.

Variations you will meet:

- **"In which situation is work done?"**: there must be a force **and** a displacement along it. Holding still, moving at right angles to the force, or sliding freely: no work.
- **Lifting at constant velocity**: the force is the weight, so $W = mgh$; a horizontal move of the same object needs no work against gravity. A weight carried along and up by a trolley has work $W \times$ (vertical rise) done on it.
- **Minimum work up a frictionless ramp**: mgh (the length of the ramp doesn't matter).
- **Work to stop an object**: equals its kinetic energy.
- **Net work** on an object moving at constant speed along a level belt: zero; the net work equals the change in E_K.
- **Work by gravity** over a path that returns to the same height: zero.

- **Gas and piston**: $W = p\Delta V$; the gas expands when it does work, so the new volume is the old one **plus** ΔV.
- **Work from a force–displacement graph**: the area under it.
- **Path choice with friction**: work against a constant friction force depends on the **distance travelled**, so a longer route costs more.

7. Energy changes and energy graphs (16 questions)

- **Name the change**: terminal velocity: $E_P \to$ thermal energy (E_K constant). Rising ball: $E_K \to E_P$. Accelerating rocket: chemical $\to E_P$ and E_K. Spring compressed by a mass: the spring gains elastic energy, and the mass on it loses E_P.
- **"Which statement is not correct?"**: check each against conservation. Total energy never increases on its own; friction doesn't reduce the **total** energy, it transfers some to thermal energy.
- E_K **against height** for a falling object: a straight line, largest at $h = 0$. E_P **against height**: a straight line through the origin, gradient mg.
- **Graph of ΔE_P against distance moved up**: a straight line means a constant force ($F = mg$, the gradient). So a graph of force against distance is a horizontal line.
- **Same ΔE_P for different masses**: $h = \frac{\Delta E_P}{mg}$, so $h \propto \frac{1}{m}$ (a curve).
- **Horizontal displacement** in a uniform vertical field: E_P doesn't change.
- **Terminal velocity graphs against time**: E_K a horizontal line, E_P a straight line falling steadily.
- **Explain a comparison without numbers**: say which energies change and where the energy goes (for example an arrow that ends lower than it started has gained E_K from lost E_P).

Worked examples

The bold codes show where marks are earned, as in Cambridge mark schemes. The number is how many marks.

- **A1** accuracy mark: for a correct answer; it needs the M mark before it.
- **B1** mark for a correct result on its own, with no method needed.

Example 1

A pump lifts water from a well to a tank 18 m higher. It delivers 0.24 m³ of water each minute. The density of water is 1000 kg m⁻³ and the pump is 60% efficient.

Calculate the input power to the pump. **[3]**

Solution

Mass per second $= \dfrac{0.24 \times 1000}{60} = 4.0$ kg s⁻¹ **C1** Useful output power $= \dfrac{mgh}{t} = 4.0 \times 9.81 \times 18 = 706$ W **C1** Input power $= \dfrac{706}{0.60} = 1180$ W $(1.18$ kW). **A1**

Example 2

A skier of mass 65 kg passes the top of a slope at 3.0 m s^{-1}. She travels 80 m along the slope and descends a vertical height of 24 m. An average resistive force of 55 N acts on her.

(a) Calculate the loss of gravitational potential energy. **[2]**

(b) Calculate the work done against the resistive force. **[1]**

(c) Determine her speed at the bottom of the slope. **[3]**

Solution

(a) $\Delta E_P = mg\Delta h$ **C1** $= 65 \times 9.81 \times 24 = 1.53 \times 10^4$ J. **A1**

(b) $W = 55 \times 80 = 4400$ J. **A1**

(c) Initial $E_K = \frac{1}{2} \times 65 \times 3.0^2 = 292.5$ J **C1** Final $E_K = 292.5 + 15\,304 - 4400 = 11\,196$ J **C1** $v = \sqrt{\dfrac{2 \times 11\,196}{65}} = 18.6$ m s^{-1}. **A1**

Example 3

(a) Define power and show that the power needed to move an object at constant velocity v against a force F is $P = Fv$. **[3]**

A van of mass 2400 kg drives up a straight road inclined at 4.0° to the horizontal at a constant speed of 15 m s^{-1}. The total resistive force on the van is 900 N.

(b) Calculate the component of the van's weight down the slope. **[1]**

(c) Calculate the output power of the engine. **[2]**

(d) The engine is 32% efficient. Calculate the input power to the engine. **[1]**

(e) The van reaches a level road and the engine keeps the same output power. Assuming the resistive force stays at 900 N, calculate the van's maximum speed on the level road. **[2]**

Solution

(a) Power is the work done per unit time. **B1** In time t the object moves $s = vt$, so work done $W = Fs = Fvt$. **B1** $P = \dfrac{W}{t} = \dfrac{Fvt}{t} = Fv$. **B1**

(b) $mg \sin 4.0^\circ = 2400 \times 9.81 \times \sin 4.0^\circ = 1640$ N. **A1**

(c) Constant speed, so the driving force $= 900 + 1642 = 2542$ N **C1** $P = Fv = 2542 \times 15 = 3.81 \times 10^4$ W. **A1**

(d) Input $= \dfrac{3.81 \times 10^4}{0.32} = 1.19 \times 10^5$ W. **A1**

(e) At maximum speed the driving force equals the resistive force, 900 N **C1** $v = \dfrac{P}{F} = \dfrac{3.81 \times 10^4}{900} = 42$ m s^{-1}. **A1**

Example 4

(a) A constant resultant force F acts on an object of mass m, initially at rest. The object reaches speed v after a displacement s. Use the work done by F to show that the kinetic energy of the object is $\frac{1}{2}mv^2$. **[3]**

(b) A ball of mass 0.50 kg is thrown vertically upwards at 12 m s^{-1}.

(i) Calculate its initial kinetic energy. **[1]**

(ii) Calculate the height it would reach if there were no air resistance. **[2]**

(iii) It actually reaches 6.4 m. Determine the average resistive force on the ball on the way up. **[3]**

Solution

(a) Work done $W = Fs$, and $F = ma$, so $W = mas$. **B1** From $v^2 = u^2 + 2as$ with $u = 0$: $as = \dfrac{v^2}{2}$. **B1** So $W = \frac{1}{2}mv^2$, and the work done on the object is its kinetic energy: $E_K = \frac{1}{2}mv^2$. **B1**

(b)(i) $E_K = \frac{1}{2} \times 0.50 \times 12^2 = 36$ J. **A1**

(ii) All the E_K becomes E_P: $mgh = 36$ **C1** $h = \dfrac{36}{0.50 \times 9.81} = 7.3$ m. **A1**

(iii) E_P gained $= 0.50 \times 9.81 \times 6.4 = 31.4$ J **C1** Work against air resistance $= 36 - 31.4 = 4.6$ J **C1** Average force $= \dfrac{4.6}{6.4} = 0.72$ N. **A1**

Common mistakes

- **Using the distance along a slope in $mg\Delta h$.** Only the vertical height counts. Raising a 12 kg box 1.5 m up a 5.0 m ramp gains $12 \times 9.81 \times 1.5 = 177$ J, not $12 \times 9.81 \times 5.0 = 589$ J. Examiner reports note the slope length being used instead of the vertical height.
- **$\frac{1}{2}m(v-u)^2$ for a change in kinetic energy.** A 1000 kg car speeding up from 10 to 20 m s^{-1} gains $\frac{1}{2} \times 1000 \times (20^2 - 10^2) = 150$ kJ, not $\frac{1}{2} \times 1000 \times 10^2 = 50$ kJ.
- **Using the resultant force in $P = Fv$.** The engine's power is **its** force × speed. With a 3500 N engine force and 1400 N resistance at 25 m s^{-1}, the engine's power is $3500 \times 25 = 87.5$ kW, not $2100 \times 25 = 52.5$ kW. Also don't multiply the weight by a horizontal speed.

- **Forgetting $mg\sin\theta$ on a slope.** Going uphill at constant speed, the engine works against the resistance **and** the weight component.
- **Getting efficiency upside down.** The input is always bigger than the useful output. To get the input, divide by the efficiency; forgetting the efficiency altogether is the other common slip.
- **Defining power as "force × velocity"** or "energy per unit time" (it is energy **transferred** per unit time, or work done per unit time). For work done, say force × **displacement in the direction of the force**.
- **Equating the wrong energies.** When there is friction, the gain in E_K is not the loss in E_P; subtract the work against friction. Examiners report the work against friction being set equal to the final E_K or to the E_P lost, and the resistive force found from an acceleration and confused with the resultant force.
- **Terminal velocity means no change in E_K.** The E_P lost goes to thermal energy, not kinetic energy.
- **Rounding too early.** Keep full values in intermediate steps (for example the E_P and E_K before subtracting), and round only the final answer.
- **In a "show that" or "derive" question**, skipping steps or not ending with the required expression. Write each line, name what you use ($W = Fs$, $F = ma$, $v^2 = u^2 + 2as$), and finish with the result exactly as asked.
- **Power-of-ten slips**: g to kg, km to m, minutes to seconds, kJ and MW.

How to get the marks

- **Definitions** (one mark each, meaning exact):
  - Work done: force × displacement in the direction of the force.
  - Power: work done per unit time.
  - Efficiency: useful energy output ÷ total energy input.
  - Principle of conservation of energy: energy cannot be created or destroyed, only transferred from one form to another; the total energy of a closed system is constant.
- **Derivations**: each step is a mark. For E_K: $W = Fs$ and $F = ma$; $v^2 = 2as$; combine to $\frac{1}{2}mv^2$. For E_P: force $= mg$, distance Δh, $W = mg\Delta h$. For $P = Fv$: $P = \frac{W}{t}$, $W = Fs$, $\frac{s}{t} = v$. The final line must be the expression asked for.
- **"Show that"**: write the formula, the substitution with numbers and a result to one more significant figure than the value quoted (for example 2.62 for a quoted 2.6).
- **Method marks (C1)** come from the formula with numbers in it. Write $\Delta E_P = mg\Delta h$ or $P = Fv$ before substituting, and you keep that mark even if the arithmetic slips.
- **"By considering energy"** or **"by considering changes of energy"**: use an energy balance, not the equations of motion. Show each energy term.
- **"State and explain"**: the effect **and** the physics reason, for example "air resistance is the same, **because** the speed is the same" or "the power increases, **because** the weight component down the slope increases".
- **Units**: J for work and energy, W for power, none for efficiency. A percentage efficiency can be given with the % sign.

- **Significant figures**: match the data (usually 2 or 3). One-significant-figure answers from 2- or 3-figure data lose marks.
- **Multiple choice**: work it out before looking at the options. The wrong options are usually the slope length instead of the height, the resultant force in $P = Fv$, a missing g, a missing square on v, or the efficiency used the wrong way up.

Quick check

1. A trolley of mass 3.0 kg speeds up from 4.0 m s^{-1} to 6.0 m s^{-1}. Calculate the increase in its kinetic energy.
2. A student of mass 55 kg climbs stairs with a vertical height of 7.5 m in 12 s. Calculate the useful power she develops.
3. A motor takes in 250 W and is 72% efficient. Calculate its useful output power and the energy it wastes in 5.0 minutes.
4. A pendulum bob is released from rest at a height of 0.35 m above its lowest point. Air resistance is negligible. Calculate its speed at the lowest point.
5. A sledge is pulled 15 m across level snow by a rope with a tension of 36 N at 25° to the horizontal. Calculate the work done by the tension.

Answers

1. $\Delta E_K = \frac{1}{2} \times 3.0 \times (6.0^2 - 4.0^2) = \frac{1}{2} \times 3.0 \times 20 = 30$ J.
2. $P = \dfrac{mg\Delta h}{t} = \dfrac{55 \times 9.81 \times 7.5}{12} = 337$ W.
3. Useful output $= 0.72 \times 250 = 180$ W. Wasted power $= 250 - 180 = 70$ W; in $5.0 \times 60 = 300$ s that is $70 \times 300 = 21\,000$ J (21 kJ).
4. $\frac{1}{2}mv^2 = mgh$, so $v = \sqrt{2 \times 9.81 \times 0.35} = 2.62$ m s^{-1}.
5. $W = Fs\cos\theta = 36 \times 15 \times \cos 25^\circ = 489$ J.

Past questions to try

- **9702/21/O/N/22 Q3**: defining power, deriving $P = Fv$, then the extra power for a lorry climbing a slope.
- **9702/22/M/J/21 Q3**: an energy balance with friction on a slide, then the speed at the top of the flight.
- **9702/22/O/N/21 Q3**: a car on a slope: work against resistance, height gained from ΔE_P, and time from the power.
- **9702/22/F/M/21 Q2**: defining work done, then the average air resistance on a falling ball from the energy lost.